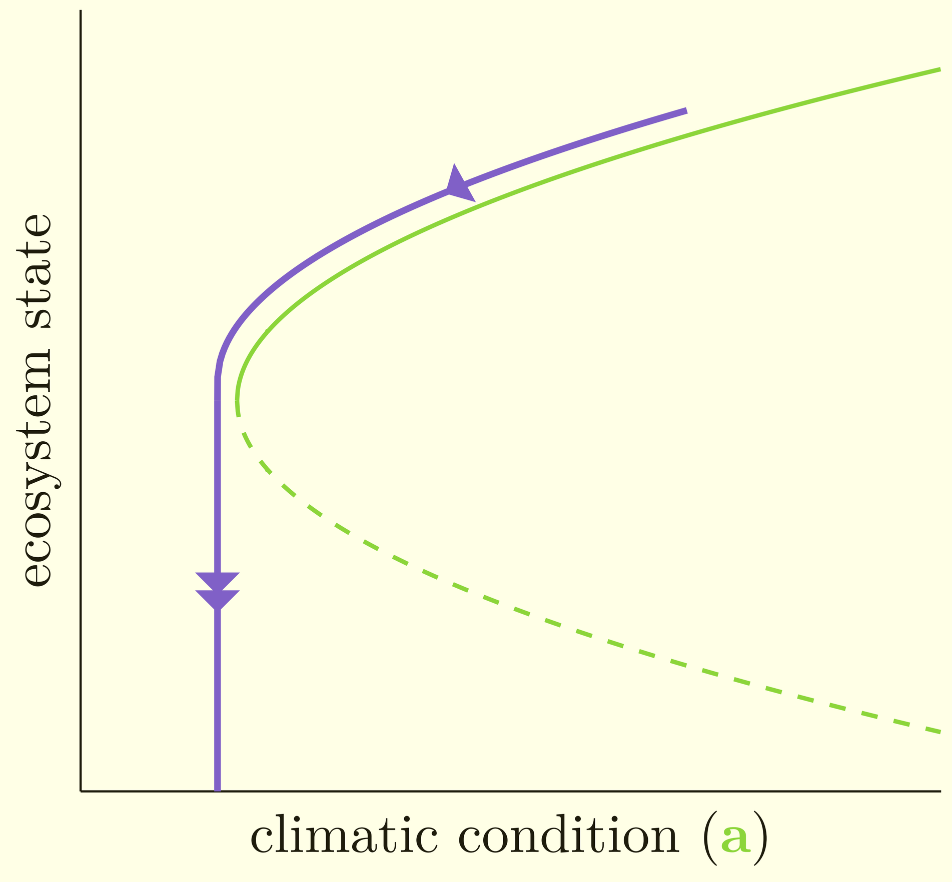
# Not one single tipping point, but a cascade of smaller transitions in patterned ecosystems

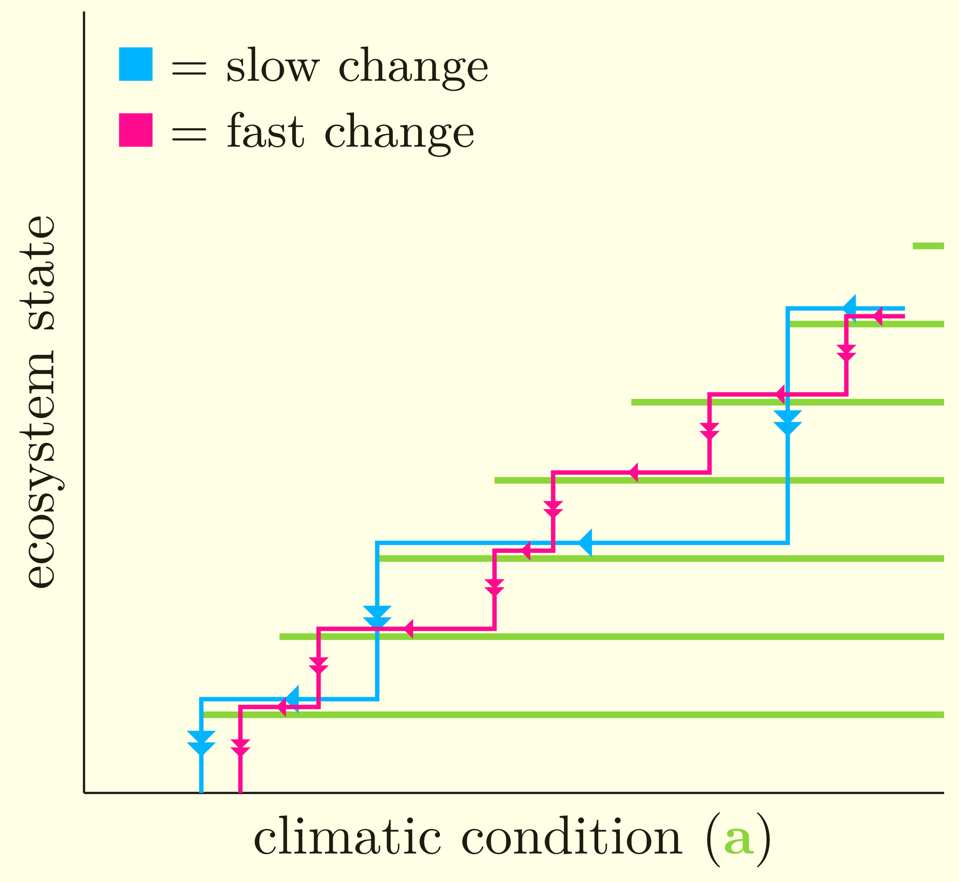
Robbin Bastiaansen

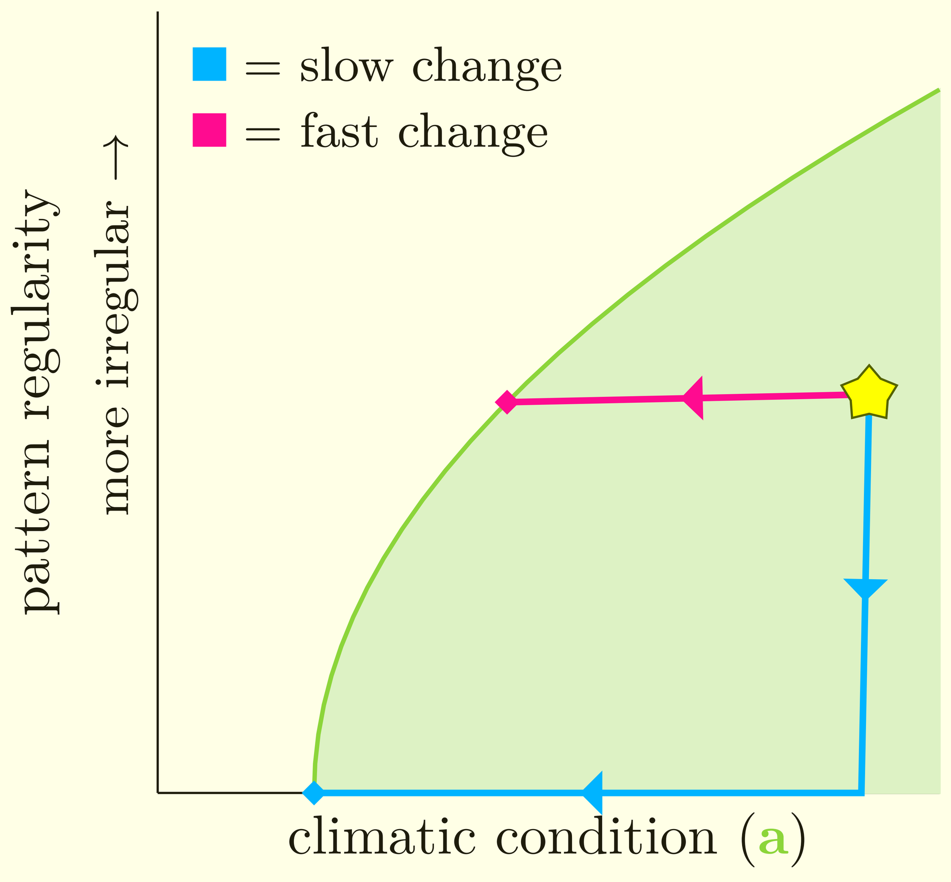
## Classical view of ecosystems [3]




- One (stable) productive state for each parameter combination
- One large, critical shift at the tipping point

## Refined view for patterned ecosystems

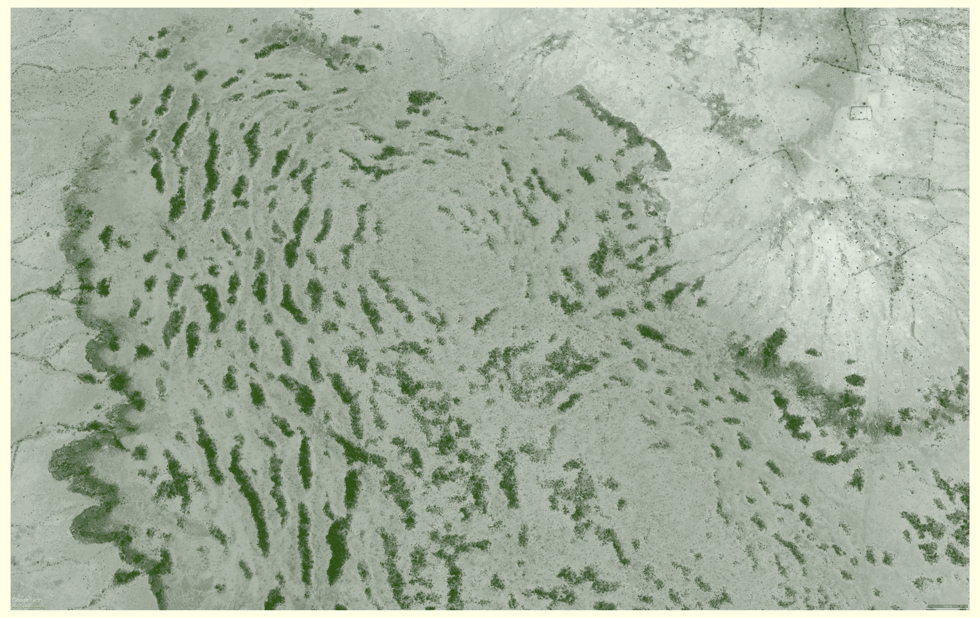


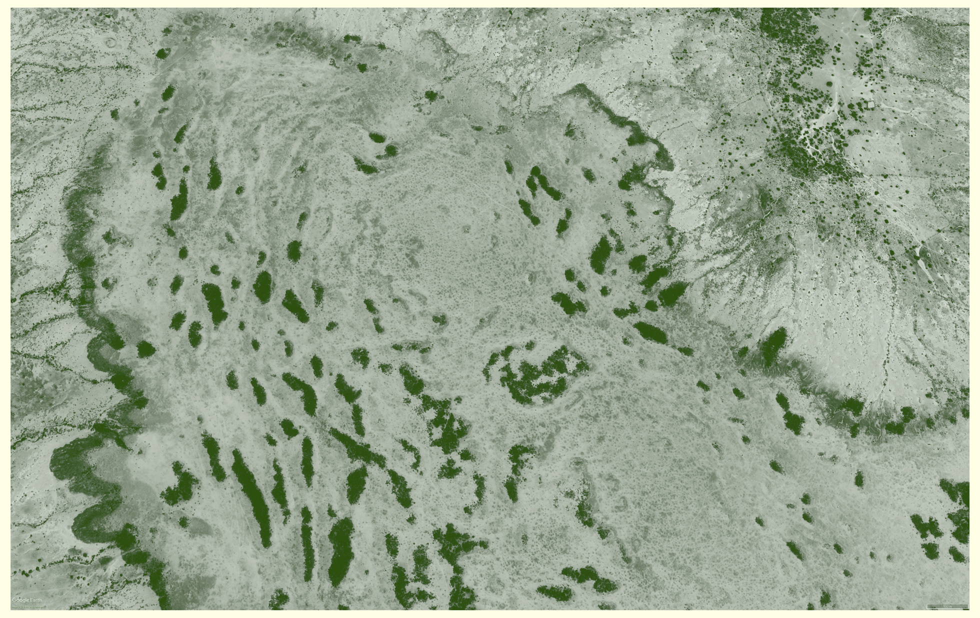

(Zoom-in on one green line from left figure)

- Multiple productive (patterned) state for each parameter combination [2]
- Multiple, smaller pattern-to-pattern transitions
- Moment and type of imminent transition depend on speed of (climatic) change

## Prototypical example: dryland ecosystems – patterns change



Niger (13°39'N, 2°27'E), 26 May 2002



Niger (13°39'N, 2°27'E), 26 February 2019

## Minimalistic model

Dryland ecosystem models describe the interplay between available water (**w**) and vegetation (**v**)

Archetype model ('extended-Klausmeier' [4]):

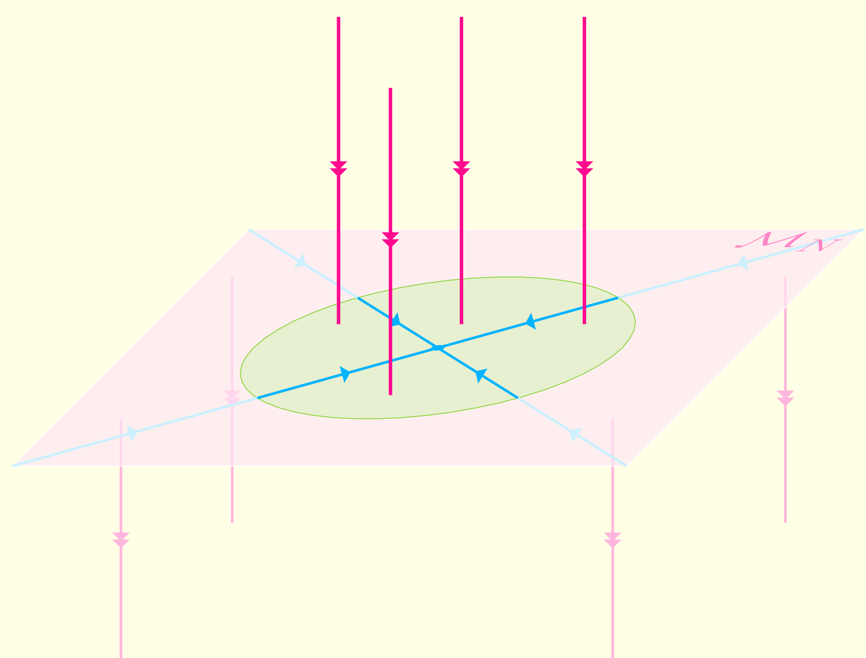
$$\frac{\partial \mathbf{w}}{\partial t} = \frac{\partial^2 \mathbf{w}}{\partial x^2} + \mathbf{a} - \mathbf{w} - \mathbf{w}\mathbf{v}^2$$
$$\frac{\partial \mathbf{v}}{\partial t} = D\,\frac{\partial^2 \mathbf{v}}{\partial x^2} - m\mathbf{v} + \mathbf{w}\mathbf{v}^2$$

This includes water movement ($\frac{\partial^2 \mathbf{w}}{\partial x^2}$), vegetation dispersion ($\frac{\partial^2 \mathbf{v}}{\partial x^2}$), rainfall ($+\mathbf{a}$), evaporation ($-\mathbf{w}$), water uptake ($-\mathbf{w}\mathbf{v}^2$), vegetation mortality ($-m\mathbf{v}$) and growth through water uptake ($+\mathbf{w}\mathbf{v}^2$)

## Dynamics of patterned solutions: model reduction

Dynamics of model are captured by two processes [1]:

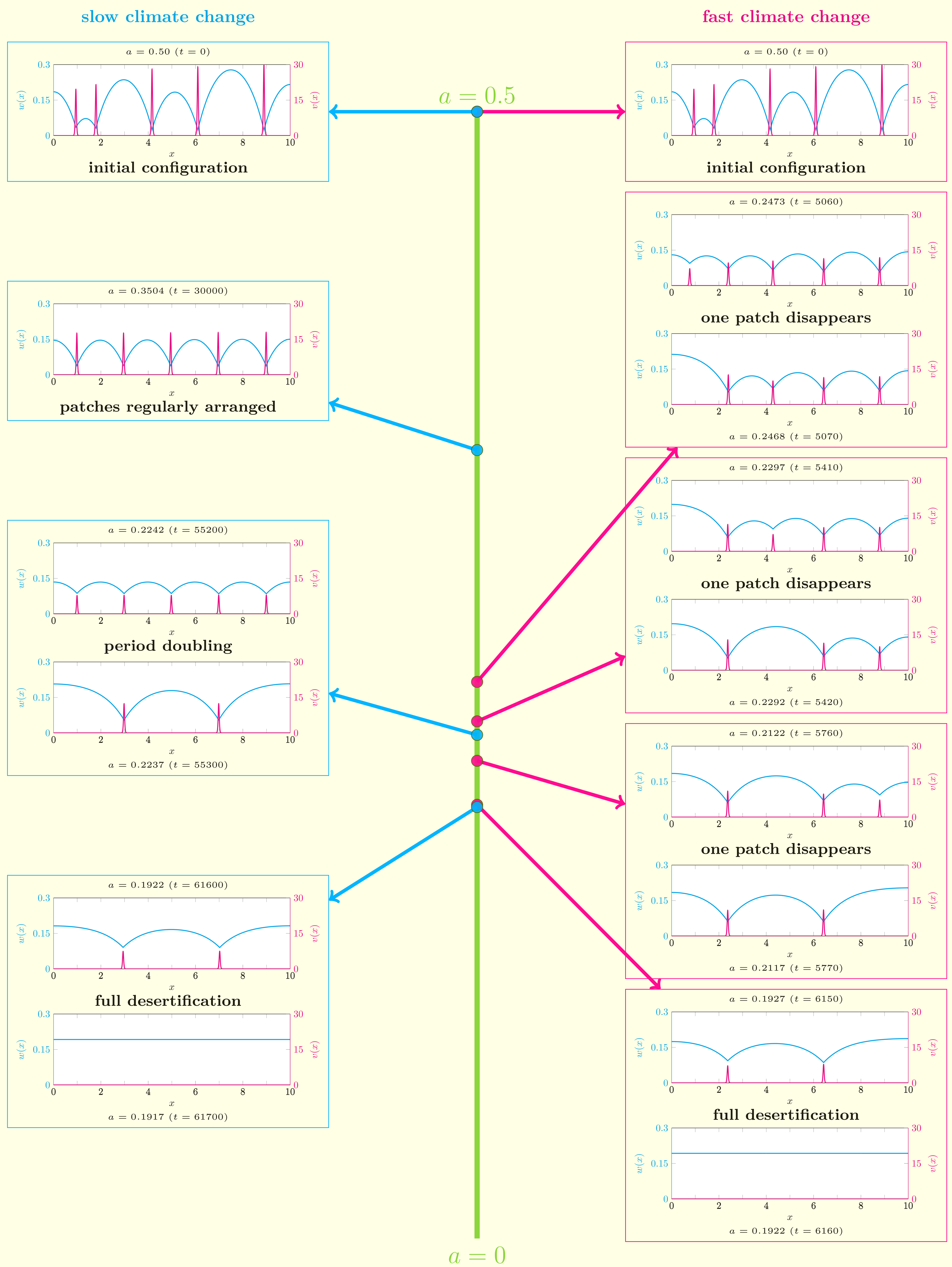
1. **Pattern rearrangement**
   Vegetation locations $P_j$ slowly drift as
   $$\frac{dP_j}{dt} = C\left[\ \mathbf{w}_x(P_j^+)^2\ -\ \mathbf{w}_x(P_j^-)^2\ \right],$$
   rearranging themselves into a regular pattern
2. **Pattern-to-pattern transitions**
   Eigenvalues and eigenfunctions indicates moment and type of transition:
   - Irregular pattern → gradual shifts
   - Regular patterns → larger shifts




**Visualisation of dynamics**

- PDE has infinite-dimensional state space
- Reduction to interlinked finite-dimensional phase portraits $\mathcal{M}_N$
- Points on $\mathcal{M}_N$ correspond to specific $N$-patch configurations
- Only some configurations feasible under climatic condition (green area)
- On $\mathcal{M}_N$: pattern rearrangement (blue)
- Between $\mathcal{M}_N$: pattern-to-pattern transitions (pink)

## Differences between ecosystem degradation during fast and slow climatic change




## Conclusions

- Indicators should signal both *when* a pattern transition is imminent and its *severity*
- Patterns are more resilient when they are more (spatially) regular
- Rate of climatic change dramatically changes the degradation process:
- Slow change → sporadic, large shifts
- Fast change → rapid sequence of smaller shifts

## References

[1] R. Bastiaansen and A. Doelman. The dynamics of disappearing pulses in a singularly perturbed reaction-diffusion system with parameters that vary in time and space. *Physica D*, 388:45–72, 2019.
[2] R. Bastiaansen *et al.* Multistability of model and real dryland ecosystems through spatial self-organization. *Proc. Natl. Acad. Sci. U.S.A.*, 115(44):11256–11261, 2018.
[3] C. S. Holling. Resilience and stability of ecological systems. *Annu. Rev. Ecol. Evol. Syst.*, 4(1):1–23, 1973.
[4] C. A. Klausmeier. Regular and irregular patterns in semiarid vegetation. *Science*, 284(5421):1826–1828, 1999.