

Climate sensitivity estimation using linear system fit: extending the Gregory method

Robbin Bastiaansen

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Current project

- postdoc @ Utrecht University
(with Anna von der Heydt & Henk Dijkstra)

- Work on WP4 '**Climate Sensitivity**':

*If we increase the atmospheric CO₂ concentration,
how much warmer does the Earth get?*

- **TODAY**: talk about endeavours since JAN 2020

My background

2009-2013 BSc in Mathematics & Physics @ Leiden

2013-2015 MSc in Applied Mathematics @ Leiden

2015-2019 PhD in Applied Mathematics @ Leiden

'Lines in the Sand:

behaviour of self-organised vegetation

patterns in dryland ecosystems'

→ Pattern formation

→ Dynamical system theory

→ Combination with ecology

lines in the sand

*Behaviour of self-organised vegetation
patterns in dryland ecosystems*



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Equilibrium Climate Sensitivity (ECS)

Experiment using climate models:

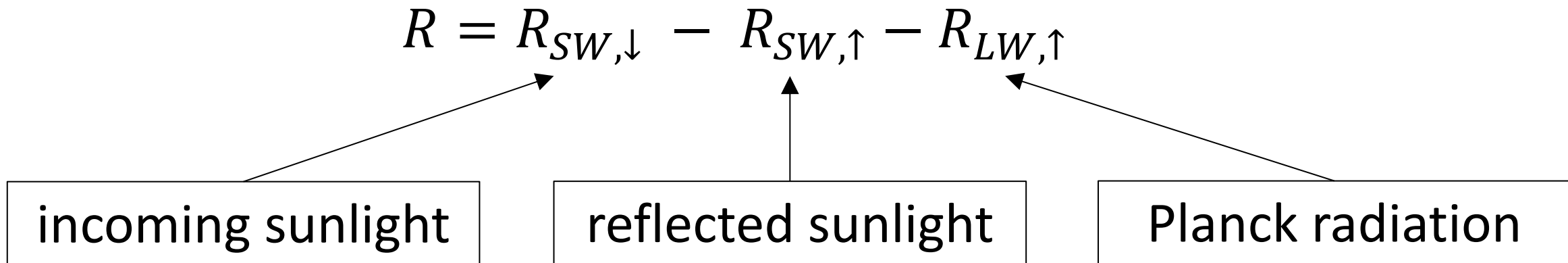
Increase CO_2 concentration

What is the increase in temperature ΔT_ in the new equilibrium?*

- Equilibrating climate models takes very, very long
(especially for e.g. very high-resolution simulations)
- Need for techniques to estimate equilibrium state/temperature

Basic Idea of Gregory method (1)

Warming is due to net positive radiative imbalance



When $\Delta R = R - R_0 = 0$ no more warming:
 \rightarrow equilibrium warming $\Delta T_* = T_* - T_0$

Basic Idea of Gregory method (2)

Assuming all feedbacks are directly temperature dependent:

$$\Delta R = R(T) - R_0$$

Close to equilibrium T_* , Taylor expansion gives approximation

$$\Delta R(T) = R'(T_*) [T - T_*]$$

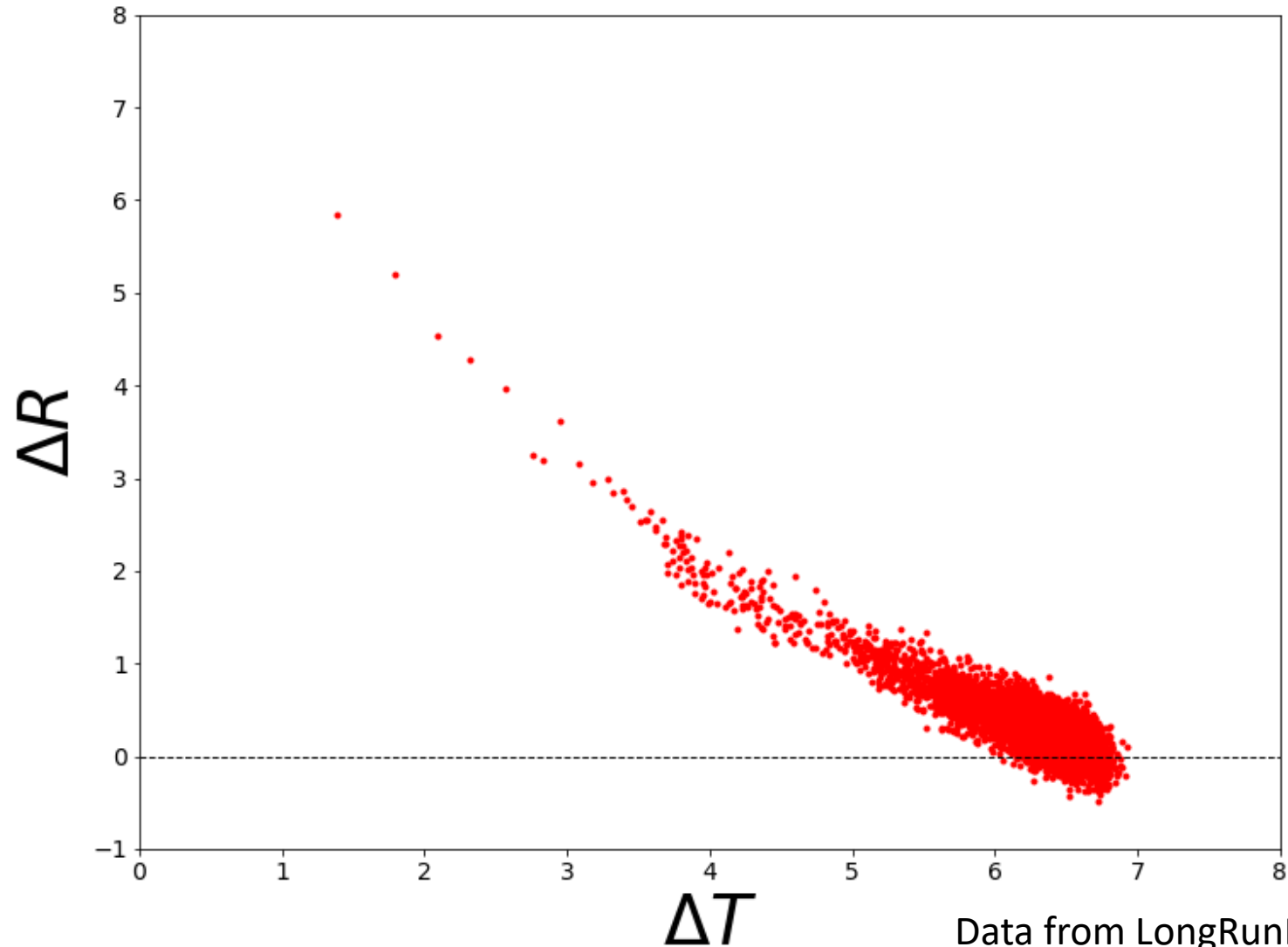
Rewriting $T = T_0 + \Delta T$ gives:

$$\Delta R(\Delta T) = R'(T_*)\Delta T - R'(T_*)\Delta T_*$$

Linear regression on data:

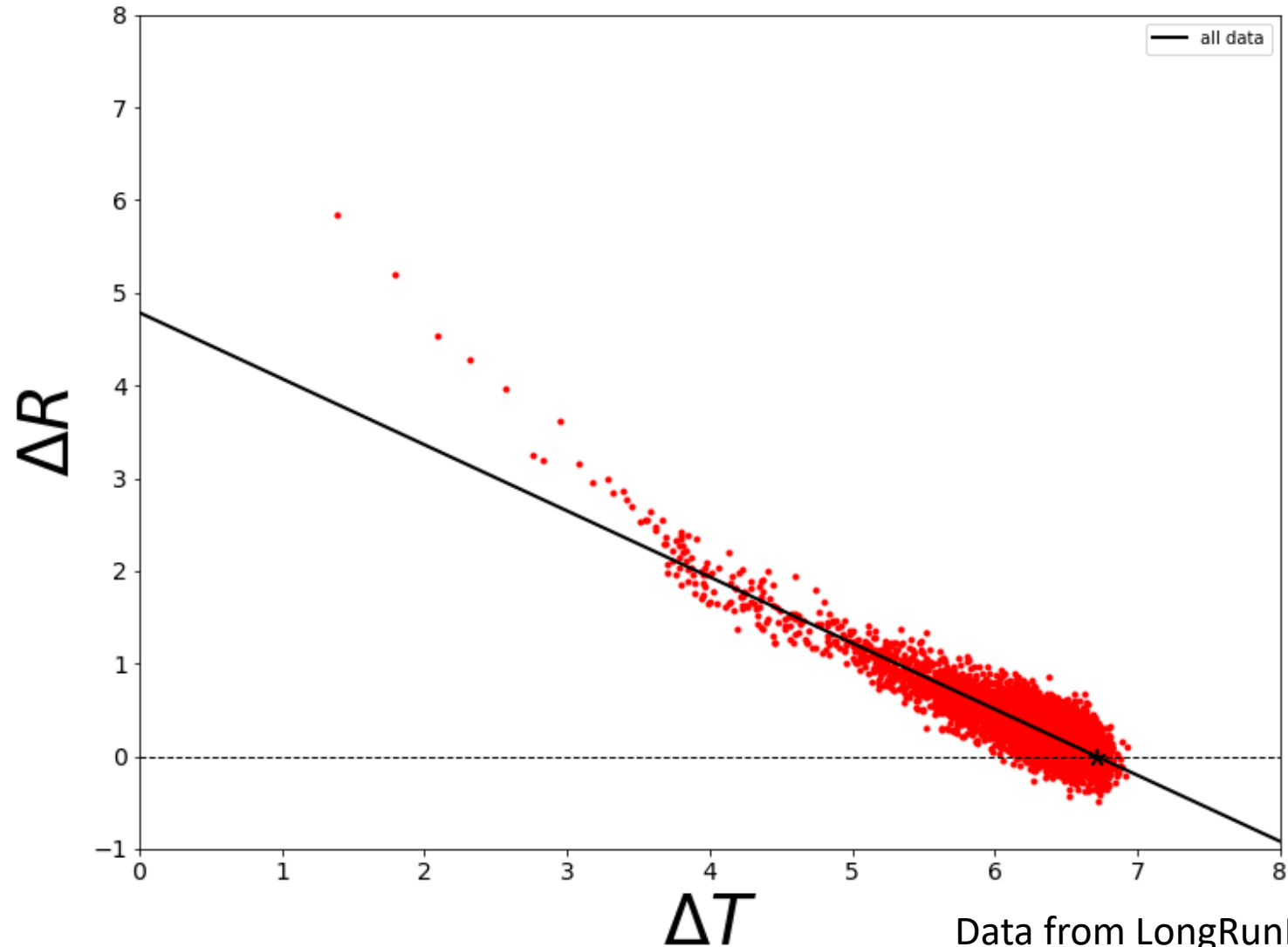
$$\Delta R = a \Delta T + f \rightarrow \Delta T_*^{est} = -a^{-1} f$$

Gregory Plot for CESM 1.0.4



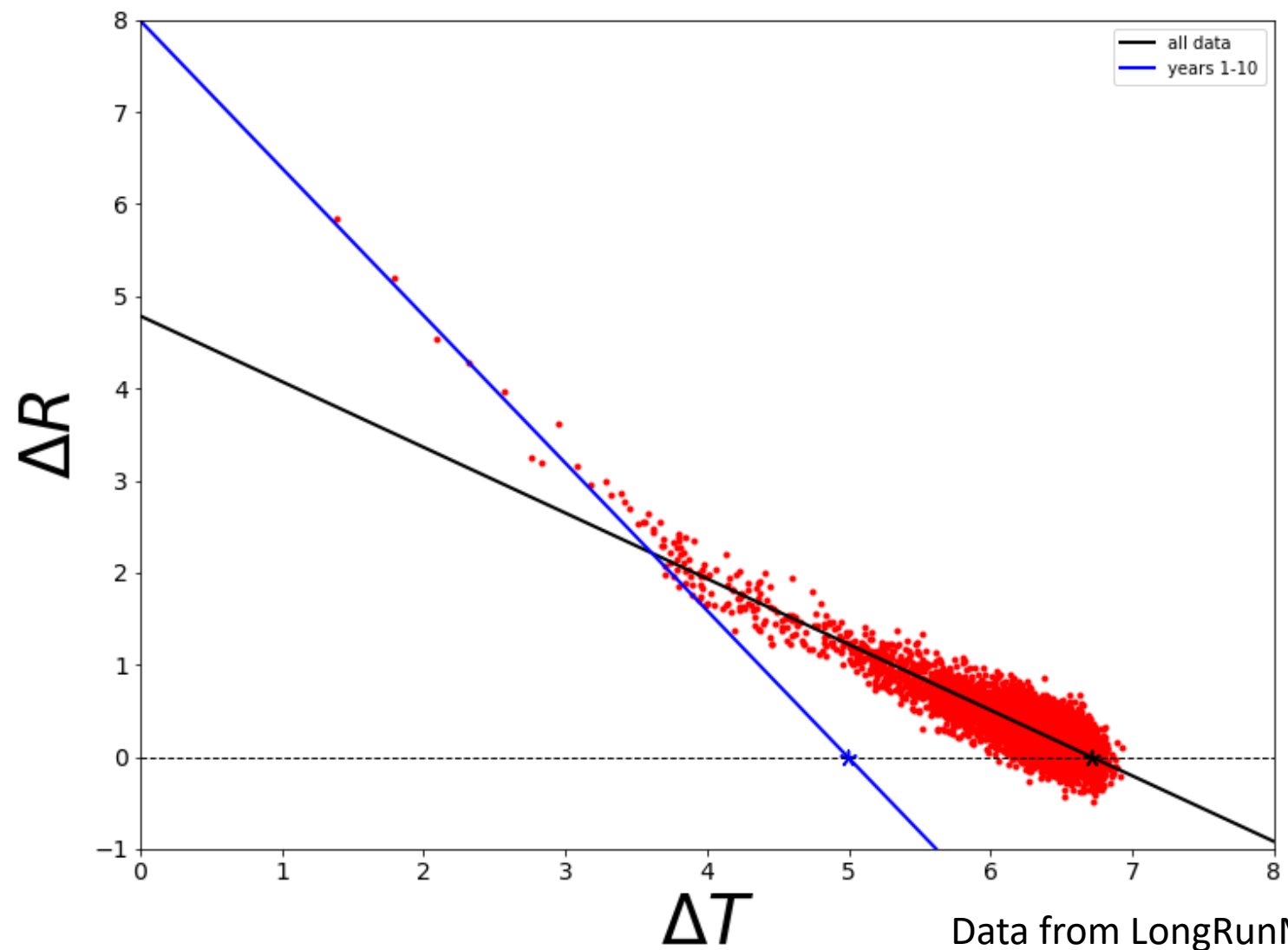
Data from LongRunMIP; [Rugenstein et al (2019)]

Gregory Plot for CESM 1.0.4



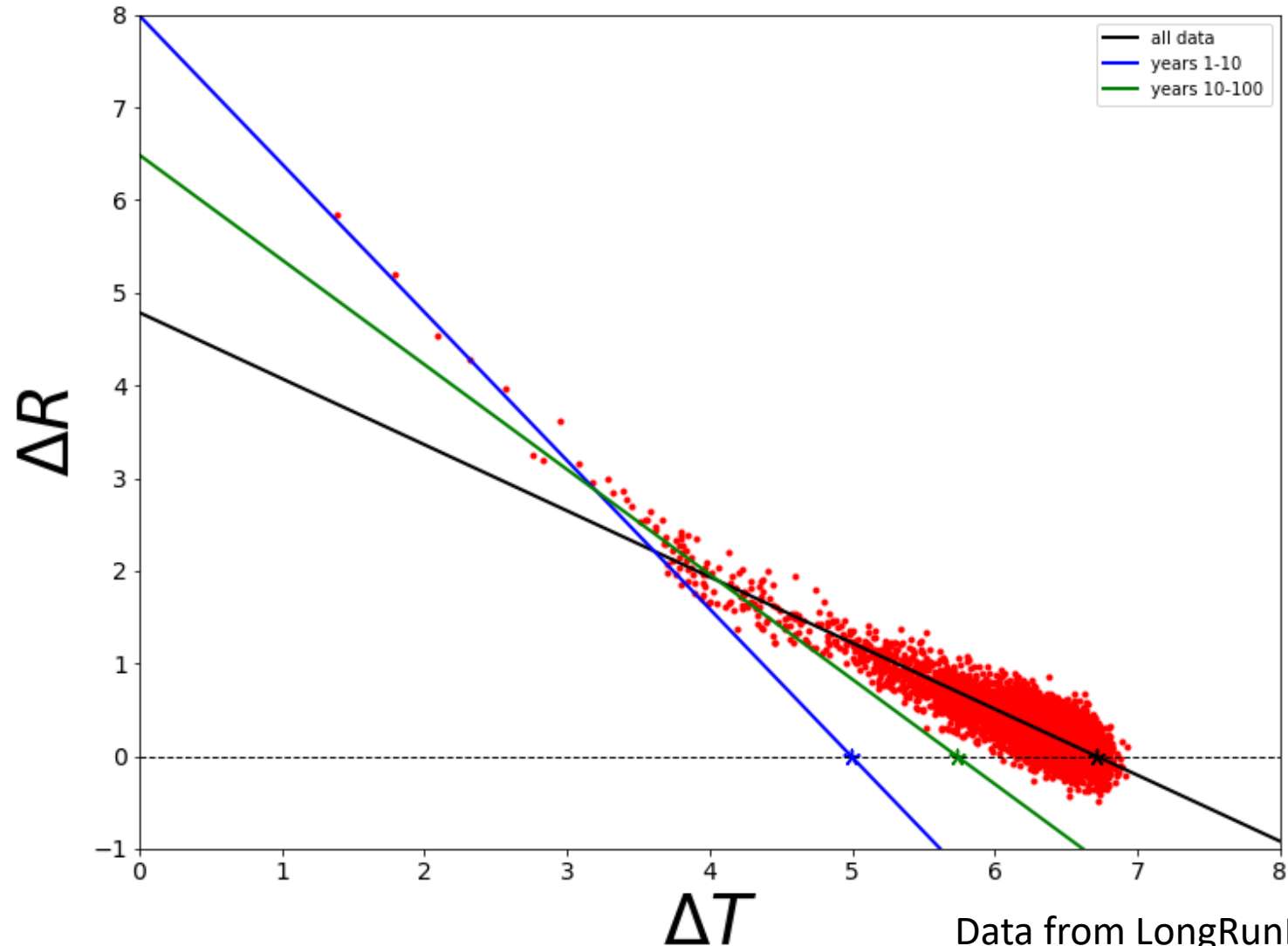
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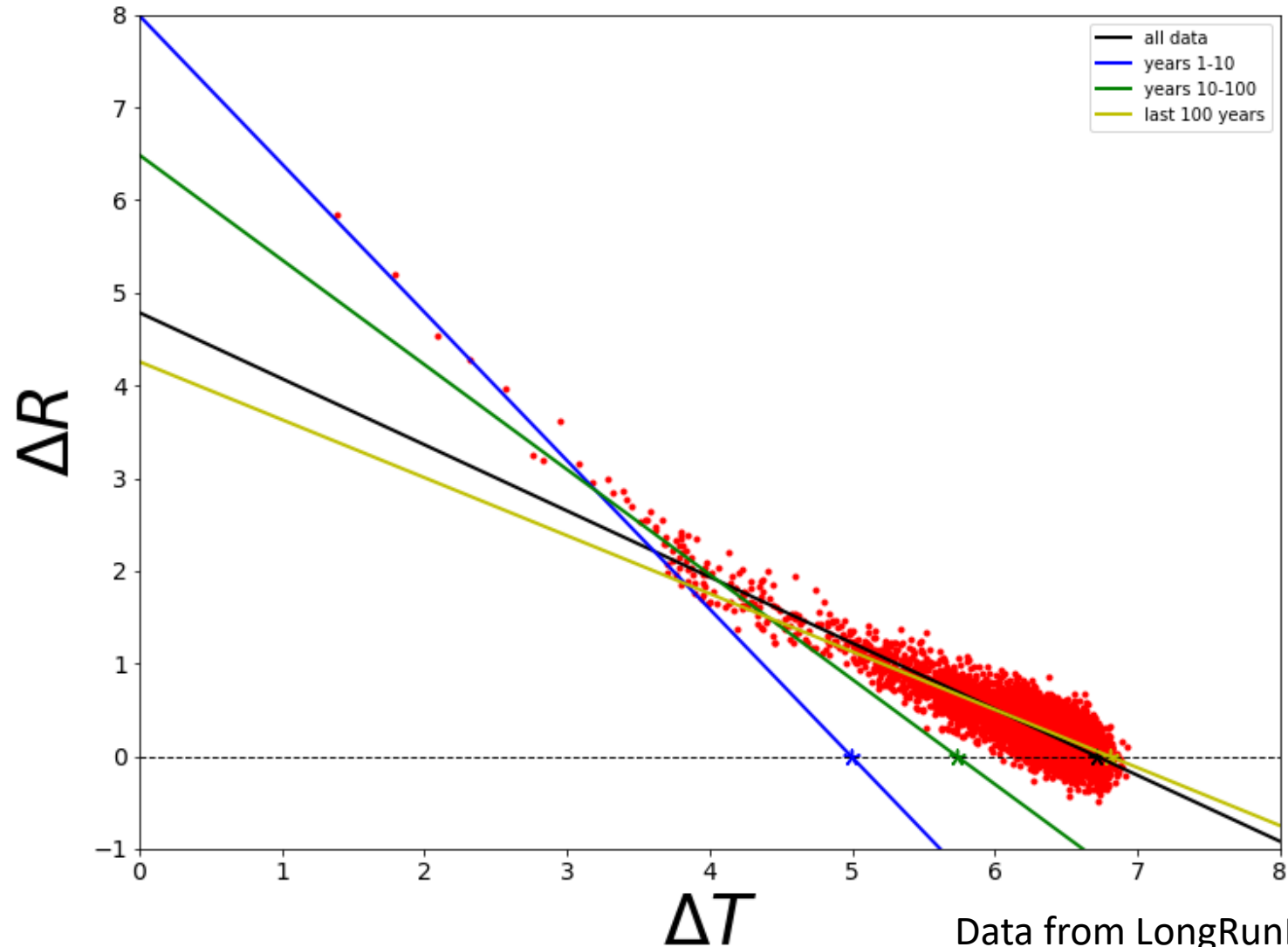
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Gregory Plot for CESM 1.0.4



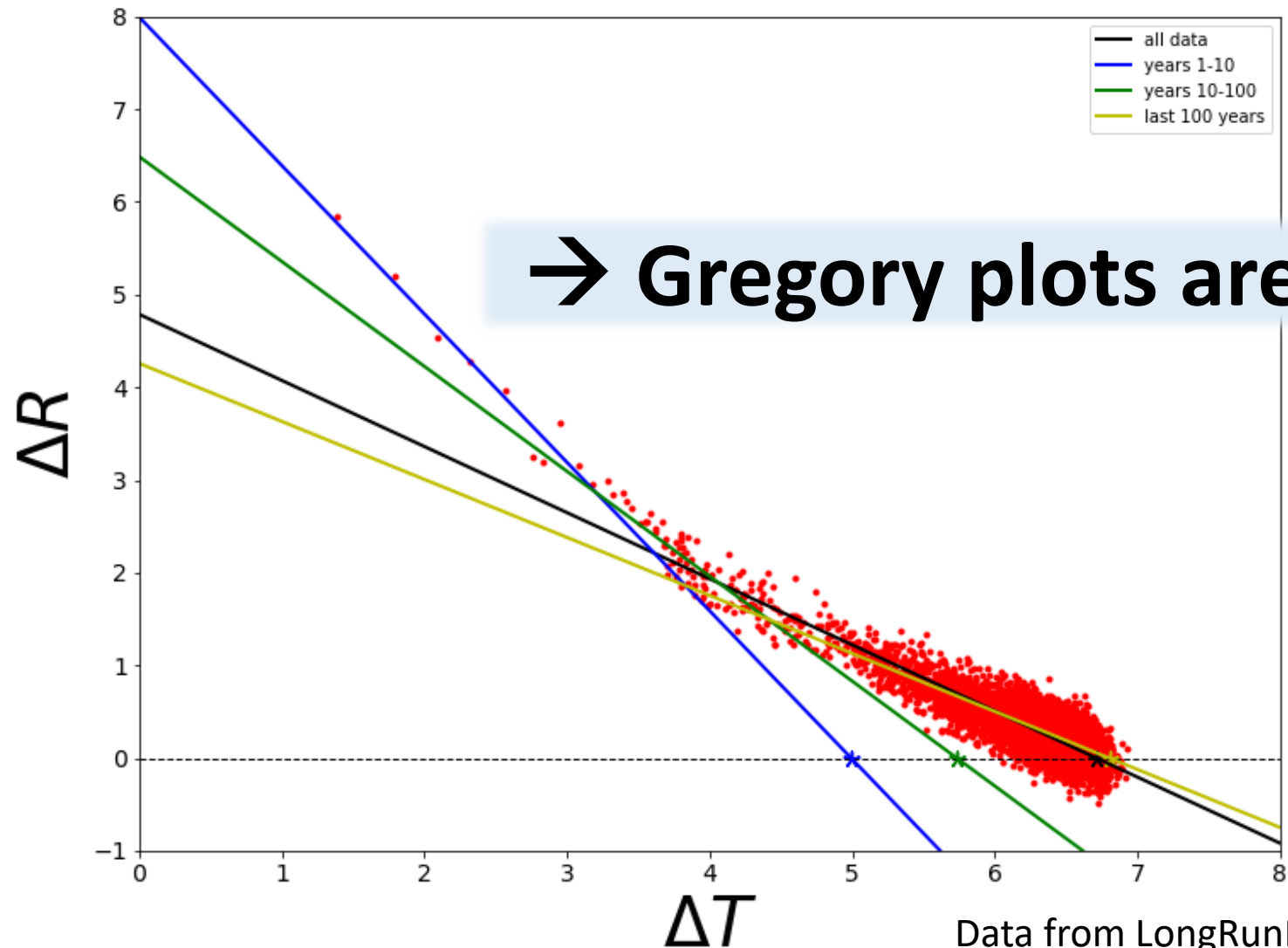
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Gregory Plot for CESM 1.0.4



Data from LongRunMIP; [Rugenstein et al (2019)]

Gregory Plot for CESM 1.0.4



→ Gregory plots are non-linear!

Step back: dynamical system point of view (1)

Nonlinearity in Gregory plot ('state-dependency'):

→ Feedbacks are not constant (& not instantaneous)

$$\cancel{R = R(T)} \rightarrow R = R(T, \alpha)$$

Let y denote (complete) system state

Dynamical system: $y' = f(y)$

Linearize around equilibrium state y_* : $y' = Df(y_*)[y - y_*]$

Thus, linear system of form

$$\Delta y' = A \Delta y + F \rightarrow \Delta y_* = A^{-1} F$$



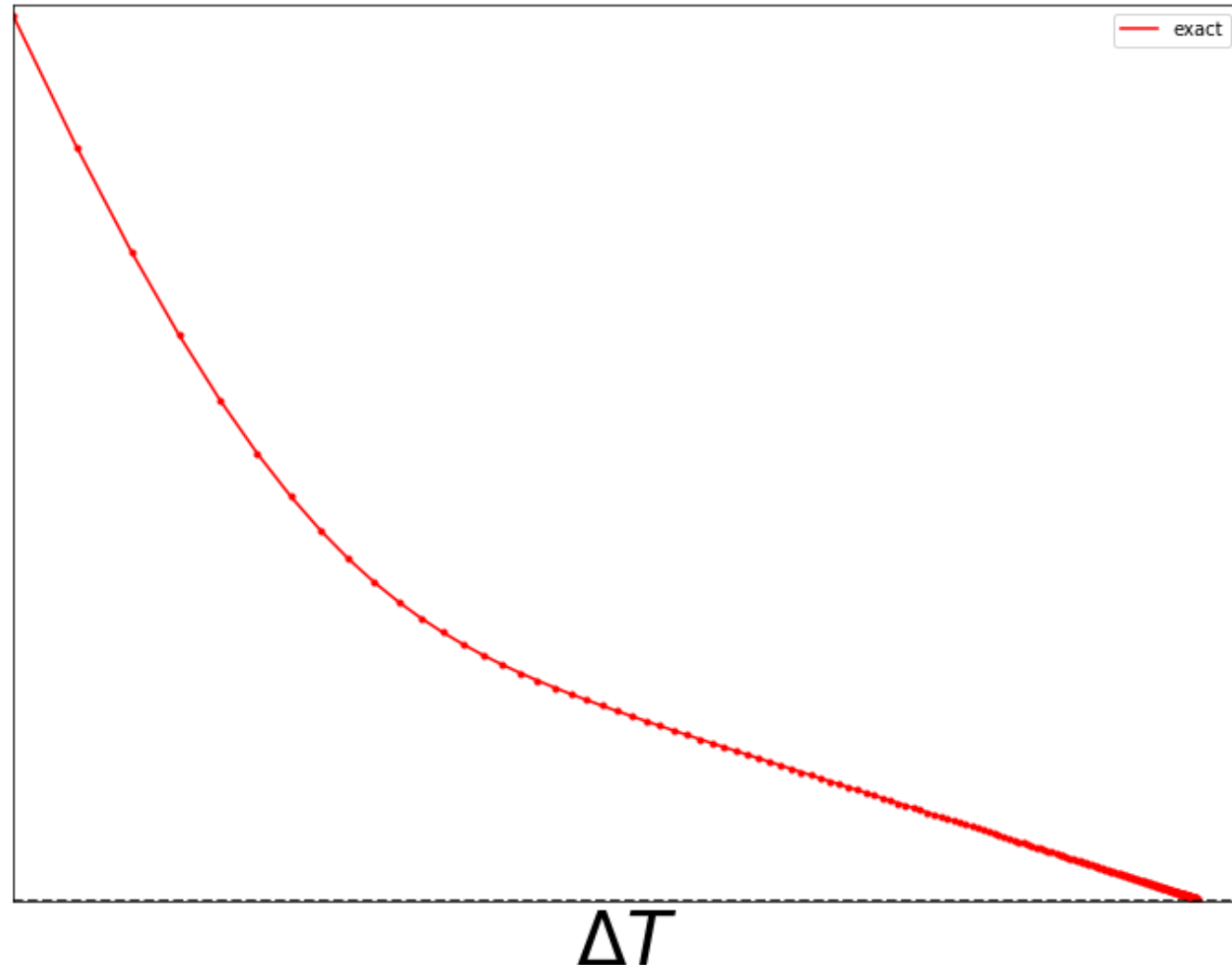
Step back: dynamical system point of view (2)

$$\Delta y' = A \Delta y + F$$

Solutions sum of exponentials

$$\Delta y(t) - \Delta y_* = \sum_{j=1}^n c_j v_j e^{\lambda_j t} \quad \Delta R$$

(v_j eigenvectors, λ_j eigenvalues of A)



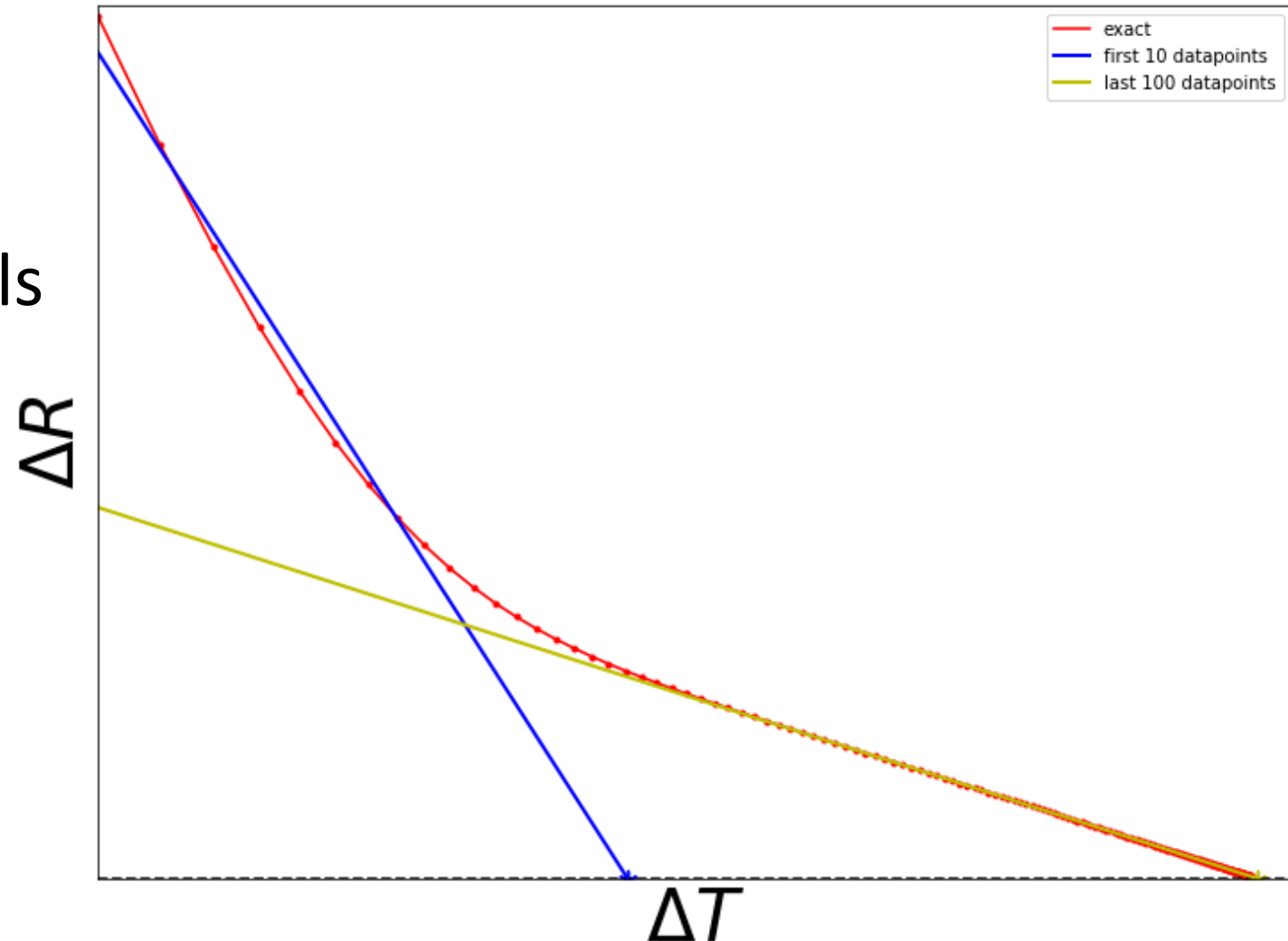
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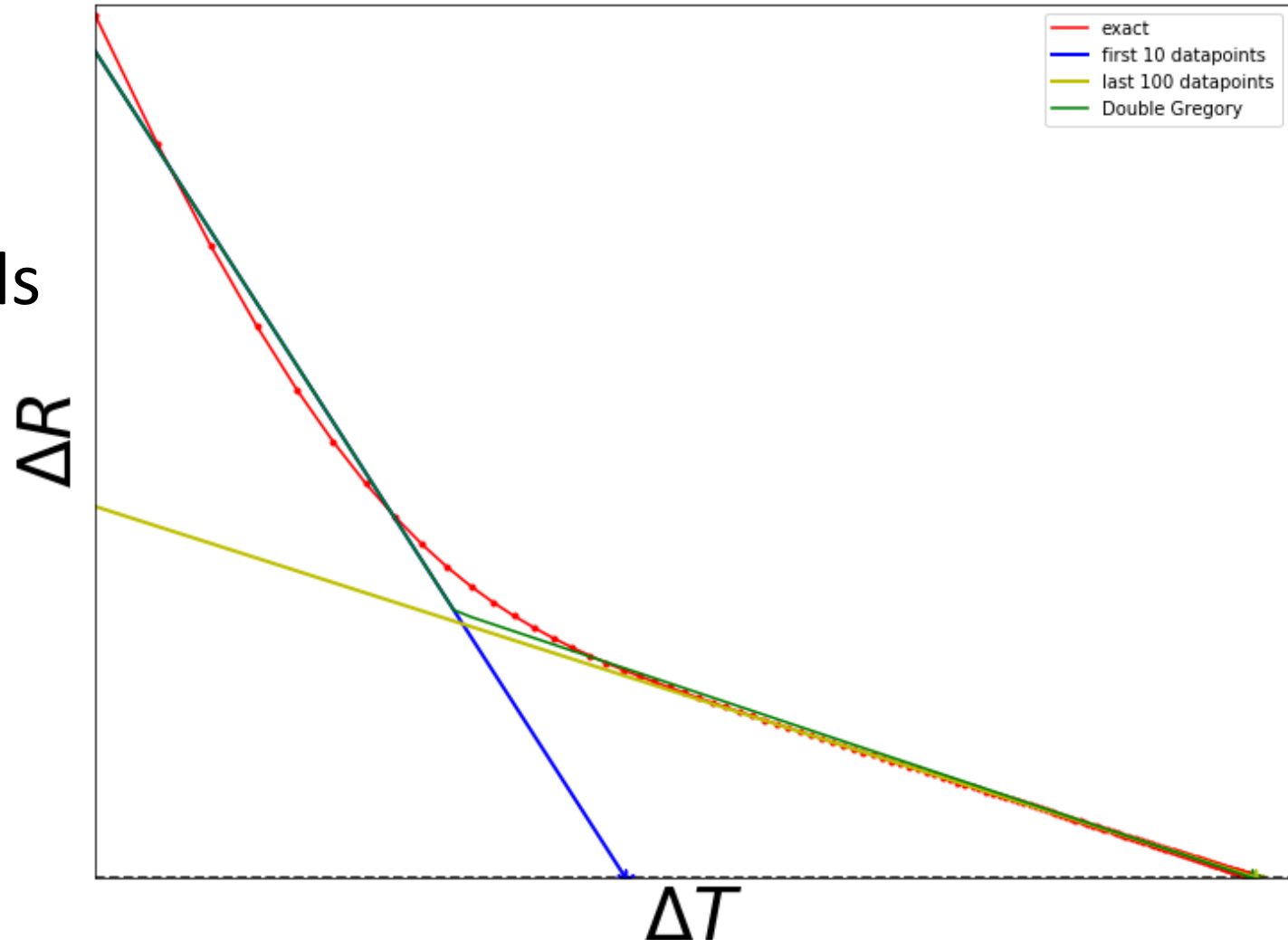
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Idea:

Fit a linear system!

$$\Delta Y = A \Delta X + F \rightarrow \Delta X_*^{est} = A^{-1} F$$

ΔY :

m observables that tend to 0 in equilibrium

ΔX :

m observables that are estimated in equilibrium

Example:

$$\begin{bmatrix} \Delta R \\ \Delta \alpha' \\ \Delta \varepsilon' \end{bmatrix} = A \begin{bmatrix} \Delta T \\ \Delta \alpha \\ \Delta \varepsilon \end{bmatrix} + F$$

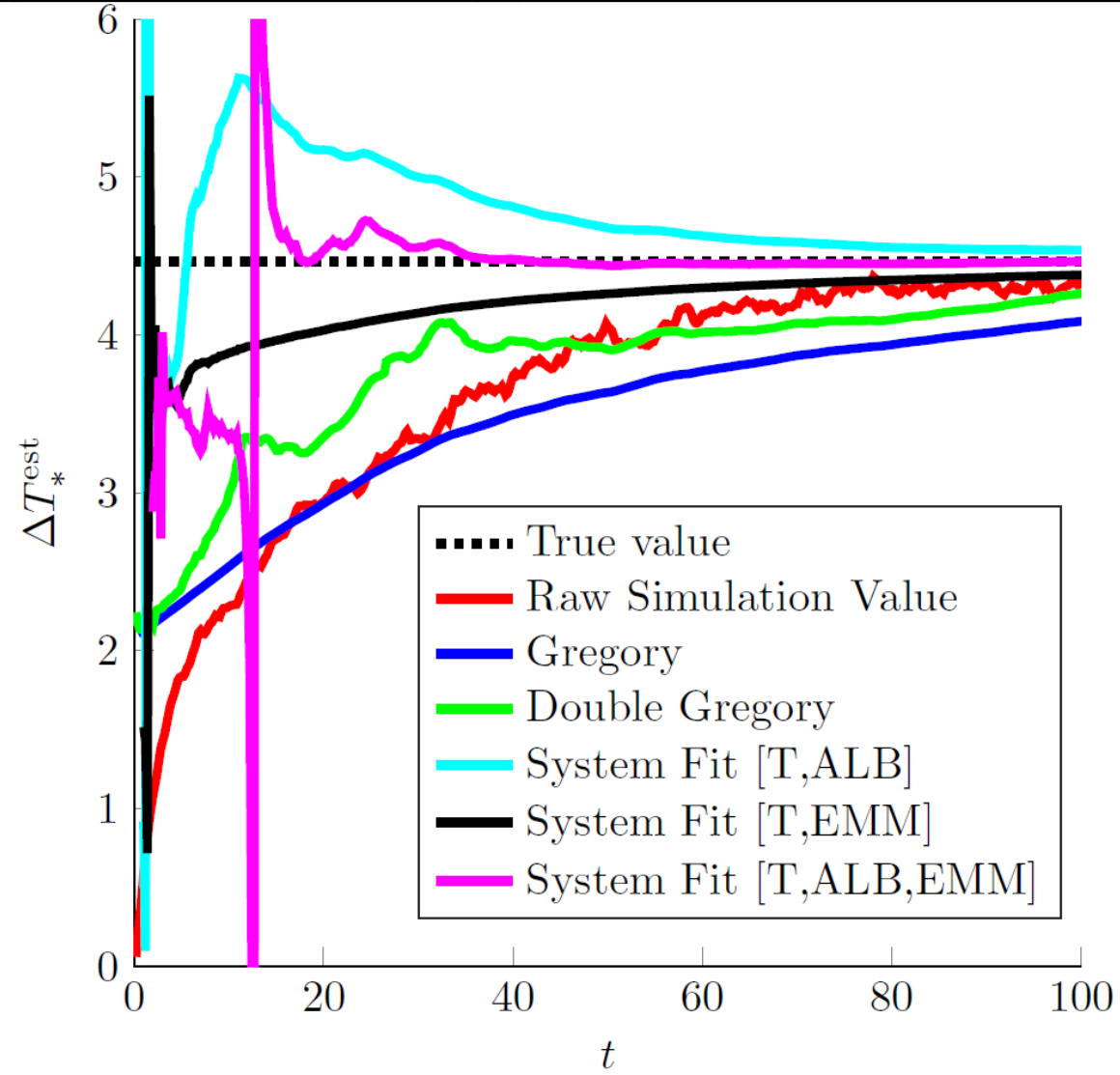
α : effective top-of-atmosphere short-wave albedo

$$\alpha = \frac{R_{SW,\uparrow}}{R_{SW,\downarrow}}$$

ε : effective top-of-atmosphere long-wave emissivity

$$\varepsilon = \frac{R_{LW,\uparrow}}{T^4}$$

Toy model: global energy balance model

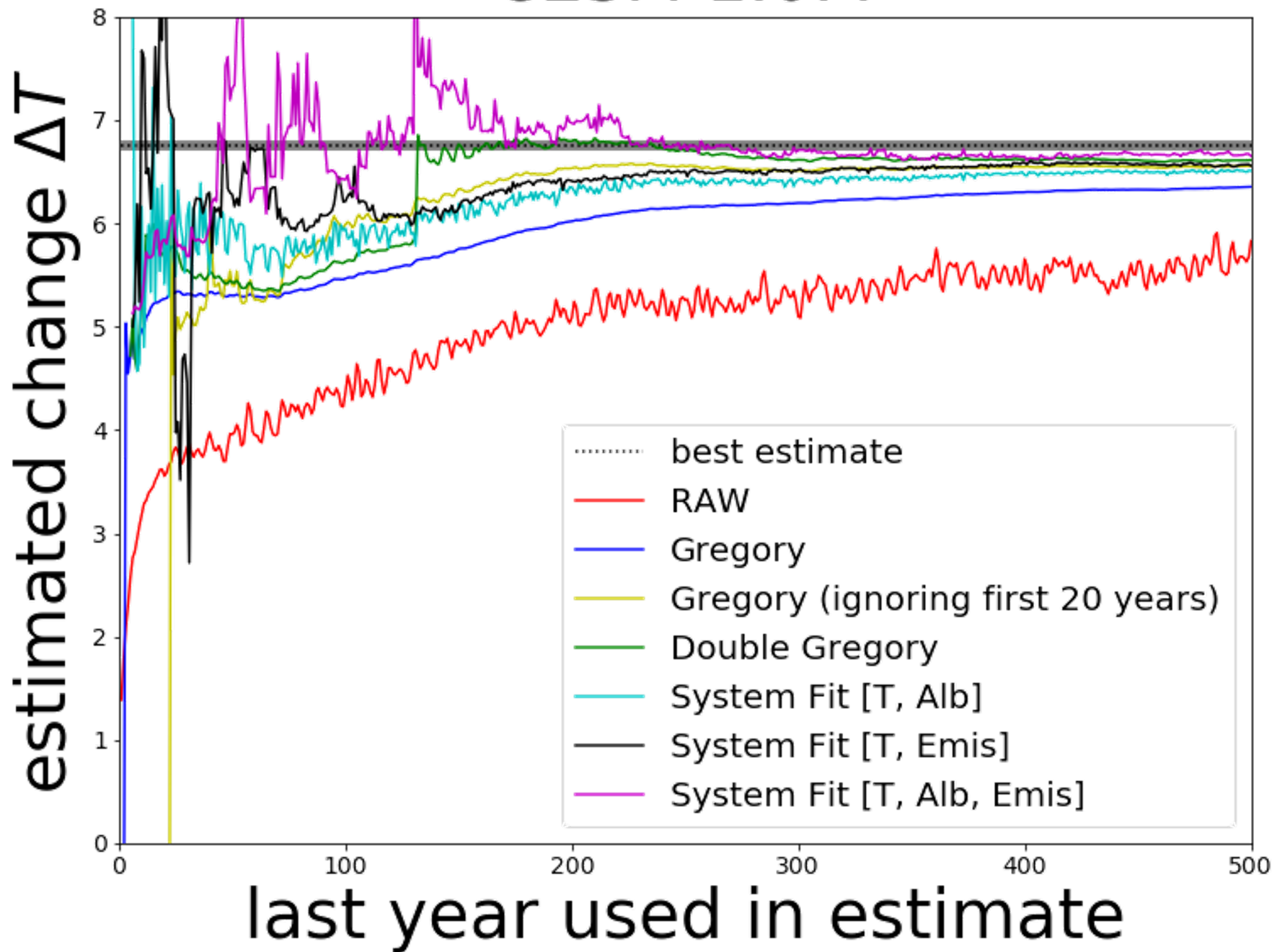


Testing on LongRunMIP data

- Models run to ‘equilibrium’ (practice: runs of at least 1000 years)
- Work with ‘abrupt-4xCO2’ forcing experiments

Experiment

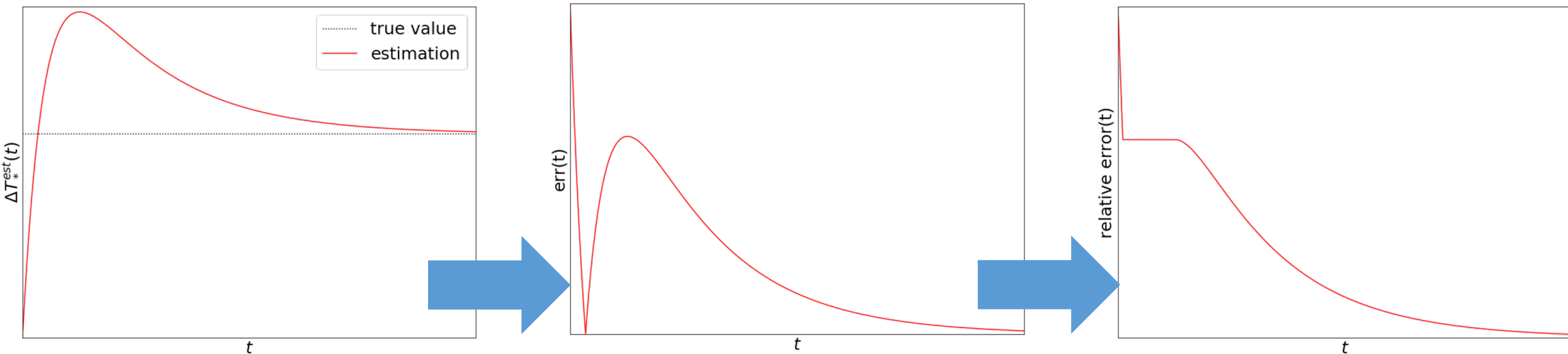
- Run estimation technique with data up to time t
- Compare with ‘equilibrium value’
- Determine effectiveness of techniques for time frame

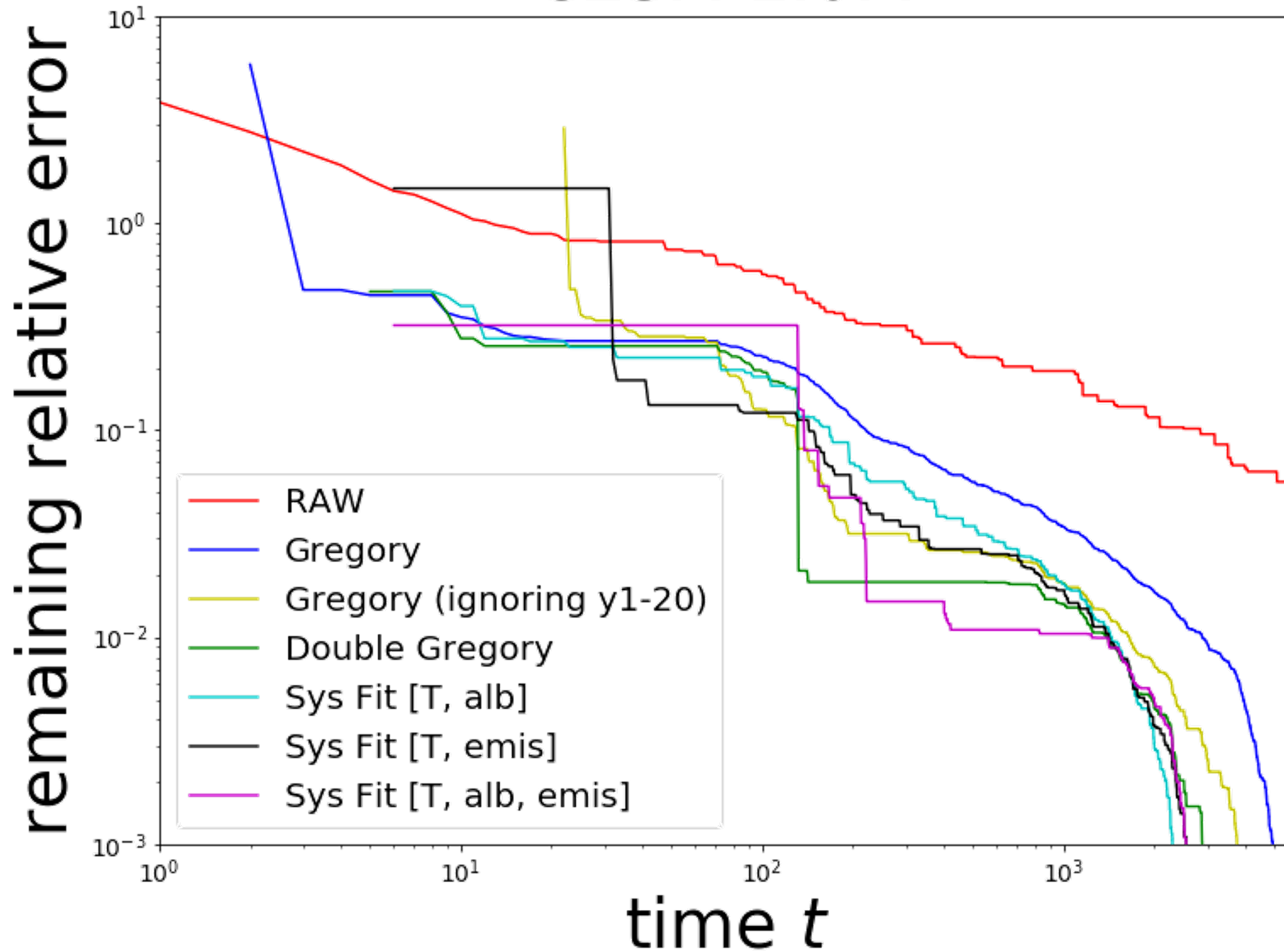


Measure for effectiveness

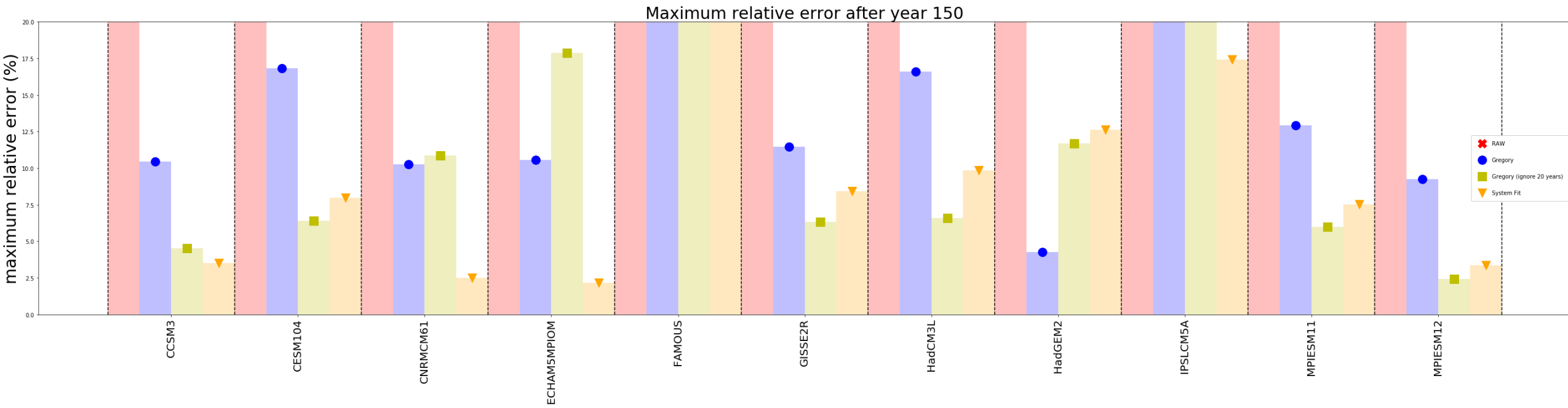
- Denote 'equilibrium' warming by ΔT_*
- Measure for maximum of relative error one ought to expect

$$\text{relative error } (t) := \max_{s \geq t} \left| \frac{\Delta T_*^{est}(s) - \Delta T_*}{\Delta T_*^{est}(s)} \right|$$

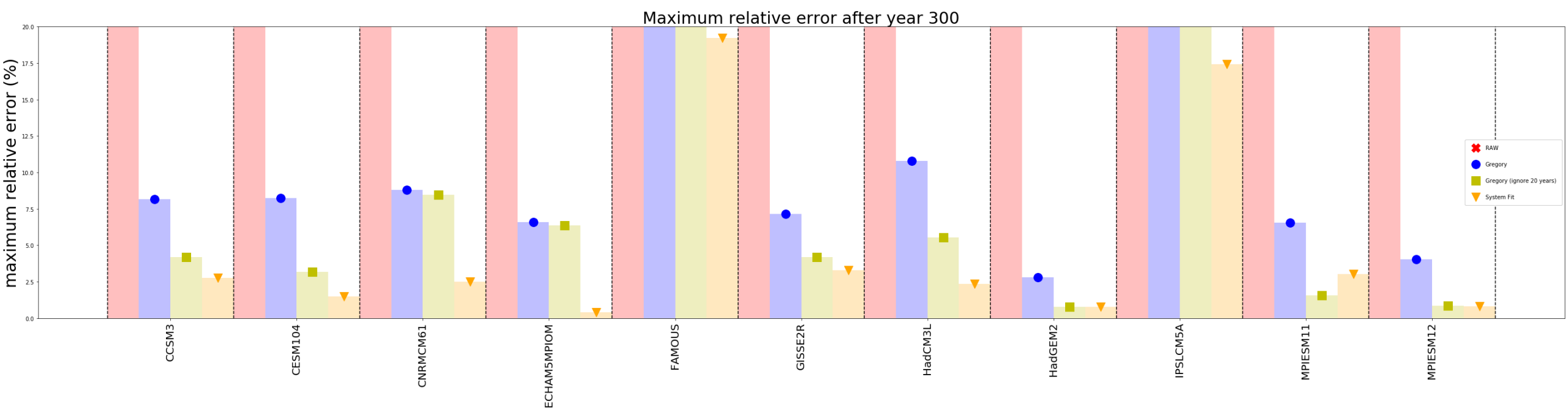




Results for all models

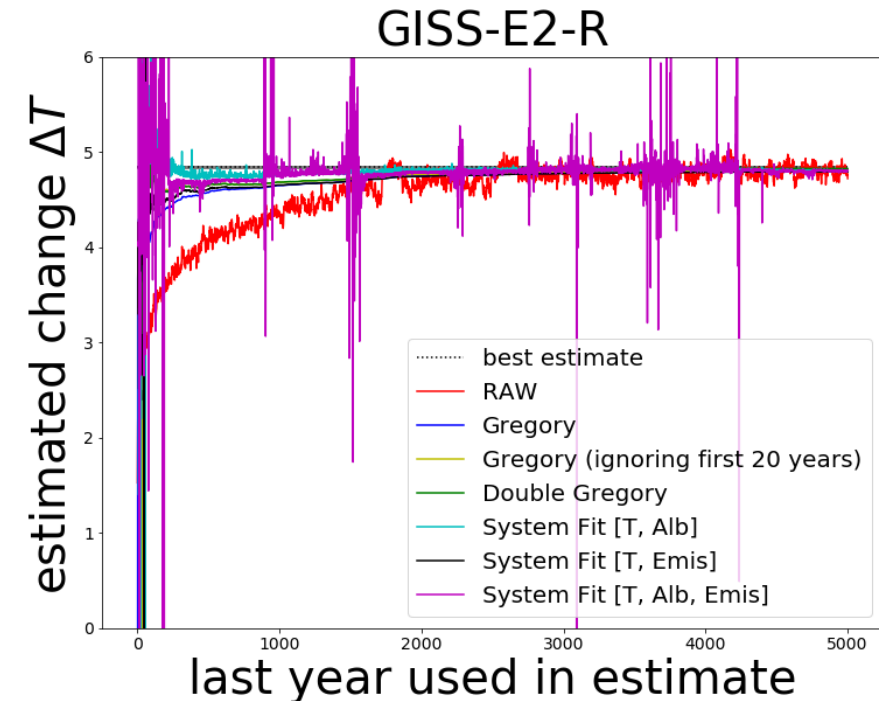
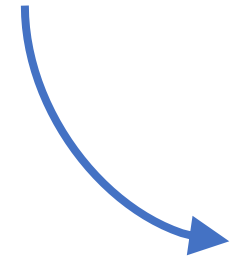


Results for all models



General observations so far (to be continued)

- Hard to find true increment ΔT_*
- Model-dependent & time-dependent which technique works best
→ However: almost always a system fit technique works best
- Some techniques rubbish on some models
→ Possibly due to model dynamics?



Possible extensions and use cases

- Can also predict other climate change indicators
 - Here: (effective top-of-atmosphere) albedo and/or emissivity
 - Other things might also be possible
- Can help to design experiments for e.g. high-resolution models
 - How long should we run those?
 - What kind of experiments/forcings/perturbations are best?