Behaviour of self-organised patterns in systems with varying parameters

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Delft Software Days, 13 November 2019

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- PhD in *Applied Mathematics*
  - ➢ pattern formation

- main application is *Ecology*
  - ➢ desertification

TODAY:
- dynamics of patterns
  - ➢ focus on general insights

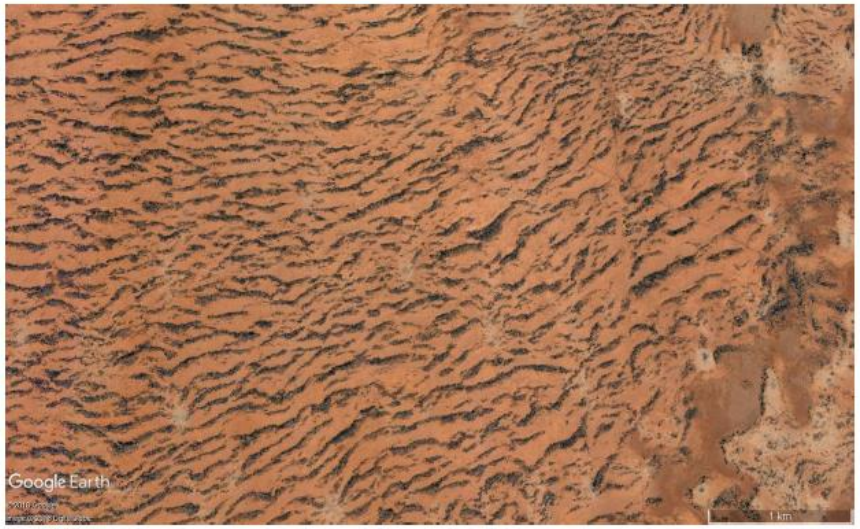
PhD thesis

hdl.handle.net/1887/74366

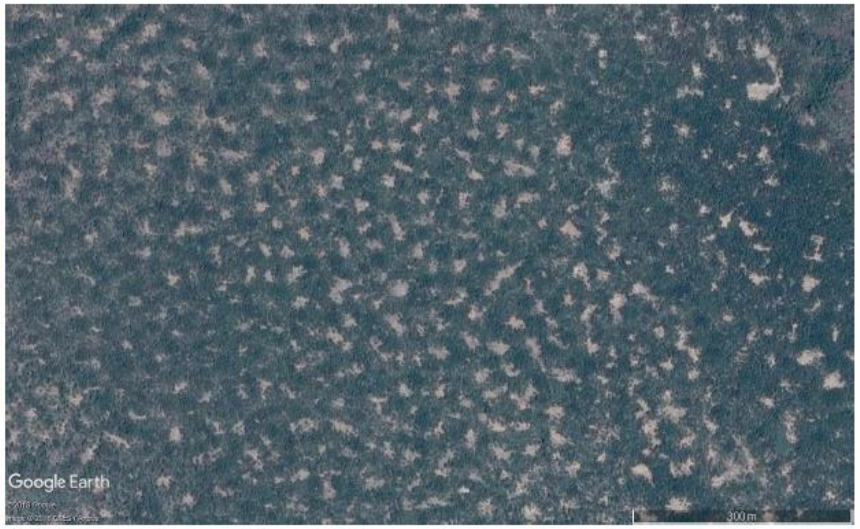
The desertification process

?

Desertification – emergent patterns

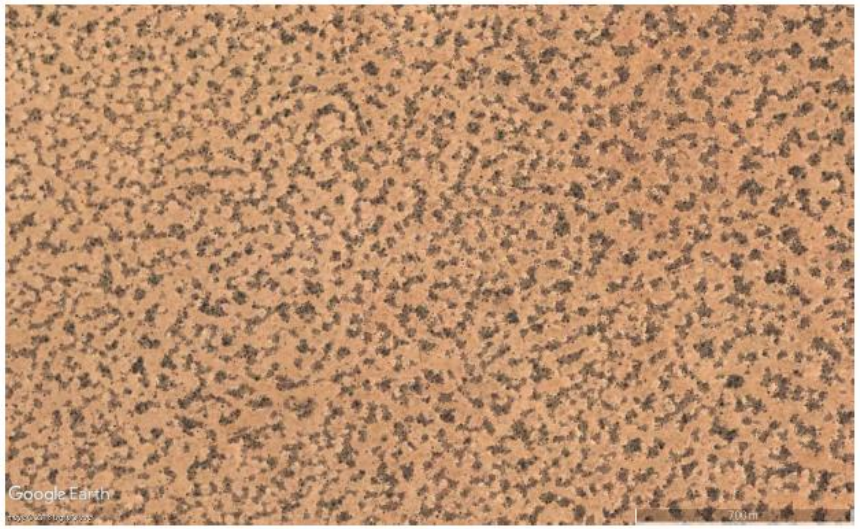


(a) Bands in Somalia



(b) Gaps in Niger

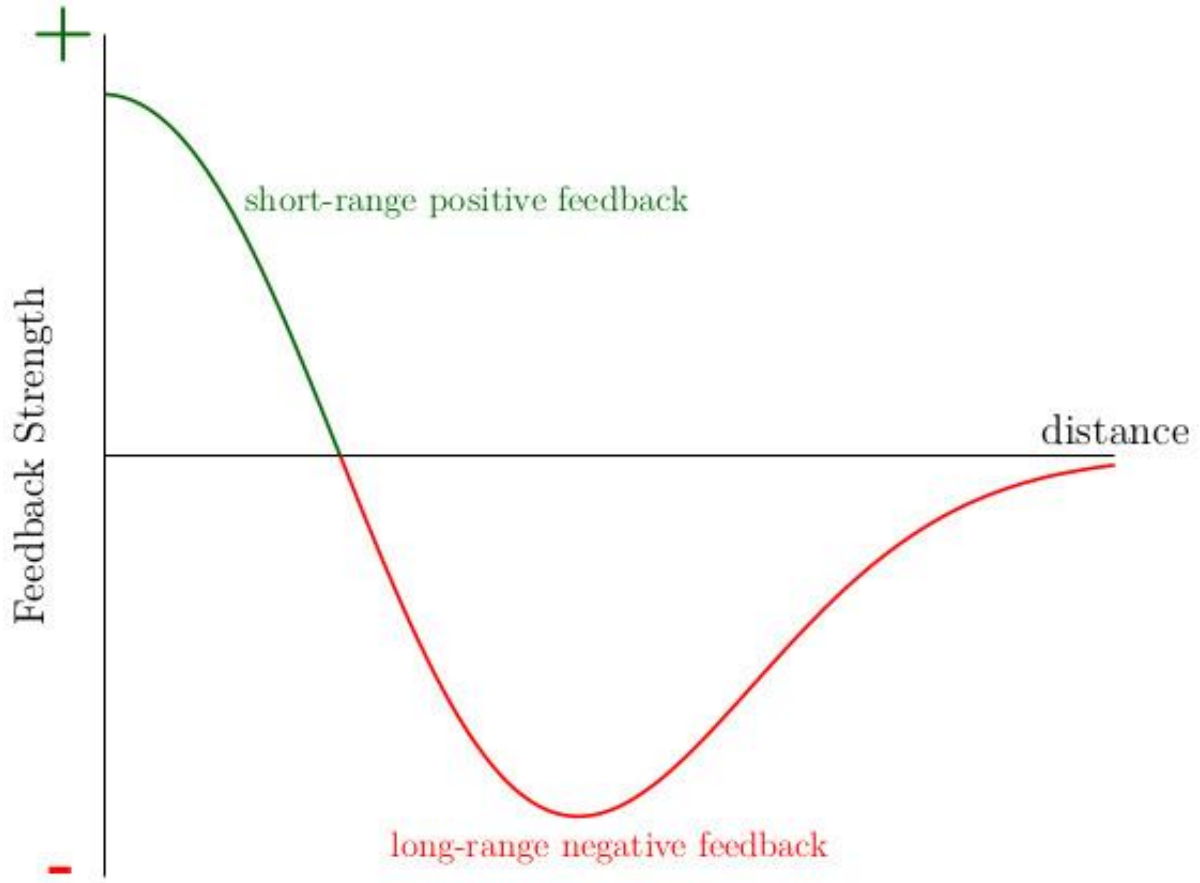
(c) Spots in Zambia



(d) Maze in Sudan

Self-organised patterns

- NO driving inhomogeneity
- BUT e.g. scale-dependent feedback



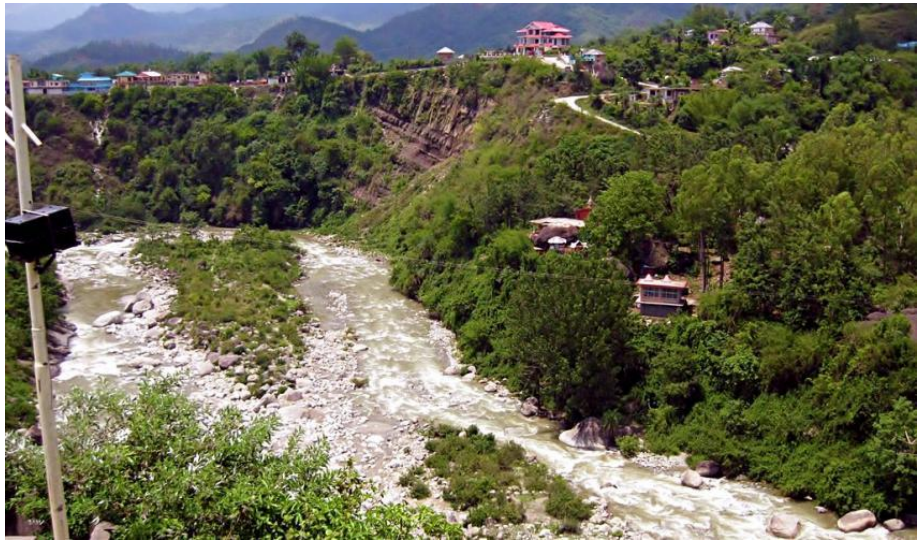
Ecology review: [Rietkerk & Van de Koppel, 2007]

Examples

mussel beds

vegetation in coastal systems

marsh formation

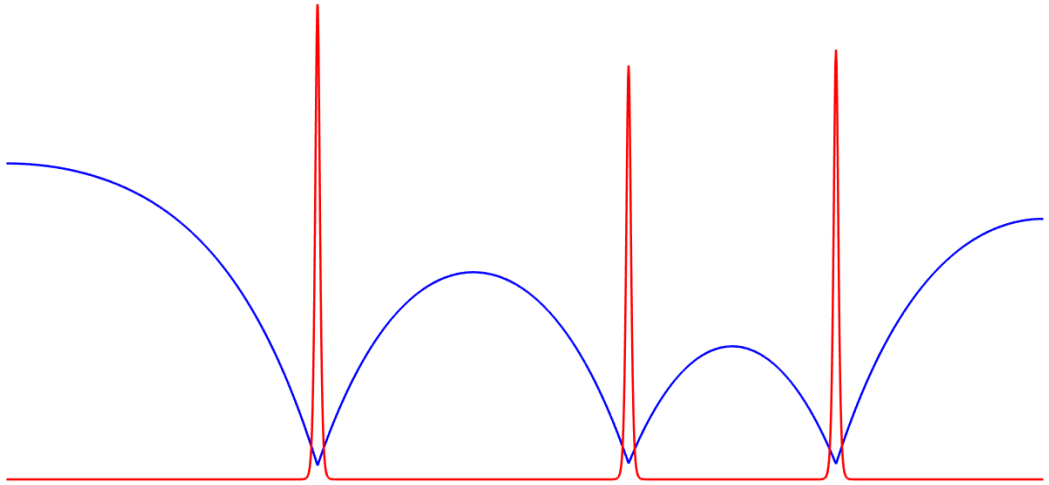


river vegetation

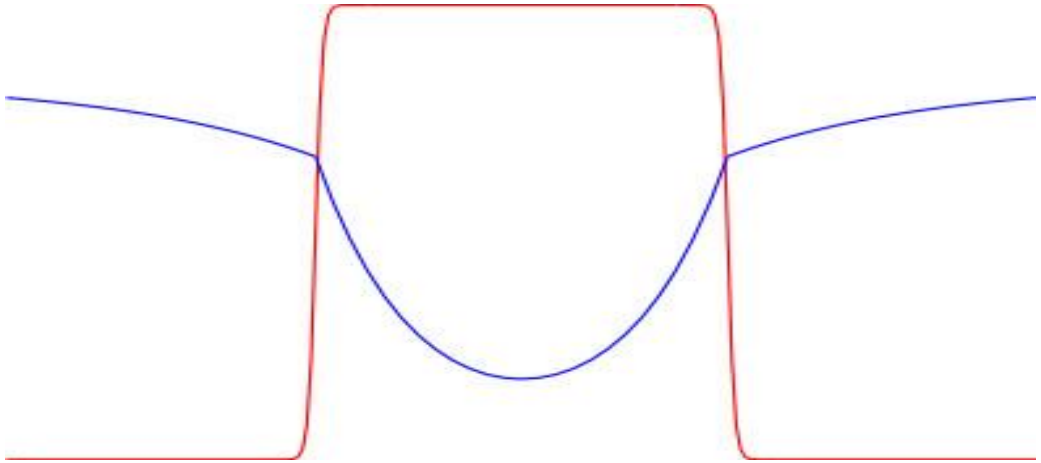
Mathematical treatment

Localized patterns ↔ localized structures

Seperation of scales ↔ small parameter

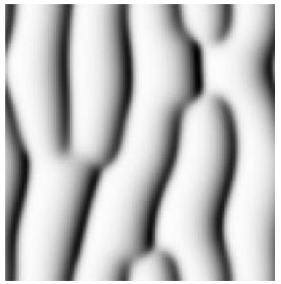


pulses

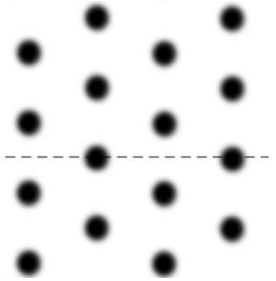


fronts

2D Reaction-diffusion models:

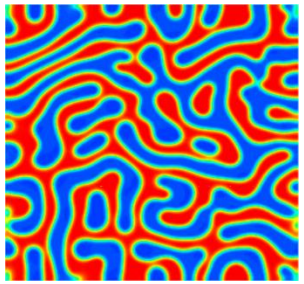


[Klausmeier, 1999]

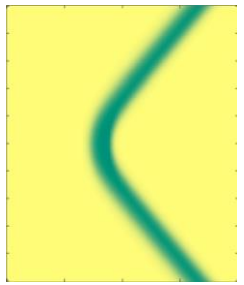


[Gilad et al, 2004]

[Rietkerk et al, 2002]



[Liu et al, 2013]



[Bastiaansen et al, 2019]

Archetypical ecosystem model

Extended-Klausmeier model

$$w_t = w_{xx} + (h(\mathbf{x})_x w)_x - w + a(\mathbf{t}) - wv^2$$
$$v_t = D^2 v_{xx} - mv + wv^2$$

w : water

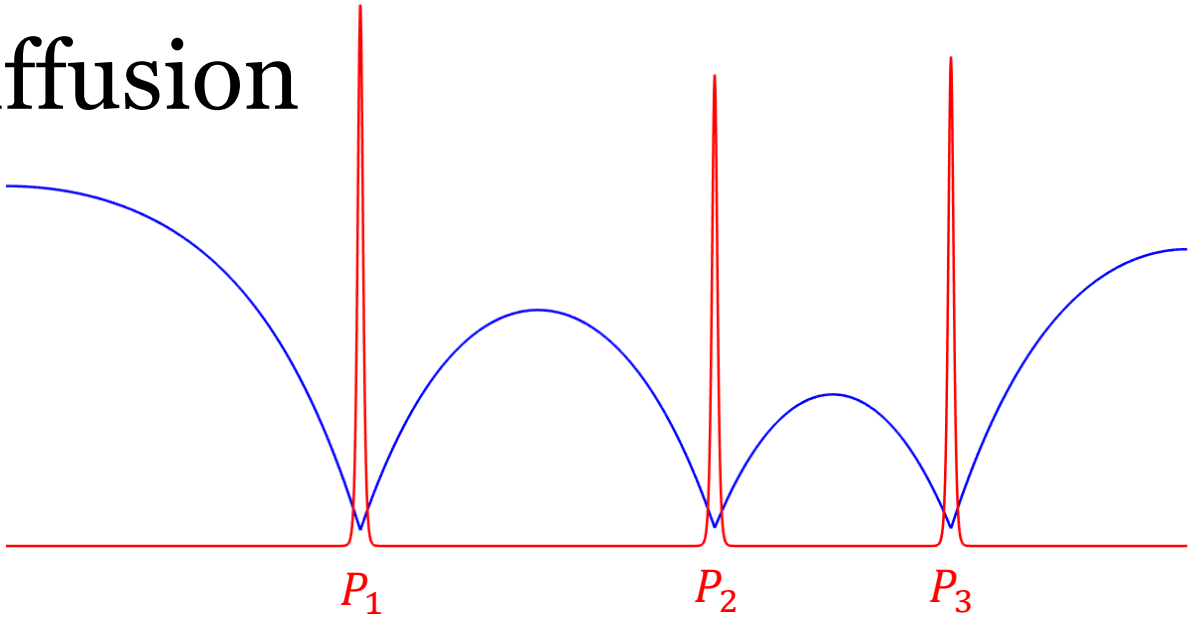
v : vegetation

h : height

D : ratio of diffusion

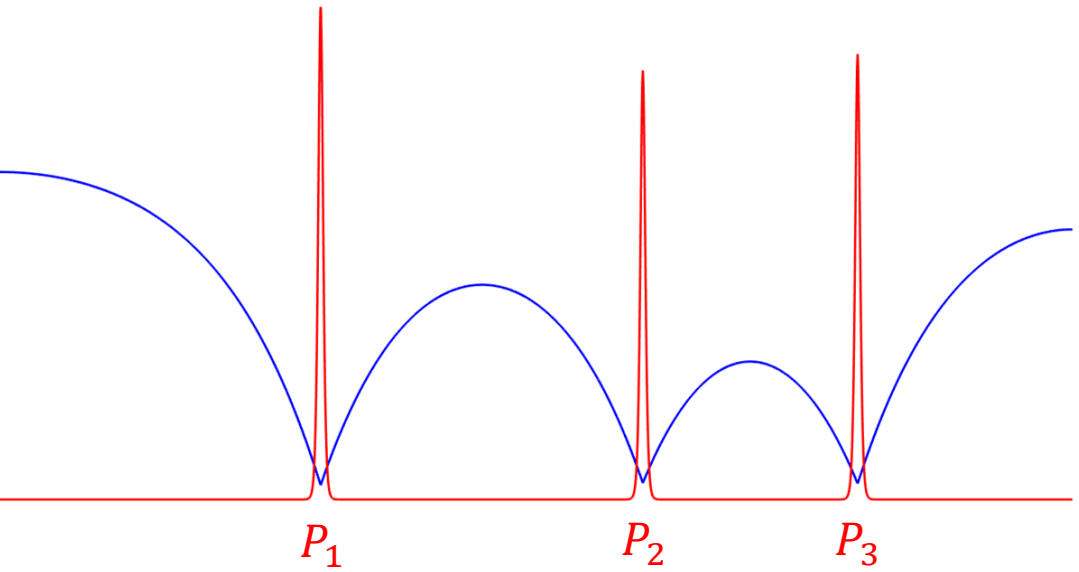
a : rainfall

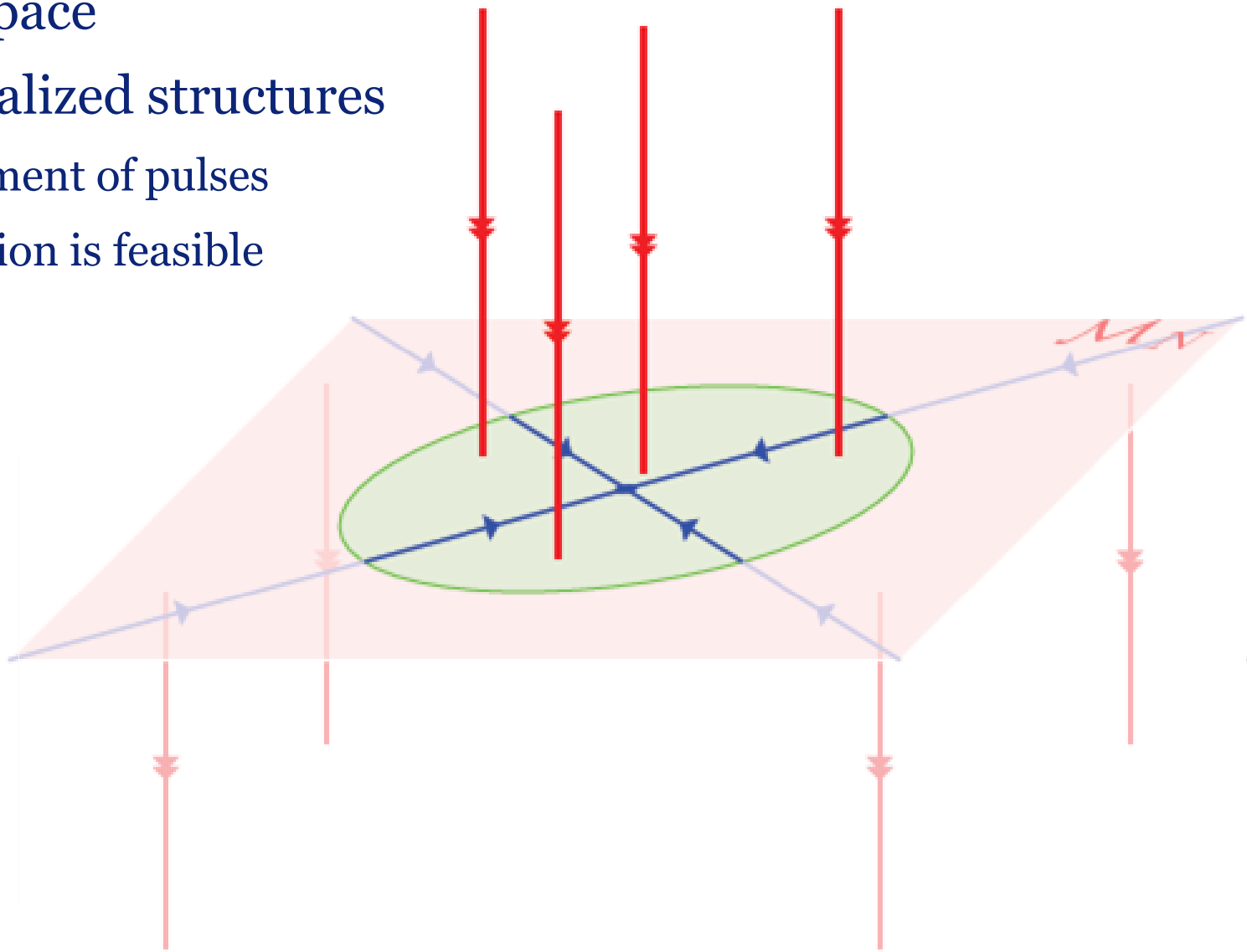
m : mortality



Understanding pulses in the model

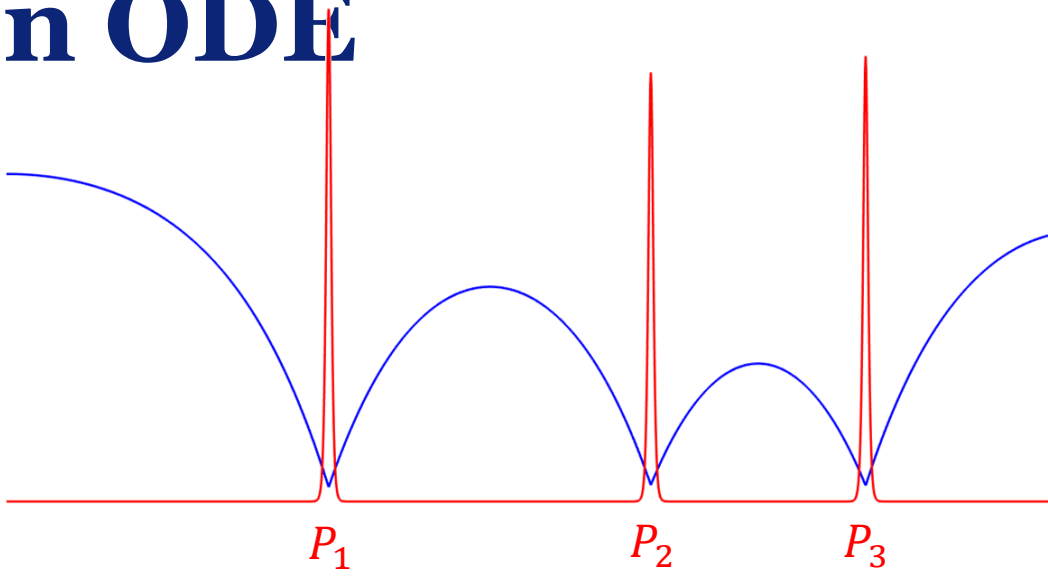
- PDE: infinite-dimensional state space
- Reduction possible because of localized structures
  1. Pulse-location ODE: describe movement of pulses
  2. Stability criterium: test if configuration is feasible

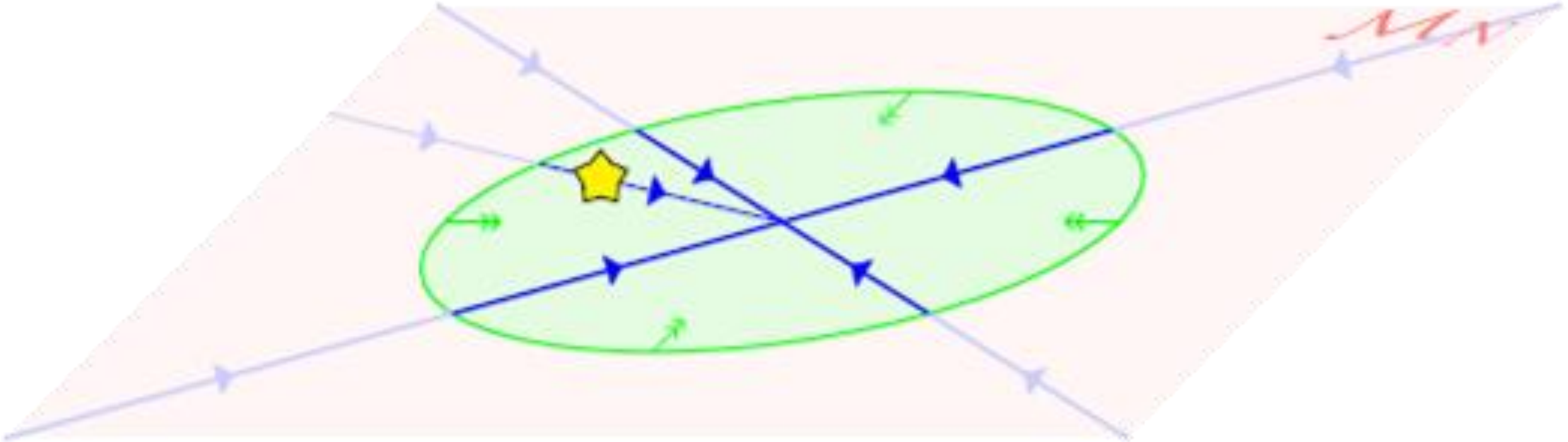




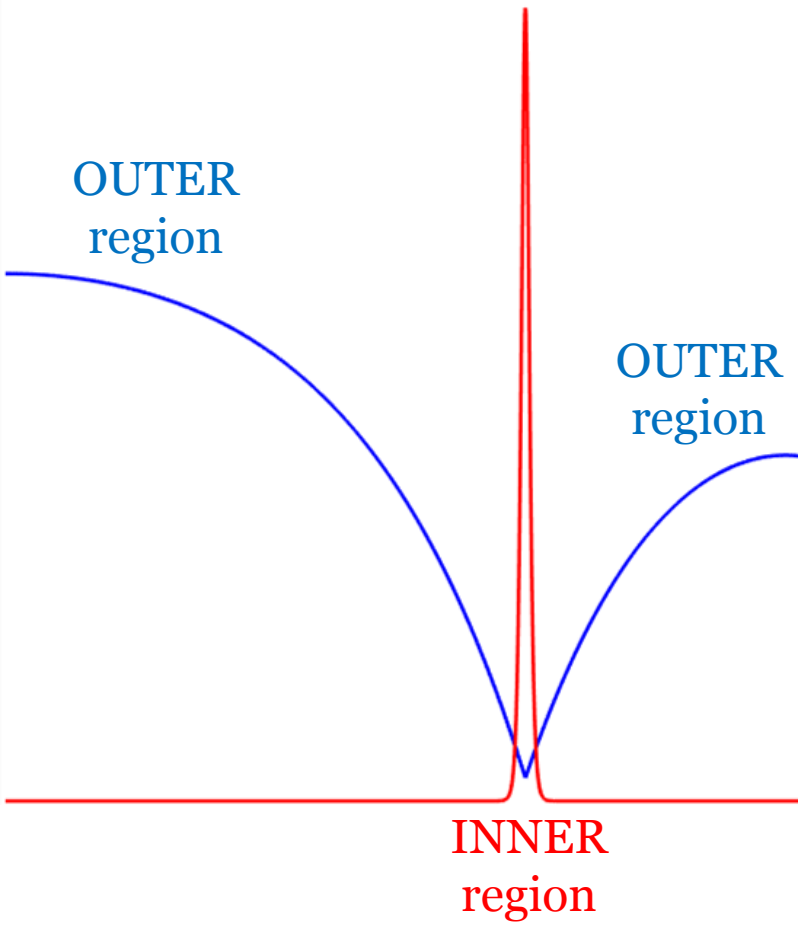
1. Pulse-location ODE

$$\frac{dP_j}{dt} = \frac{Da^2}{m\sqrt{m}}\left[w_x(P_j{}^+)^2 - w_x(P_j{}^-)^2\right]$$





How to derive the ODE?



$$\begin{aligned} w_t &= w_{xx} + (h(\mathbf{x})_x w)_x \; - w + a(\mathbf{t}) \; - w v^2 \\ v_t &= D^2 v_{xx} \qquad\qquad\quad - m v \qquad\quad + w v^2 \end{aligned}$$

INNER regions:

$$0 = D^2 v_{xx} \; - m v \; + w v^2$$

$$\Rightarrow \; v_p\left(x - P_j(\mathbf{t})\right)$$

OUTER regions:

$$0 = w_{xx} + (h(\mathbf{x})_x w)_x \; - w + a(\mathbf{t})$$

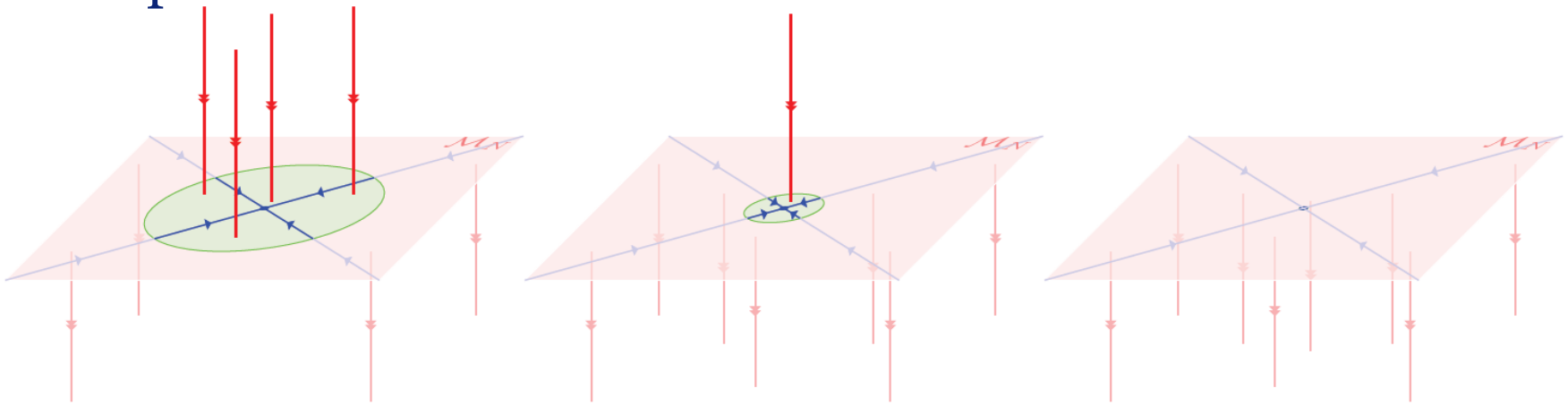
Match solutions at boundaries:

$$\Rightarrow \; \frac{dP_j}{dt} = \frac{Da^2}{m\sqrt{m}} \left[w_x(P_j{}^+)^2 - w_x(P_j{}^-)^2 \right]$$

2. Stability criterium

Enough resources to sustain all vegetation patches?

Depends on **amount of rainfall** and **distance between patches**



high rainfall

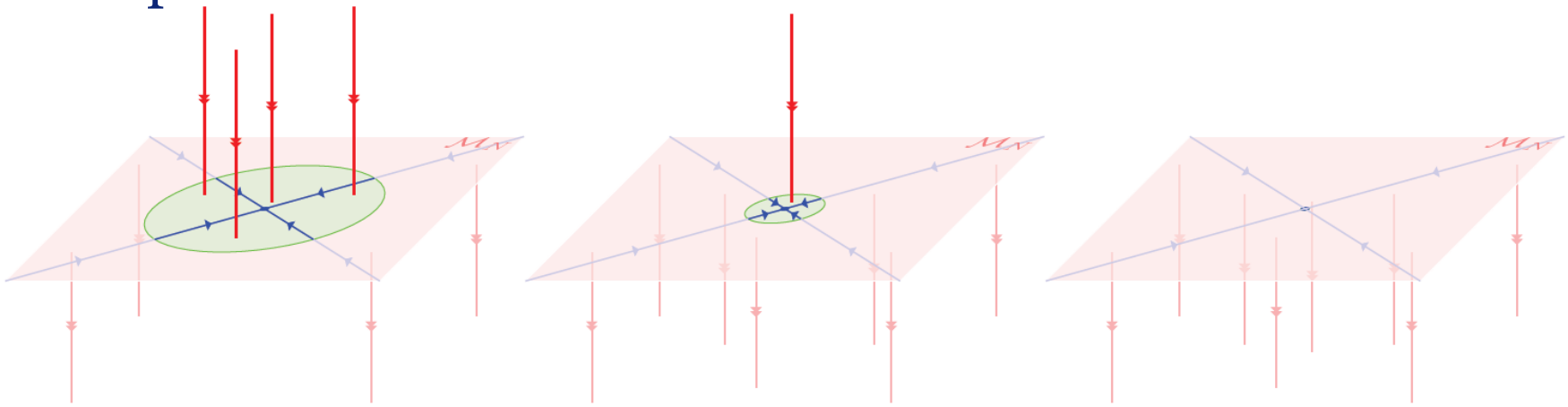
medium rainfall

low rainfall

2. Stability criterium

Enough resources to sustain all vegetation patches?

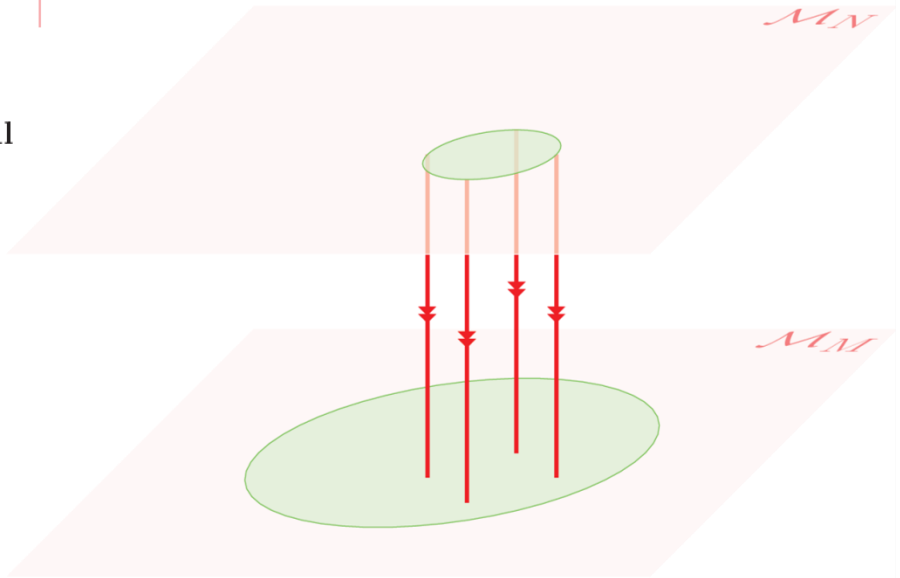
Depends on **amount of rainfall** and **distance between patches**



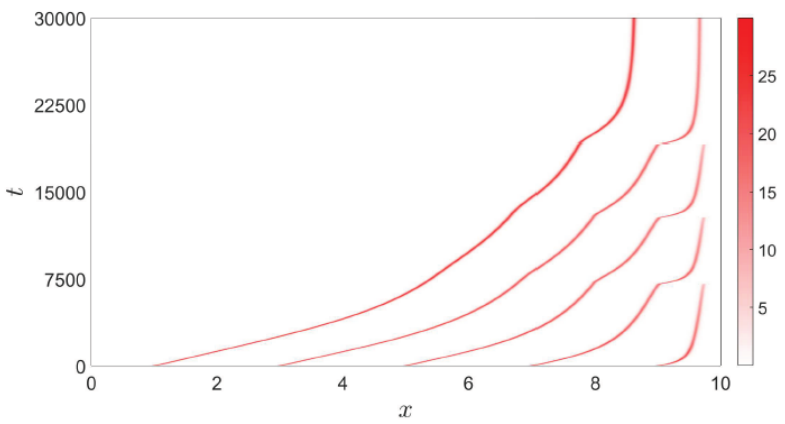
high rainfall medium rainfall low rainfall

What happens when outside feasible region?

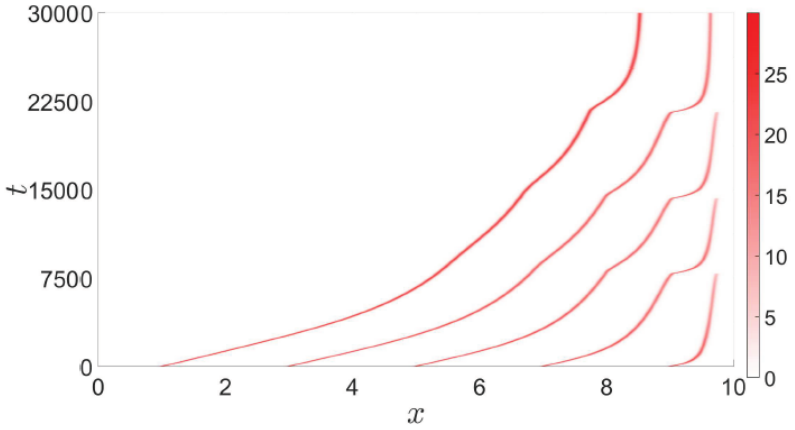
irregular configuration:	**One** patch disappears (least amount of biomass)
regular configuration:	**Half** of the patches disappears (wavelength doubling)



Dynamics of disappearing pulses

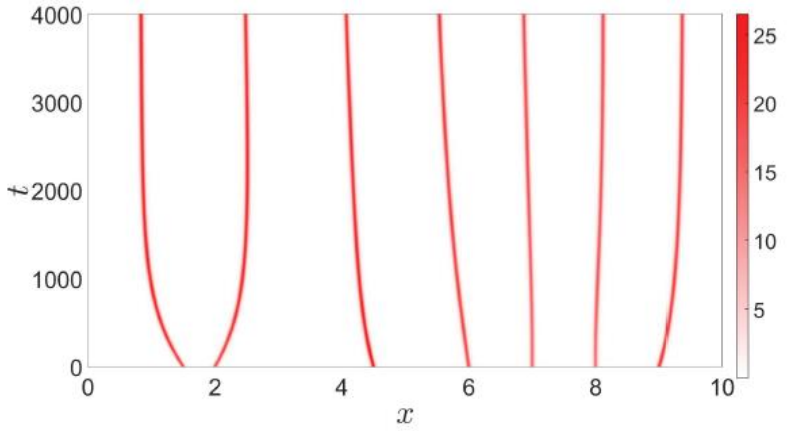


(a) ODE

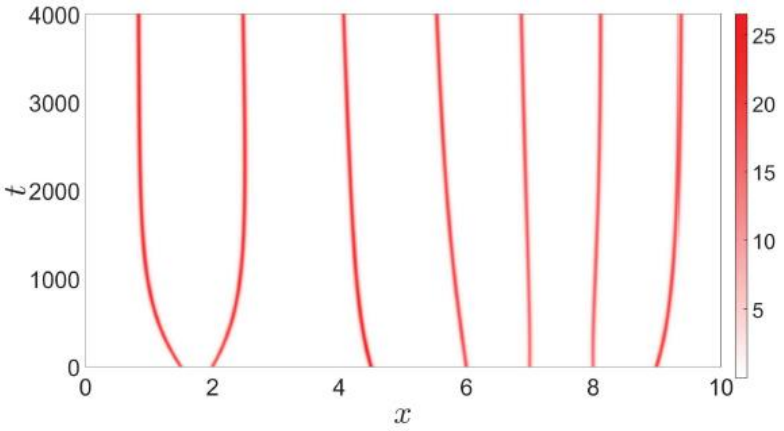


(b) PDE

Numerical simulations support theory

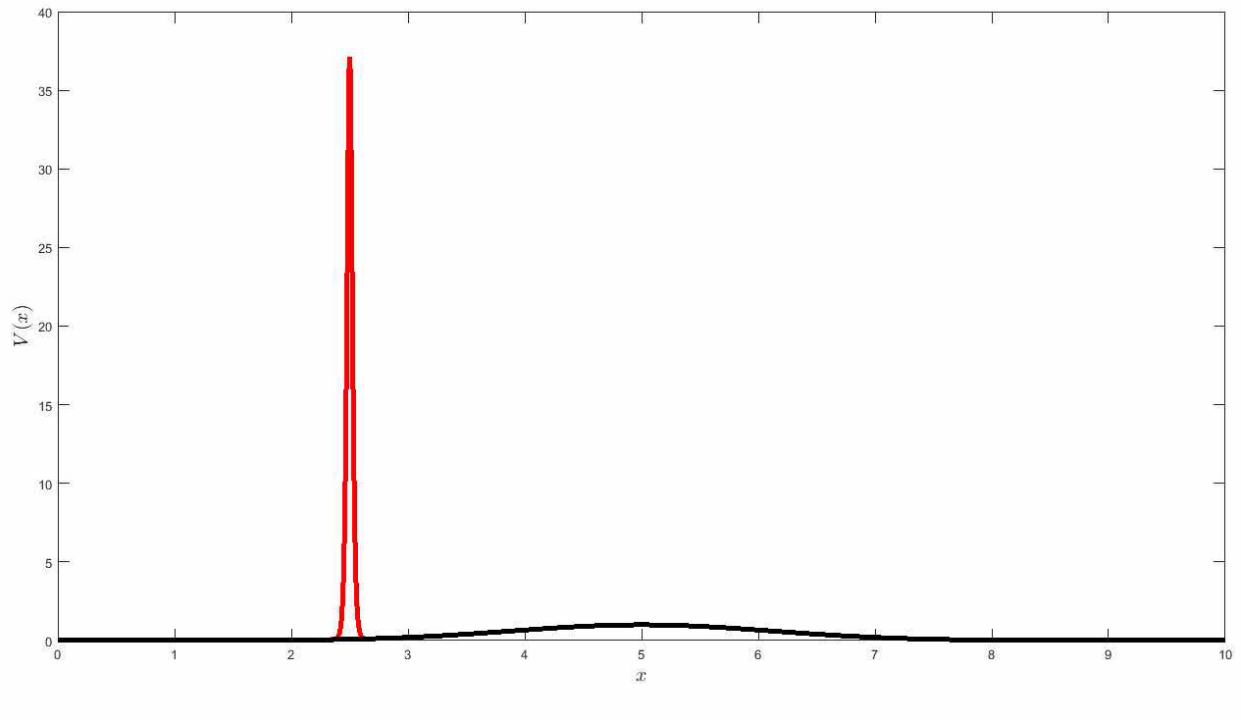


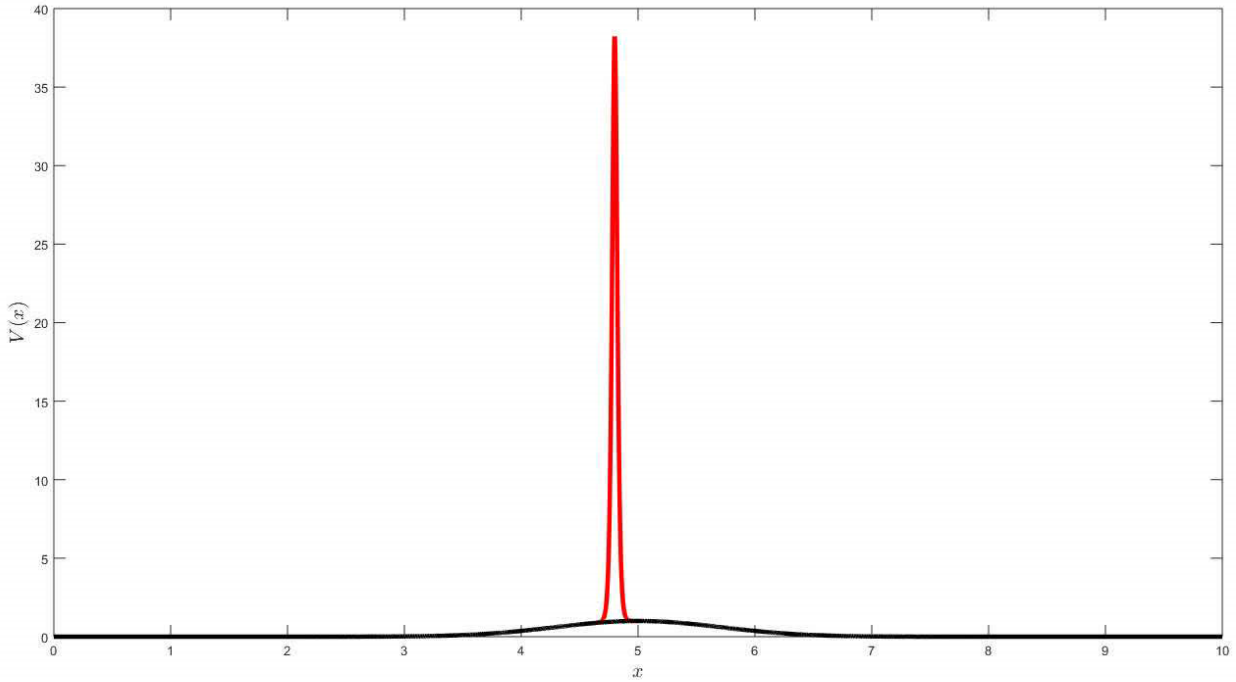
(a) ODE



(b) PDE

New insight: the effect of topography





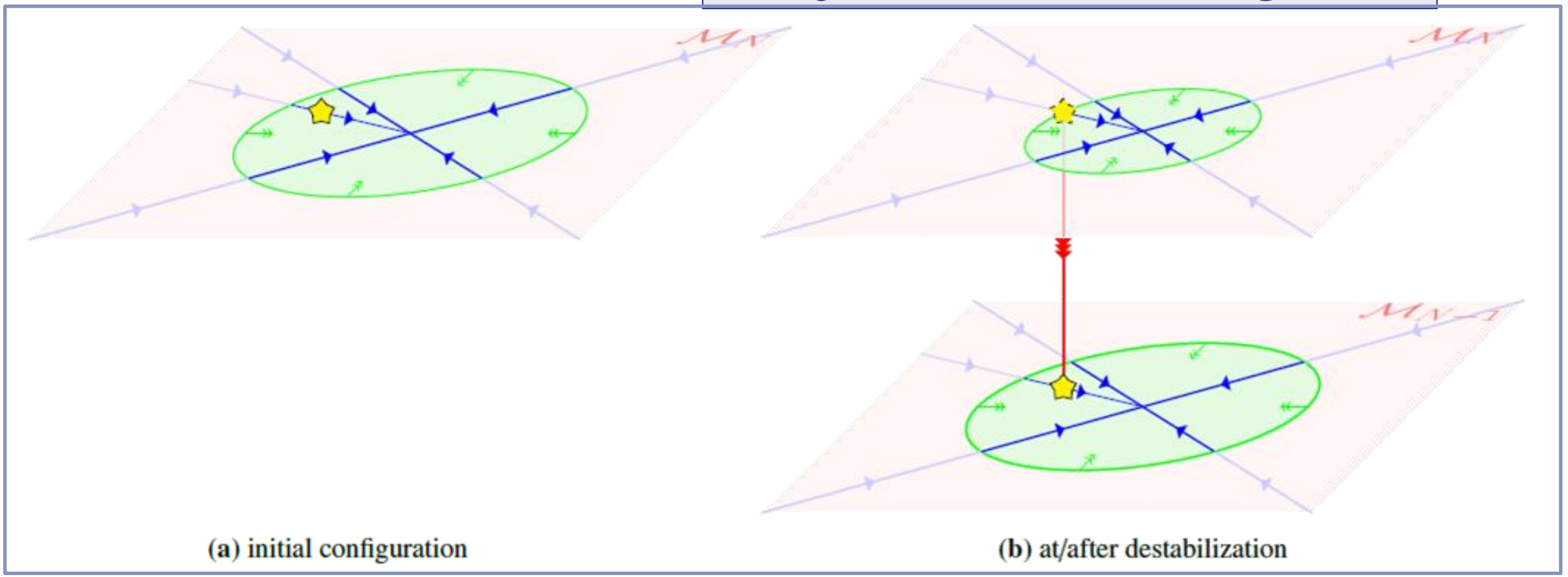
Vegetation pulses can move uphill and downhill

Pulses during climate change (1)

Competition of two effects:

1. Pulse rearrangement
2. Shrinking of feasible region

fast climate change

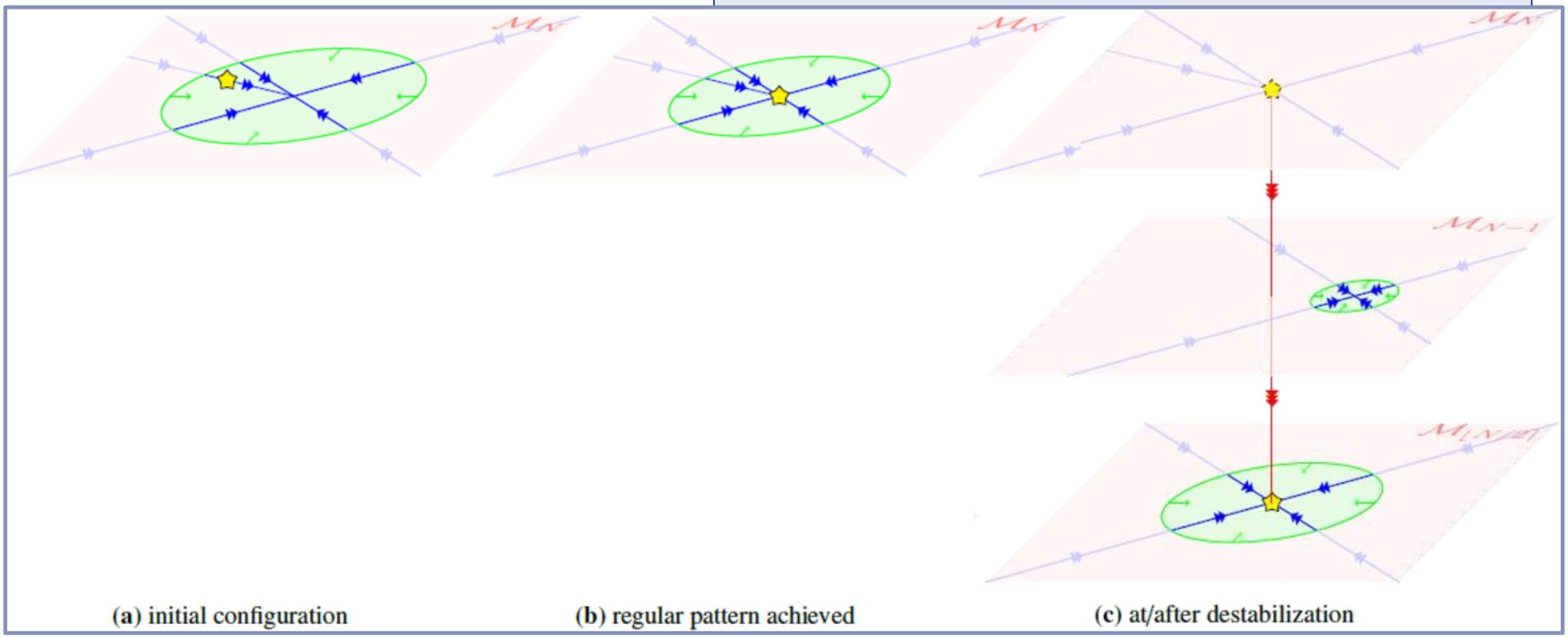


Pulses during climate change (2)

Competition of two effects:

1. Pulse rearrangement
2. Shrinking of feasible region

slow climate change

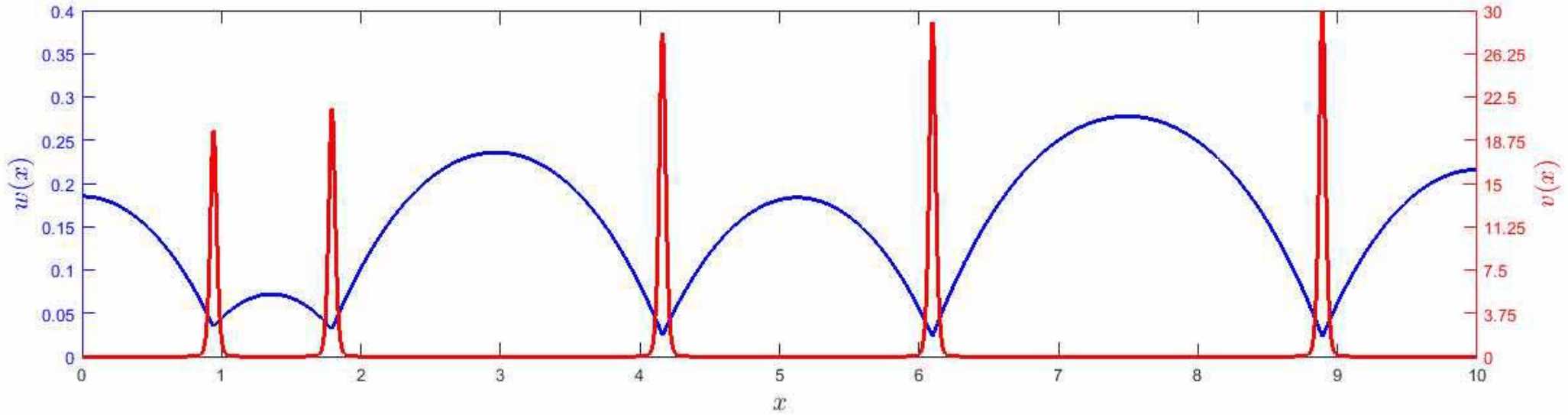


(a) initial configuration (b) regular pattern achieved (c) at/after destabilization

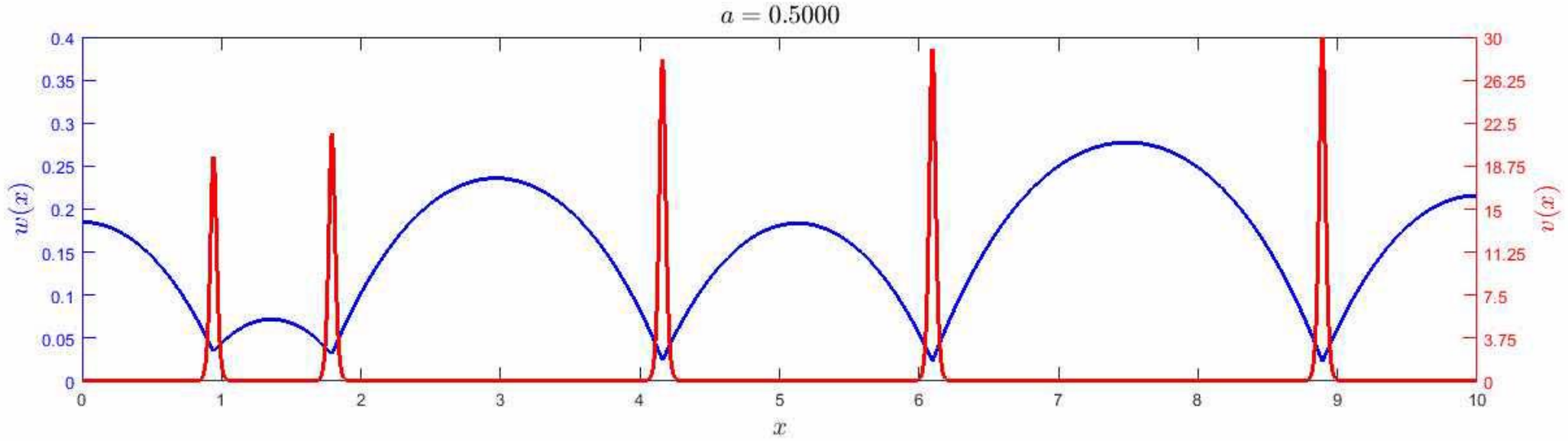
Pulses during climate change (3)

Rate of climate change

FAST



SLOW



Summary

PDE to ODE reduction captures

i. Pattern rearrangement
ii. Pattern transitions

Reduction works for *Localized* patterns

- Applicable in many systems
- BUT: reduction needs to be redone

Example: insights in desertification

- Influence of topography
- Importance of rate of change

~~~ **LIST OF COAUTHORS** ~~~

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Ecologists (theorists)	Maarten Eppinga (Zürich)	Max Rietkerk (Utrecht)
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