TIPES
TIPPING POINTS IN THE EARTH SYSTEM

Co-financed by the Connecting Europe Facility of the European Union

Multivariate Climate Projections

More Accurate Equilibrium Estimations
&
Evolution of Climate Feedbacks

Robbin Bastiaansen

Climate Response

The change in observable due to climate forcing (e.g. CO_2)

Global Climate models are computationally expensive

- 🖳 not all time scales can be simulated
- 🖳 not all forcings can be simulated

» Need for extrapolation and estimation techniques

Problem:

Classic methods are univariate

☹ Everything is assumed to relate linearly to temperature

Solution:

Incorporate additional observables in multivariate methods

- ☻ Leads to more accurate estimates
- ☻ Leads to multivariate projections

Robbin Bastiaansen

- Background in (Applied) Mathematics
- 2015-2019:
  PhD @ Leiden University on *Pattern Formation* and *Desertification*
  (with Arjen Doelman, Martina Chirilus-Bruckner & Max Rietkerk)
- Since JAN 2020:
  PostDoc @ IMAU, Utrecht University on *Climate Sensitivity*
  (with Anna von der Heydt & Henk Dijkstra)

  Work within H2020 project TiPES: Tipping Points in the Earth System

Most used Climate Sensitivity Metrics

Equilibrium Climate Sensitivity (ECS)

change in equilibrium temperature
due to (instantaneous) doubling of CO_2

Transient Climate Response (TCR)

change in temperature after 100 years
with 1% CO_2 increase per year (until doubling)

Some Details

- Dedicated experiments with climate models
- Start from equilibrium with pre-industrial levels of CO_2
- Change compared to control run

Mathematical Context

$$\frac{dy}{dt} = f(\, y \,;\, \mu(t) \,)$$

y : state variable
μ : forcing parameter

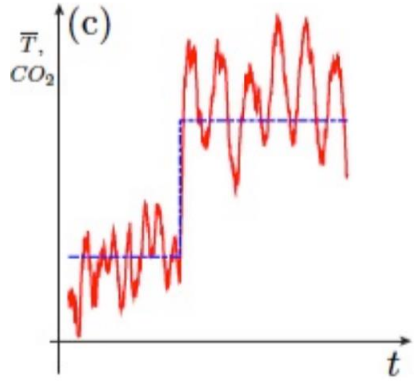


Image source: [Ghil and Lucarini, 2020]

Mathematical Idea behind Classic Estimation Techniques

Idea behind estimation methods (1)

Warming is due to net positive radiative imbalance

$$\Delta N = N_{SW,\downarrow} - N_{SW,\uparrow} - N_{LW,\uparrow}$$

incoming sunlight | reflected sunlight | Planck radiation

When $\Delta N = 0$ no more warming:

→ equilibrium warming $\Delta T_* = T_* - T_0$

Idea behind estimation methods (2)

Express imbalance as function of system state

$$\Delta N(t) = \Delta N(\,y(t)\,, \mu(t)\,)$$

Near equilibrium y_* (with $\mu = \mu_*$) a Taylor expansion gives

$$\Delta N(t) = \Delta N(y_*, \mu_*) + \left.\frac{\partial \Delta N}{\partial \mu}\right|_* \Delta\mu(t) + \left.\frac{\partial \Delta N}{\partial y}\right|_* \Delta y(t) + h.o.t.$$

Equals Zero

Radiative Forcing

Climate Response

Assumed to be small

$$\Delta N(t) = F(t) + \Delta R(t)$$

Implicit assumption: relevant climate dynamics are **approximately** a **linear** system

Idea behind estimation methods (3)

Climate Response ΔR is sum of feedback contributions:

$$\Delta R(t) = \sum_{j \in \mathcal{F}} \Delta R_j(t)$$

$$\Delta R_j(t) := \left.\frac{\partial \Delta N}{\partial y_j}\right|_* \Delta y_j(t)$$

$\mathcal{F}$ is set of **Climate Feedbacks:**

- Planck Feedback
- Lapse Rate Feedback
- Surface Albedo Feedback
- Water Vapour Feedback
- Cloud Feedback

Then, express as function of temperature T:

$$\Delta R_j(t) = \Delta \mathrm{R_j}(T(t))$$

Taylor expansion gives

$$\Delta R_j(t) = \left.\frac{\partial \Delta R_j}{\partial T}\right|_* \Delta T + h.o.t.$$

That is,

$$\Delta R_j(t) = \lambda_j \, \Delta T(t)$$

Classic Estimation Method:

$$\Delta N(t) = F(t) + \left(\sum_{j \in \mathcal{F}} \lambda_j\right) \Delta T(t)$$

Implicit assumption: relevant climate dynamics play on **approximately one mode**

Part A
More Accurate Equilibrium Estimation

Equilibrium Estimations

Experiment in model:

- start in equilibrium at pre-industrial CO2
- instant CO2 quadrupling from start

Gregory Method

Classic Estimation Method:

$$\Delta N(t) = F + \lambda \, \Delta T(t)$$

where $\lambda = \sum_{j \in \mathcal{F}} \lambda_j$

Equilibrium Estimation:

$$\Delta T_*^{est} := -\lambda^{-1} F$$

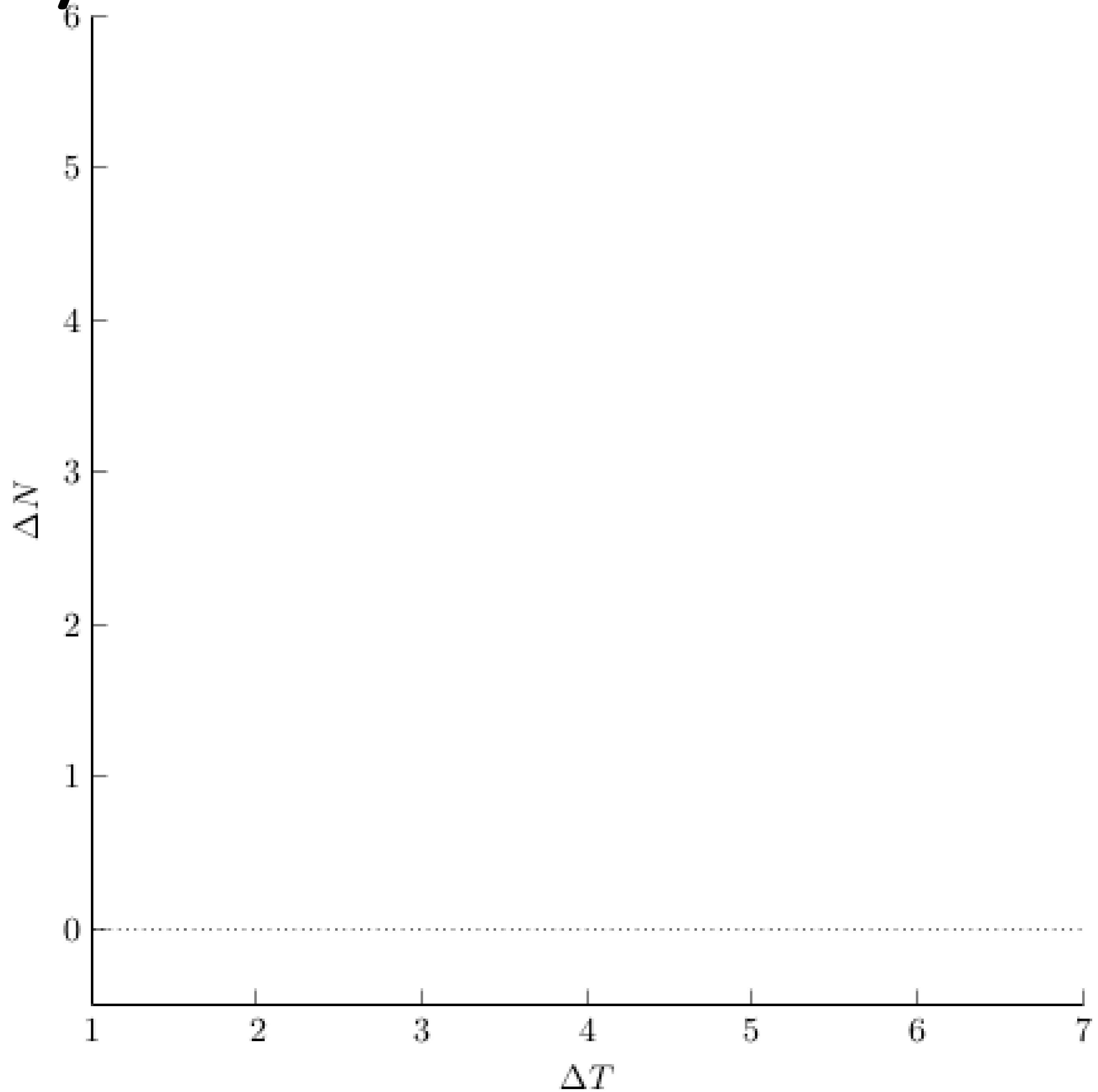
Gregory Method

Regress data to

$$\Delta N(t) = \boldsymbol{F} + \boldsymbol{\lambda}\, \Delta T(t)$$

Since $\Delta N_* = 0$ in equilibrium, ECS estimation is

$$\Delta T_*^{est} = -\, \boldsymbol{\lambda}^{-1}\, \boldsymbol{F}$$



[Gregory et al, 2014]

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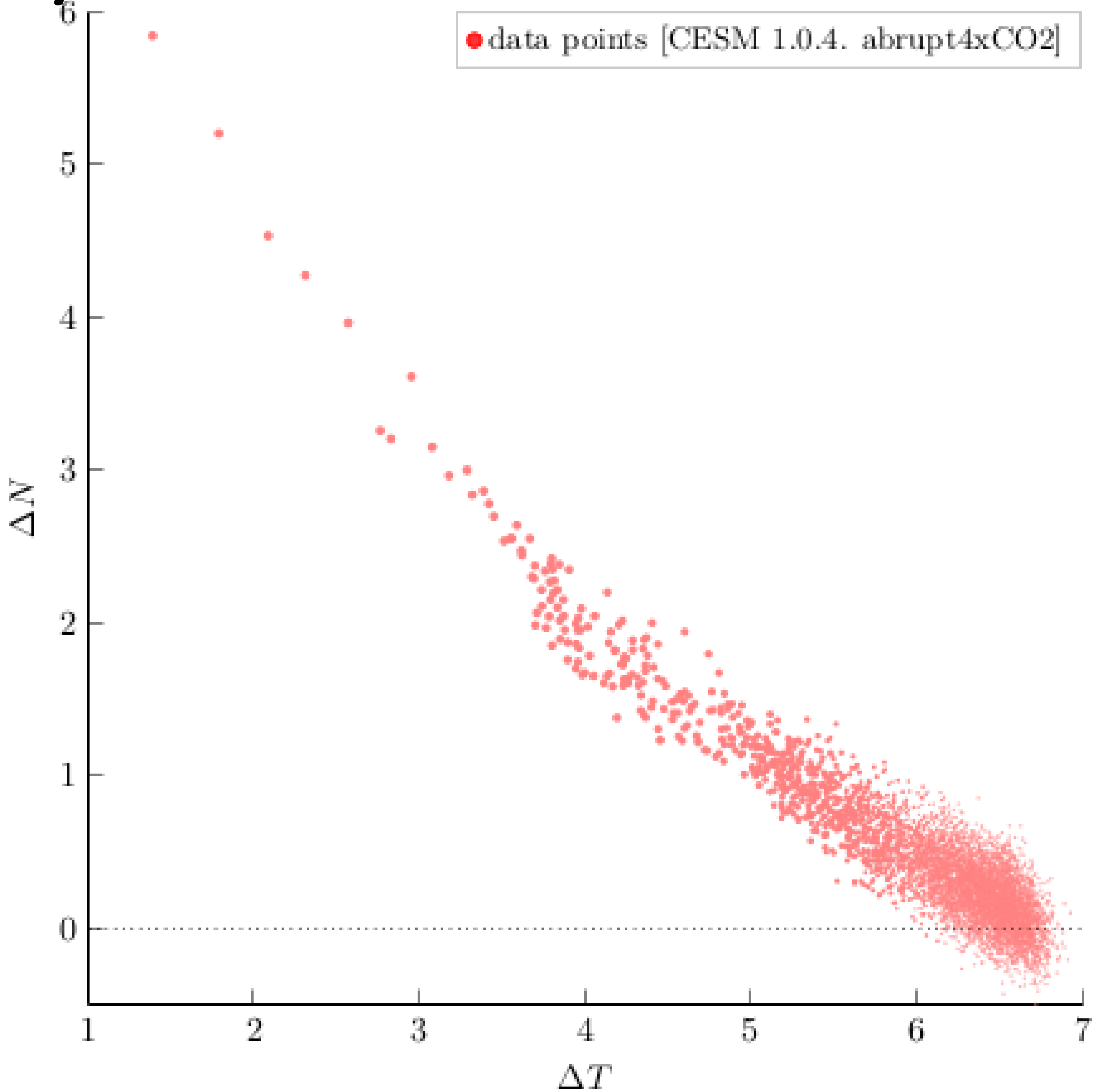

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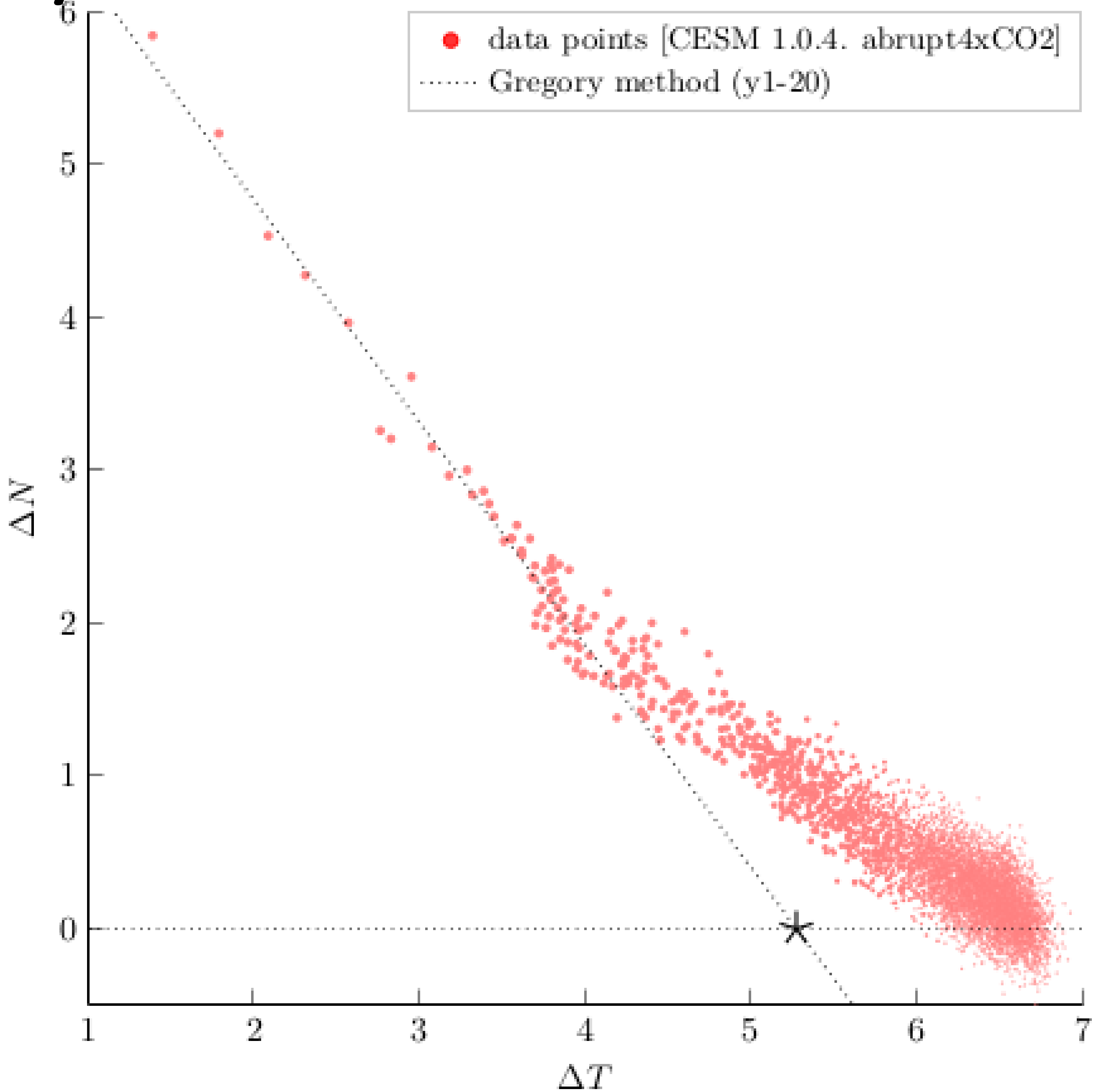

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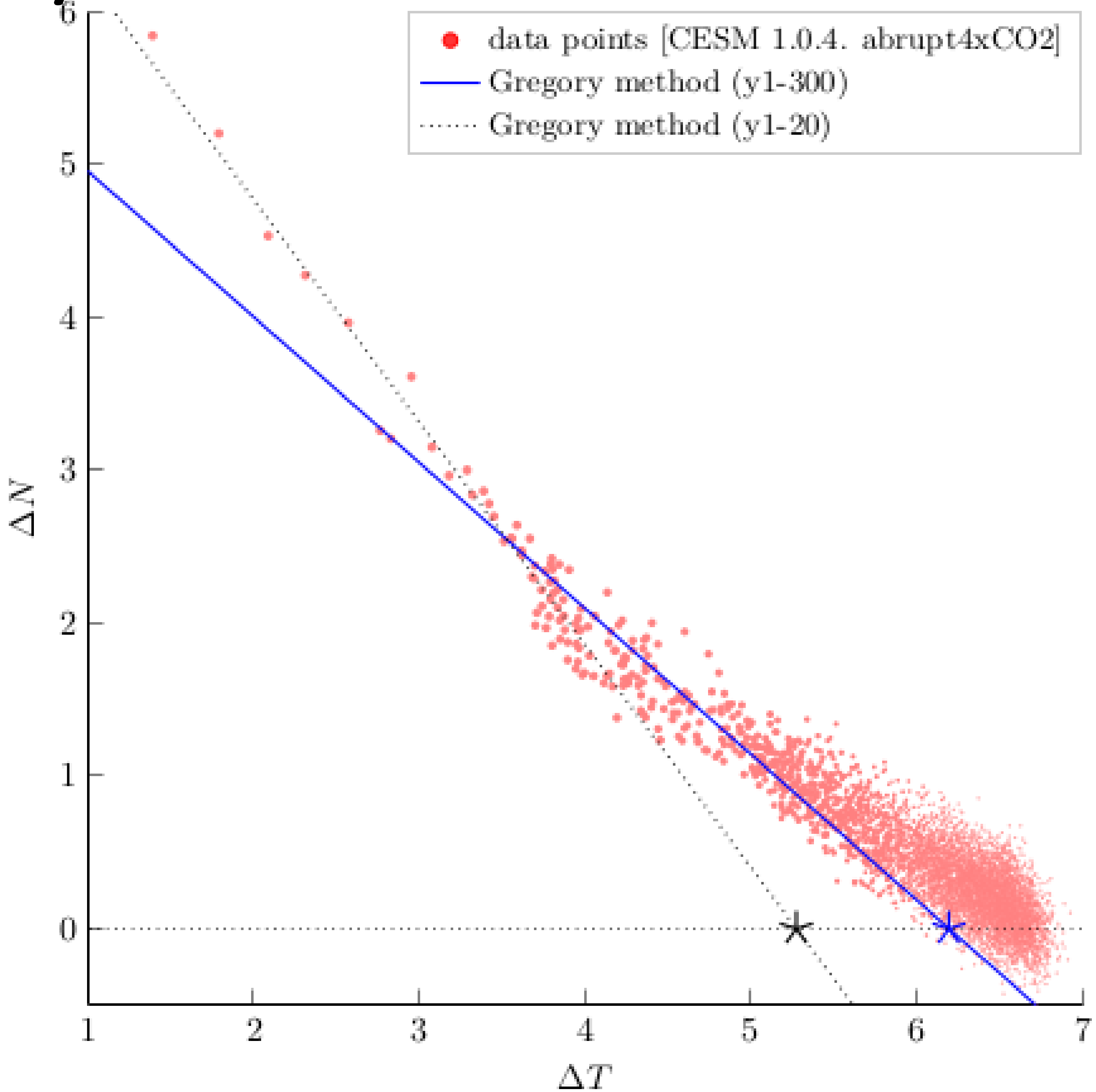

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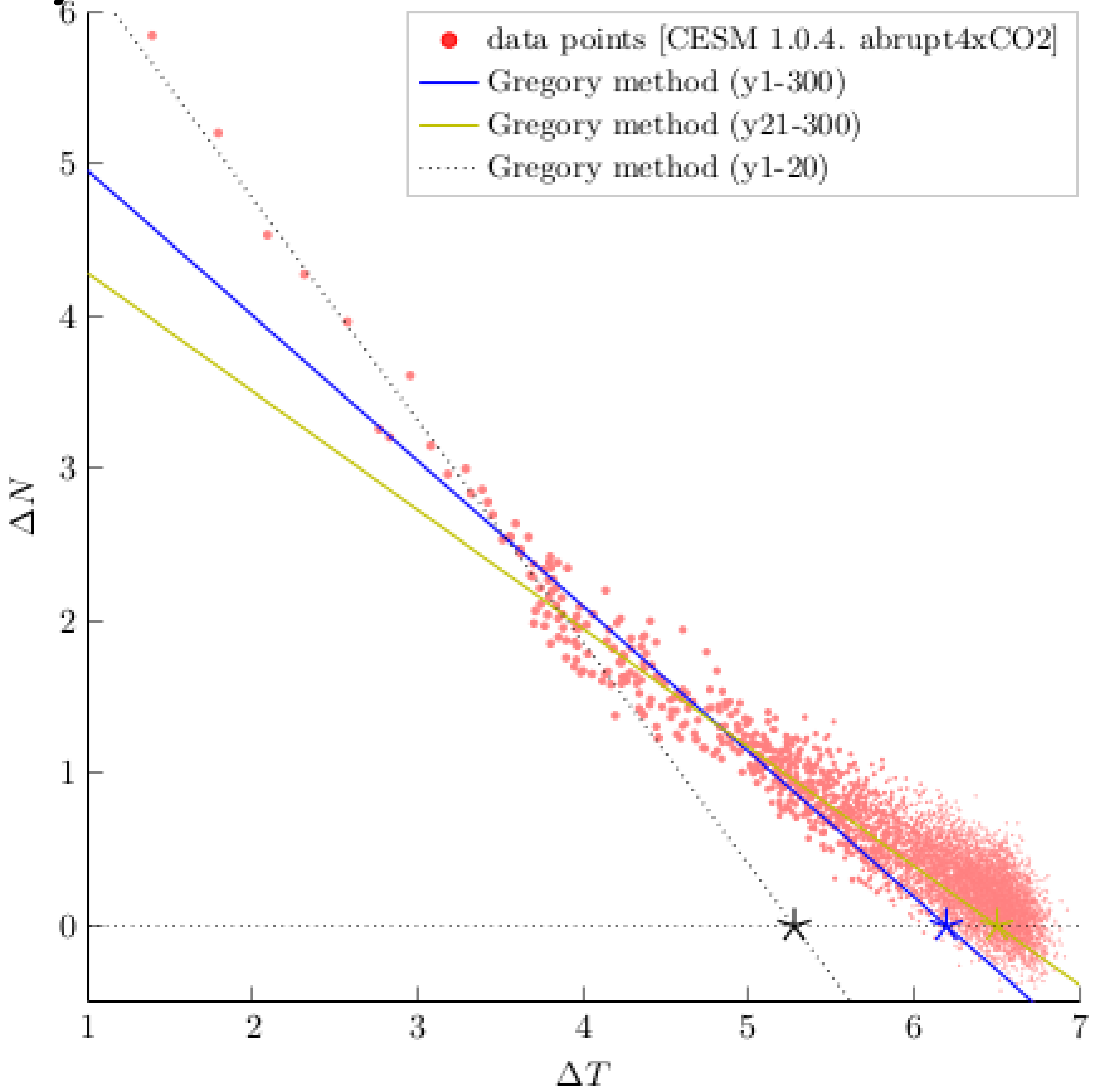

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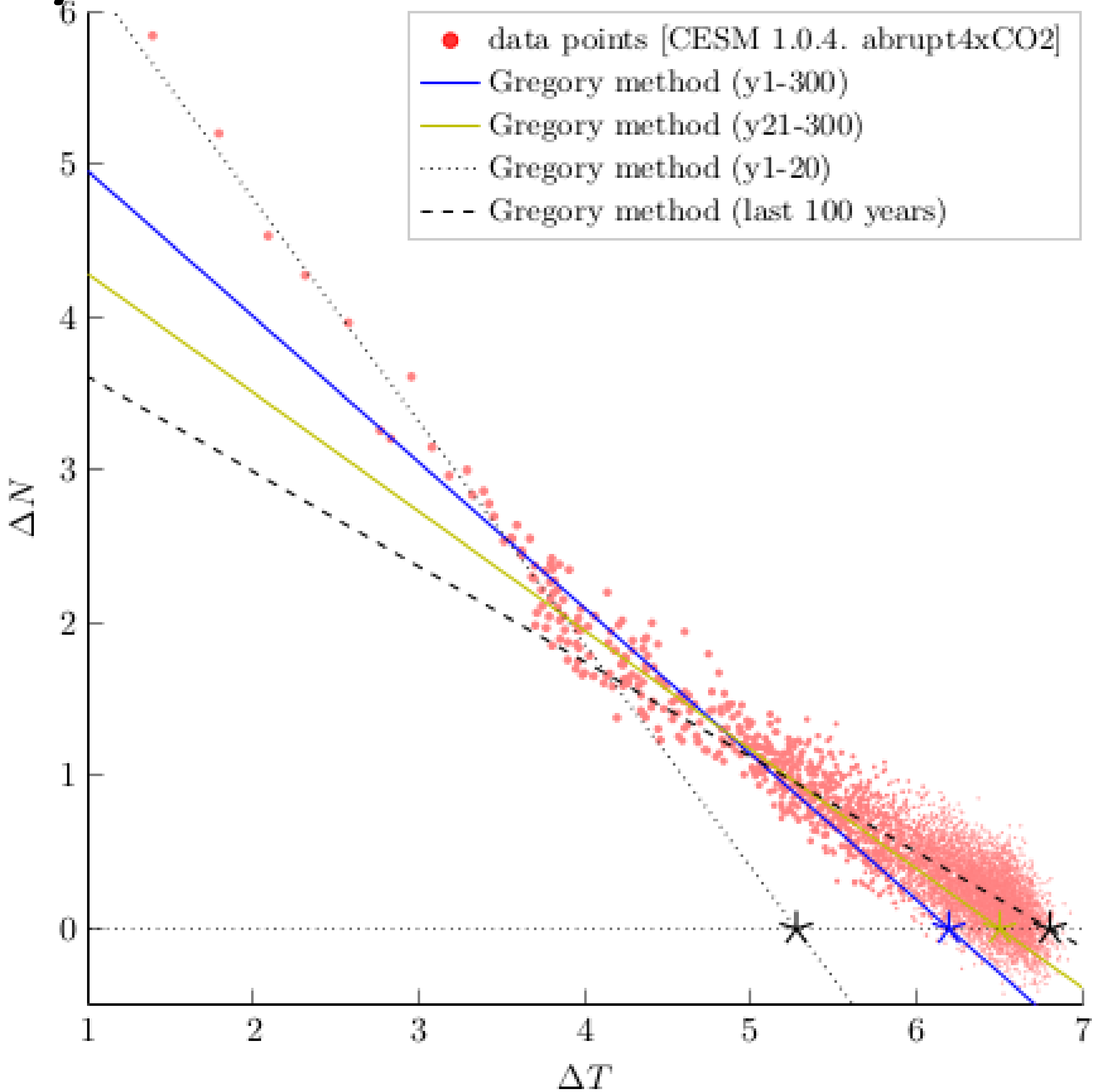
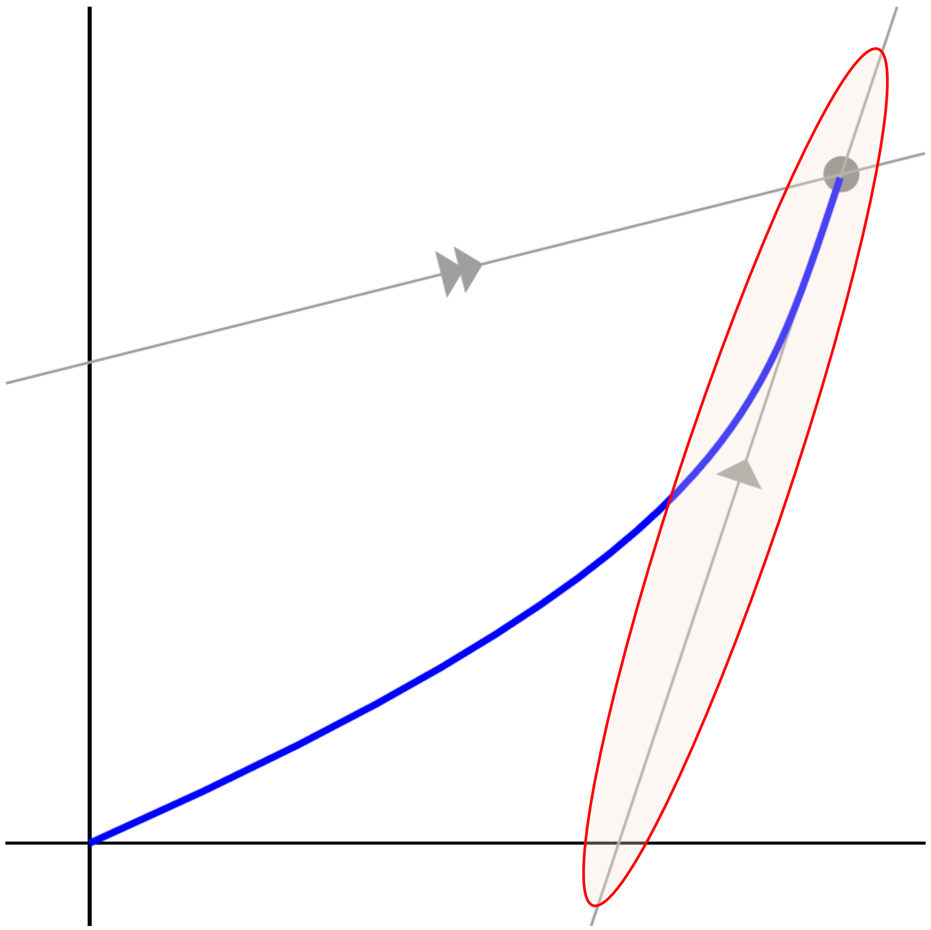


[Gregory et al, 2014]

What is the problem?

Assumption 1: relevant climate dynamics are **approximately** a **linear** system



Assumption 2: ~~relevant climate dynamics play on~~ **~~approximately~~ one mode**

New Multicomponent Linear Regression Method

Use additional observables!

Regress to:

$$\overrightarrow{\Delta Y} = \boldsymbol{A}\,\overrightarrow{\Delta X} + \vec{\boldsymbol{F}}$$

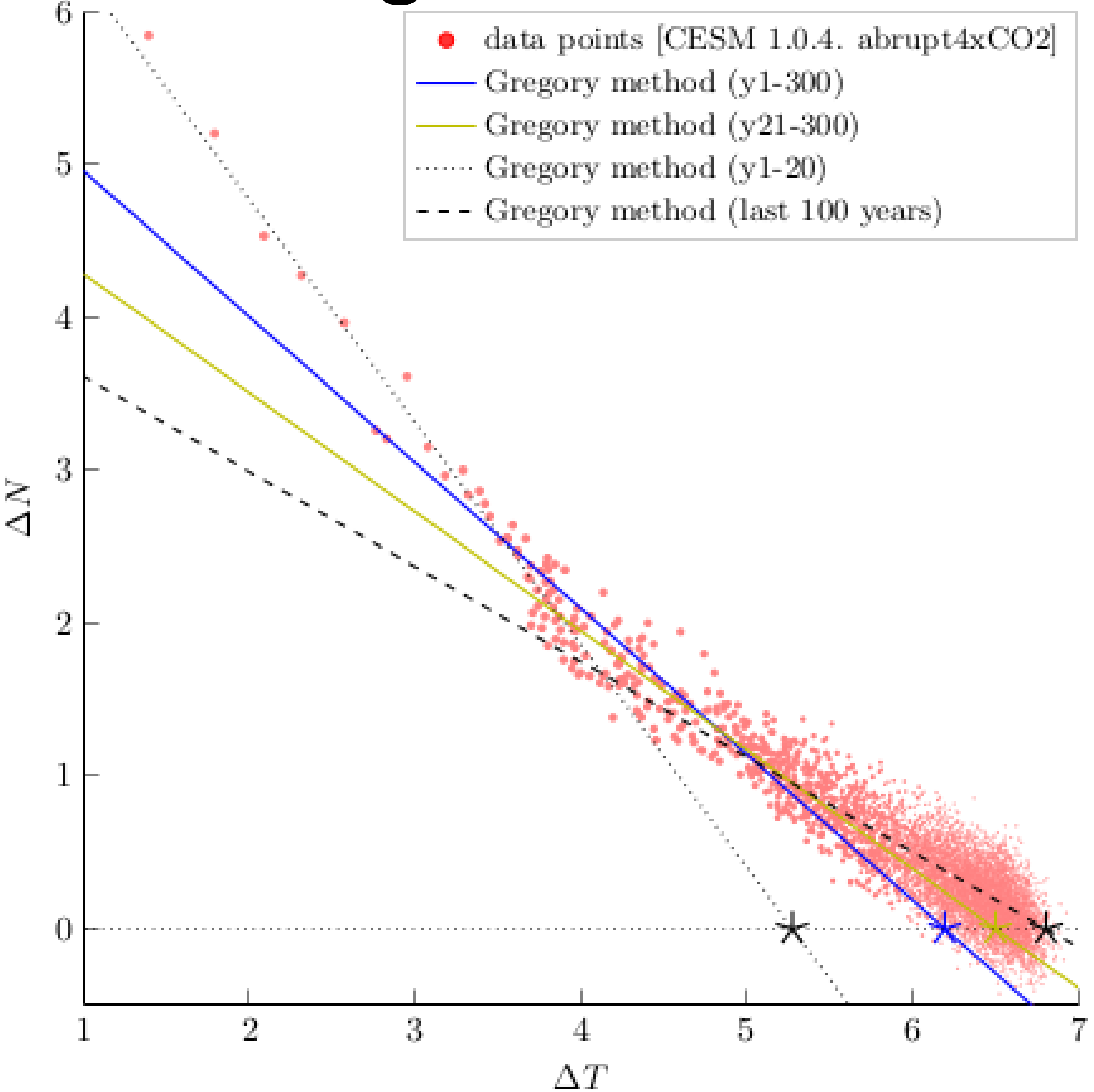
$\overrightarrow{\Delta Y}$: observables that **tend to 0** in equilibrium

$\overrightarrow{\Delta X}$: observables that **get estimated** in equilibrium

Multivariate ECS estimation is

$$\overrightarrow{\Delta X}_*^{est} = -\boldsymbol{A}^{-1}\,\vec{\boldsymbol{F}}$$

[Bastiaansen et al, 2021]



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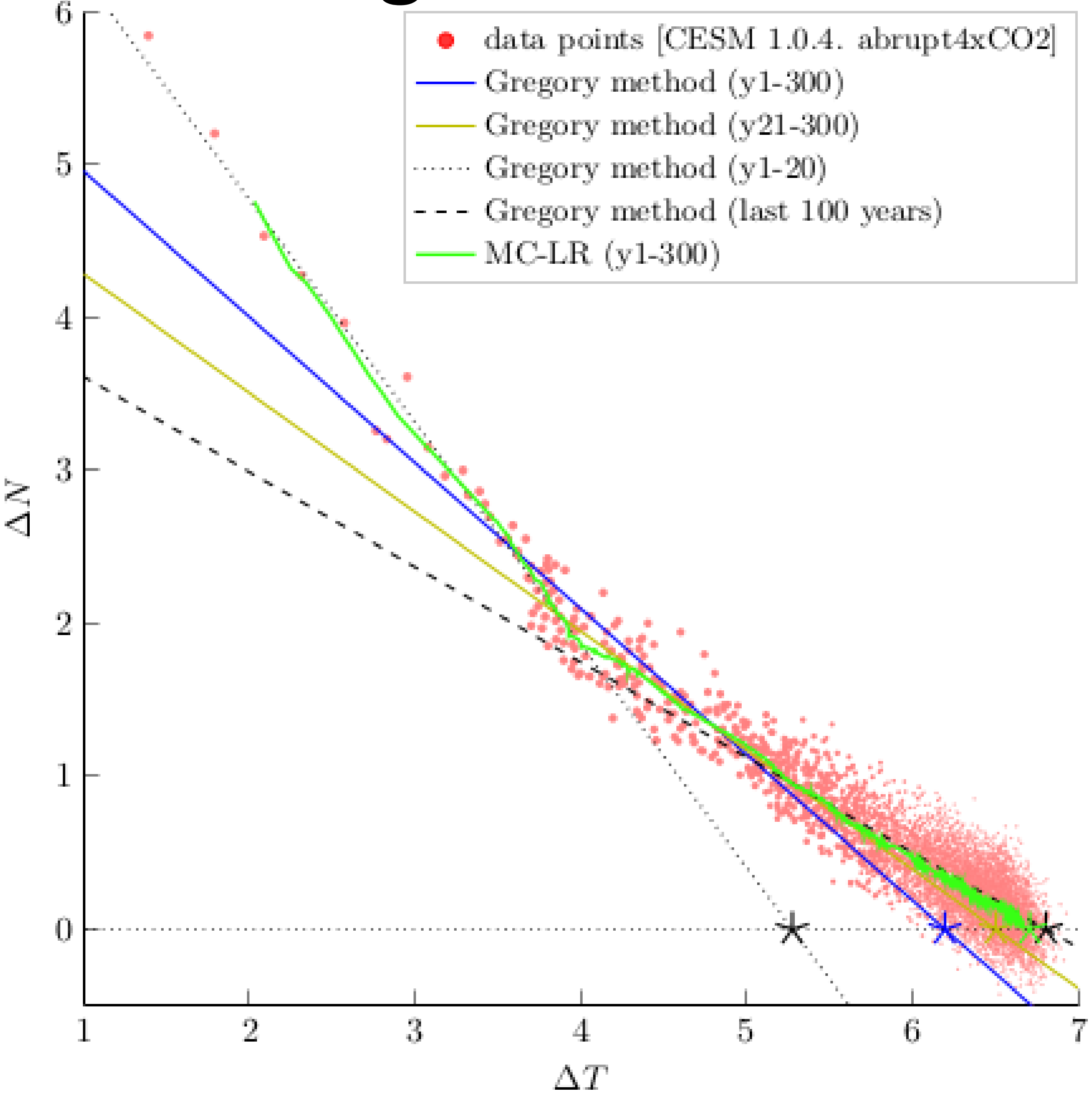
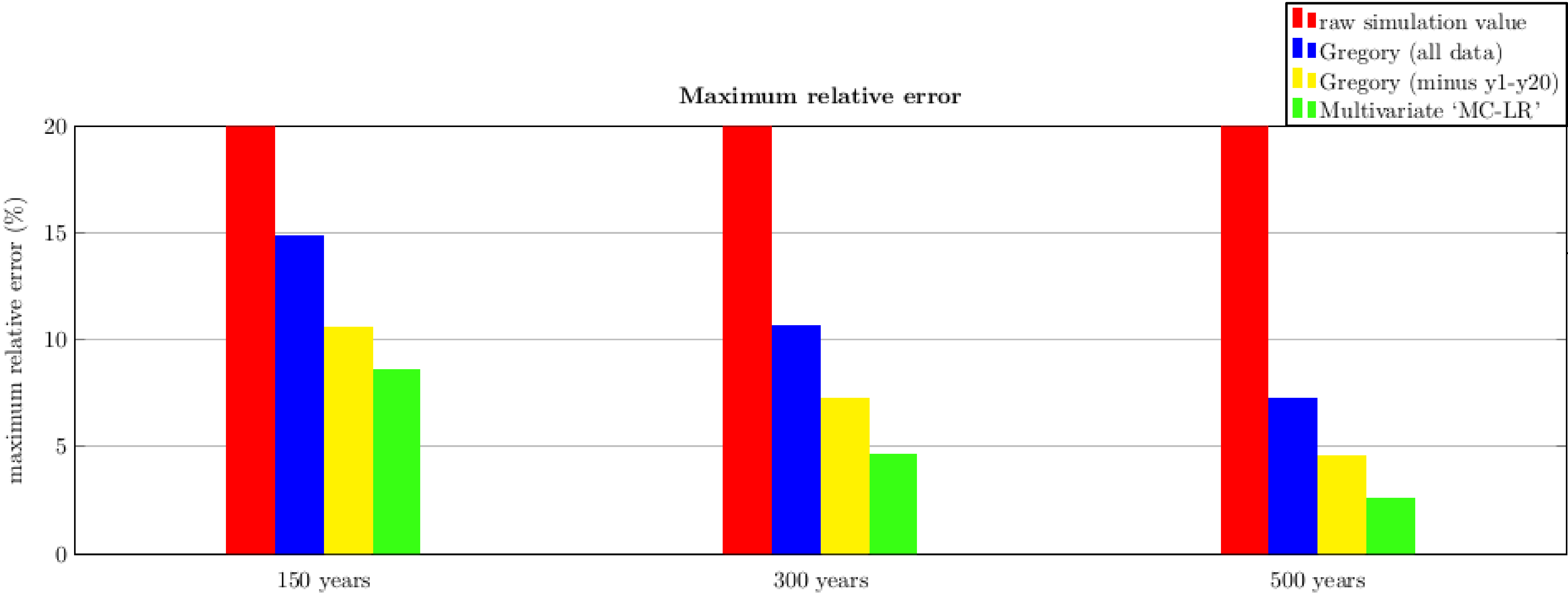


[Bastiaansen et al, 2021]

Other Global Climate Models



Computed as average of 11 millenia-long runs from LongRunMIP [Rugenstein et al, 2019]

[Bastiaansen et al, 2021]

Multivariate Estimations of Equilibrium Sensitivity

SUMMARY OF METHOD

Multicomponent Linear Regression

$$\overrightarrow{\Delta Y} = \boldsymbol{A}\,\overrightarrow{\Delta X} + \overrightarrow{\boldsymbol{F}}$$

Multivariate Estimate

$$\overrightarrow{\Delta X}_{*}^{est} = -\boldsymbol{A}^{-1}\,\overrightarrow{\boldsymbol{F}}$$

- ☻ More accurate estimates from short transient warming simulations

CAVEAT:

- ☹ Finding observables is an art

Part B:
Evolution of Climate Feedbacks

Climate Feedback Contributions

Back to

$$\Delta N(t) = F(t) + \Delta R(t)$$

Climate Response ΔR is sum of feedback contributions:

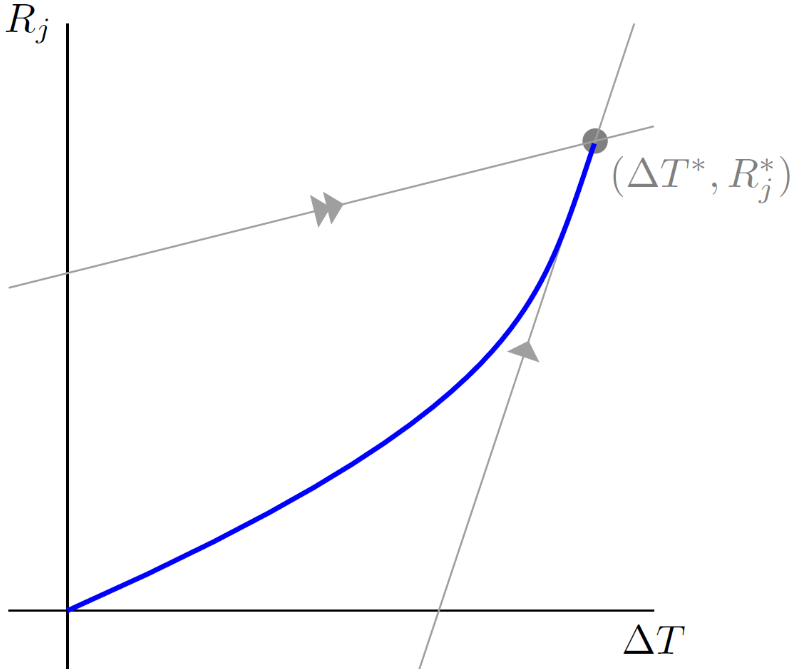
$$\Delta R(t) = \sum_{j \in \mathcal{F}} \Delta R_j(t)$$

$\mathcal{F}$ is set of **Climate Feedbacks:**

- Planck Feedback
- Lapse Rate Feedback
- Surface Albedo Feedback
- Water Vapour Feedback
- Cloud Feedback

Classical treatment of feedbacks:

$$\Delta R_j(t) = \lambda_j \, \Delta T(t)$$



Evolution of Observables

$$\frac{dO}{dt} = \mathcal{L}O + g(t)$$

Linear Response Theory (& Koopman Theory):

$$\Delta O(t) = \left(G^{[O]} * g\right)(t) = \int_0^t G^{[O]}(s)\, g(t-s)\, ds$$

Approximation of Green Function:

$$G^{[O]}(t) = \sum_{m=1}^{M} \beta_m^{[O]}\, e^{-t/\tau_m}$$

So:

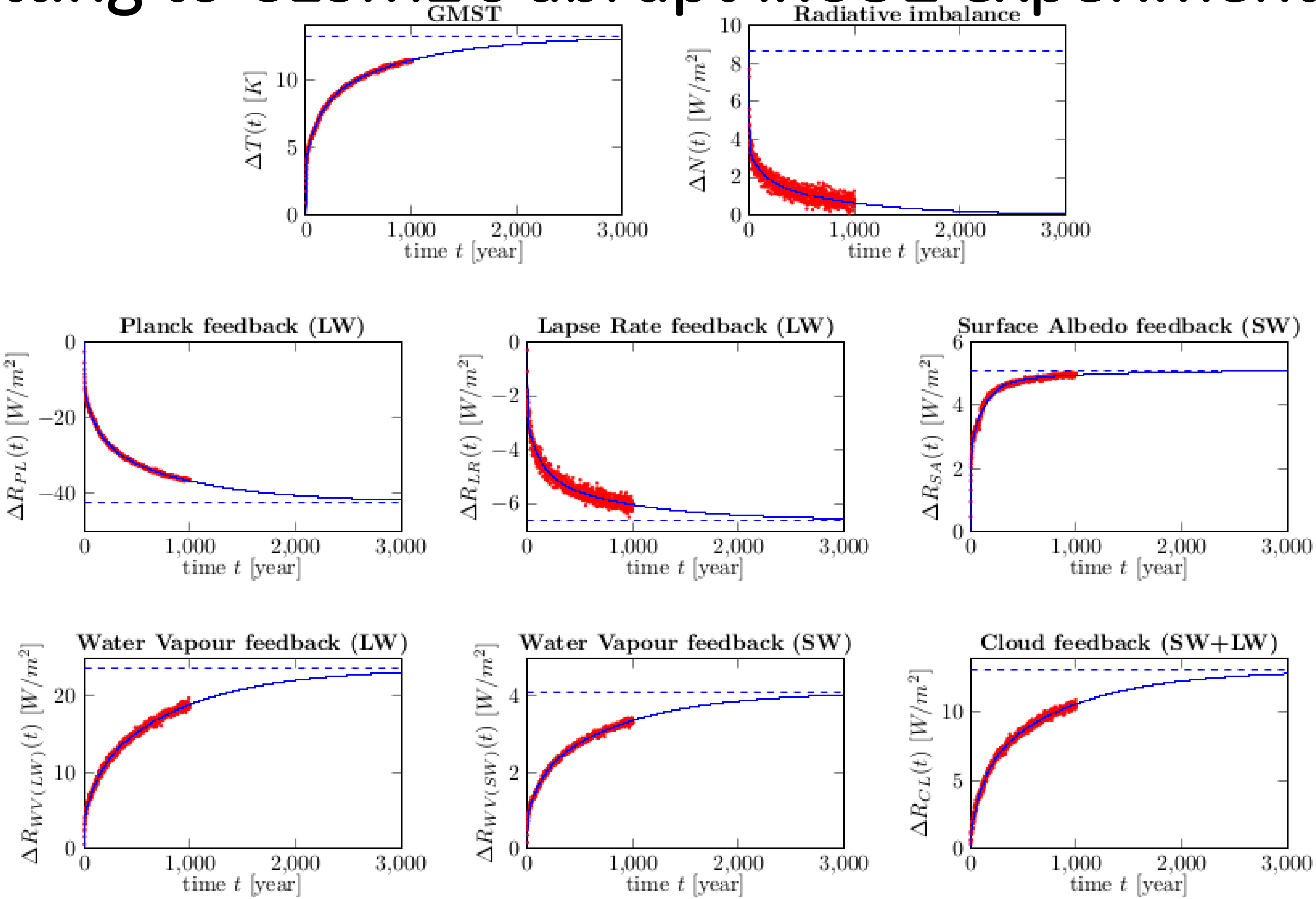
$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]}\, \mathcal{M}_m^{g}(t)$$

$$\mathcal{M}_m^{g}(t) = \int_0^t e^{-s/\tau_m}\, g(t-s) ds$$

all
observable
dependency

all
forcing (and time)
dependency

Fitting to CESM2’s abrupt4xCO2 experiment



Analysis of CESM2's abrupt4xCO2 experiment

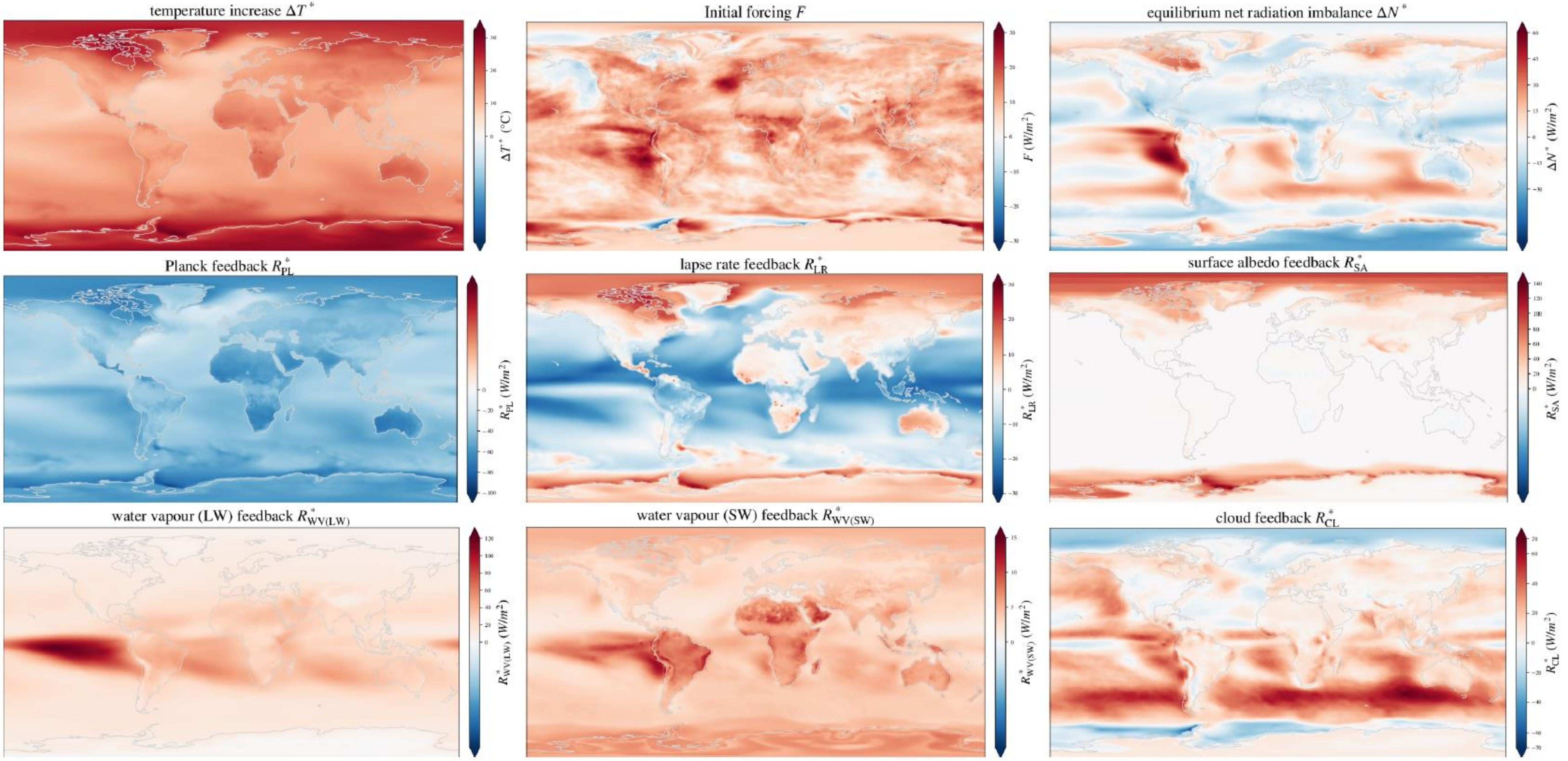
$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]} \mathcal{M}_m^g(t)$$

Define feedback strength per mode $\lambda_j^m := \frac{\beta_m^{[R_j]}}{\beta_m^{[T]}}$

	Mode 1	Mode 2	Mode 3	Equilibrium
τ_m	4.5 (± 0.1)	127 (± 3.8)	889 (± 50)	-
λ_m	−1.28 (± 0.08)	−0.38 (± 0.03)	−0.37 (± 0.02)	−0.66 (± 0.03)
Planck (LW)	−3.16 (± 0.02)	−3.24 (± 0.02)	−3.23 (± 0.01)	−3.21 (± 0.05)
Lapse Rate (LW)	−0.73 (± 0.03)	−0.50 (± 0.03)	−0.32 (± 0.03)	−0.50 (± 0.01)
Surface Albedo (SW)	+0.62 (± 0.04)	+0.56 (± 0.02)	+0.08 (± 0.10)	+0.39 (± 0.01)
Water Vapour (LW)	+0.97 (± 0.03)	+1.38 (± 0.02)	+2.71 (± 0.01)	+1.79 (± 0.04)
Water Vapour (SW)	+0.21 (± 0.09)	+0.26 (± 0.05)	+0.43 (± 0.02)	+0.31 (± 0.01)
Clouds (SW + LW)	+0.27 (± 0.36)	+1.19 (± 0.02)	+1.43 (± 0.01)	+1.00 (± 0.03)
sum	−1.82 (± 0.37)	−0.36 (± 0.07)	+1.09 (± 0.11)	−0.22 (± 0.08)
missing	+0.54 (± 0.38)	−0.02 (± 0.08)	−1.46 (± 0.11)	−0.43 (± 0.08)

Spatial Response – Equilibrium Estimates

$$\Delta O(\boldsymbol{r},t) = \sum_{m=1}^{M} \beta_m^{[O]}(\boldsymbol{r})\ \mathcal{M}_m^{g}(t)$$



Projections for other forcings

$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]} \mathcal{M}_m^g(t)$$

$$\mathcal{M}_m^g(t) = \int_0^t e^{-s/\tau_m} g(t-s)ds$$

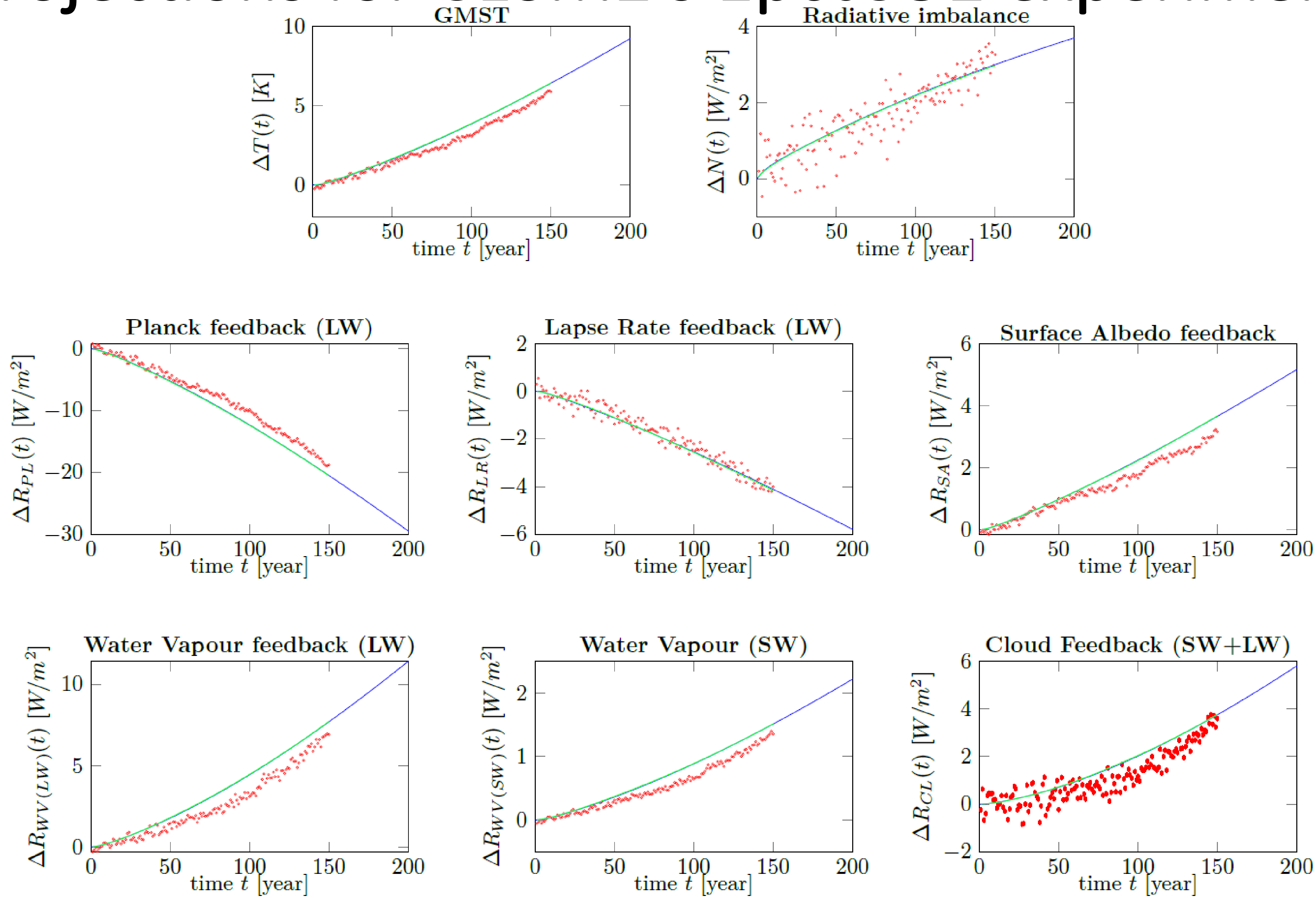
only this gets changed!

Linear Response Theory – CAVEATS:

i. forcings & responses should be ‘small enough’

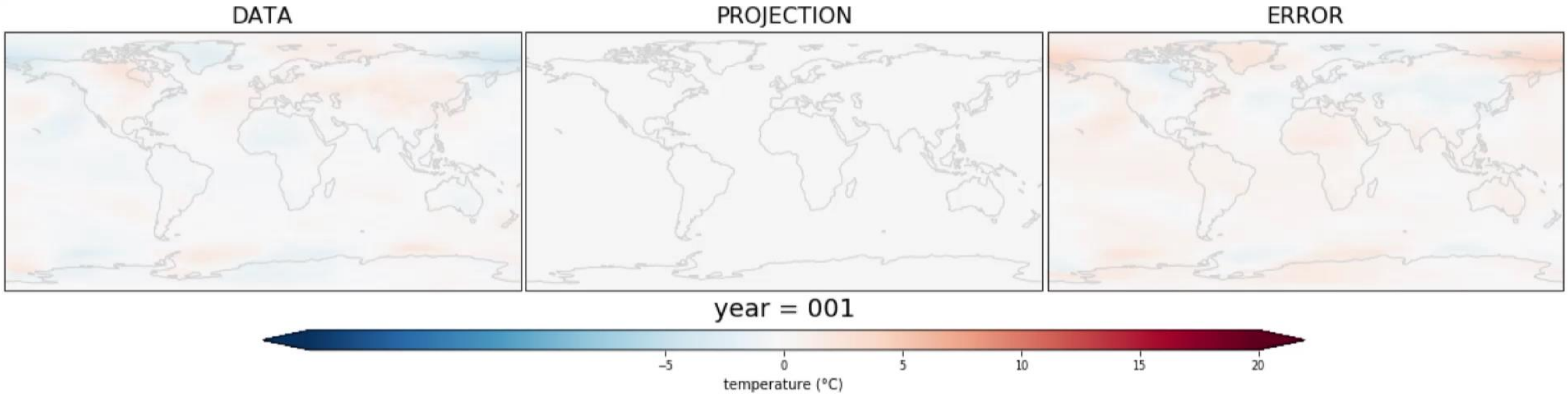
ii. should look at ensemble means

Projections for CESM2's 1pctCO2 experiment



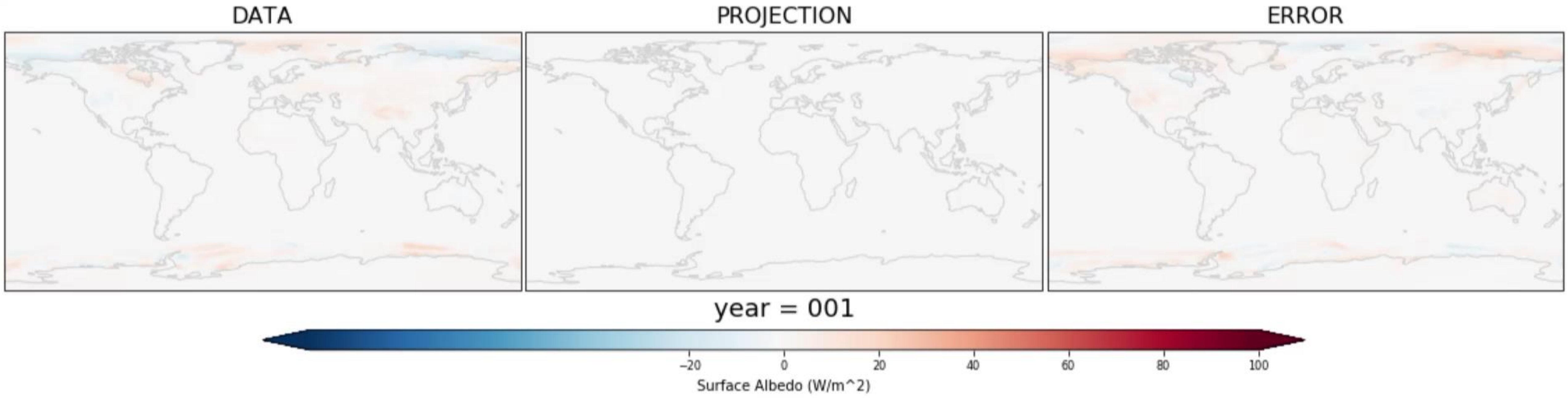
Spatial projections for 1%CO2 experiment

TEMPERATURE



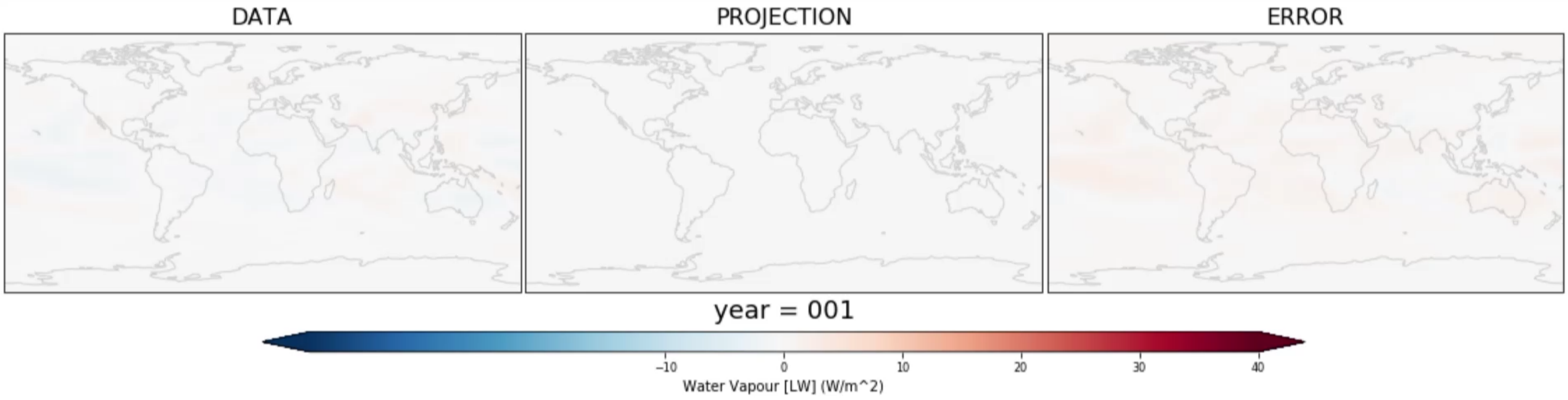
Spatial projections for 1%CO2 experiment

SURFACE ALBEDO FEEDBACK CONTRIBUTION



Spatial projections for 1%CO2 experiment

WATER VAPOUR (LW) FEEDBACK CONTRIBUTION



Projections of Climate Feedbacks

SUMMARY OF METHOD

Evolution of observable

$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]} \mathcal{M}_m^g(t)$$

$$\mathcal{M}_m^g(t) = \int_0^t e^{-s/\tau_m} g(t-s) ds$$

- Allows dissecting feedback contributions over time
  - Feedback strength per mode
  - Feedback missing on long time scale?

- Projections for other forcings possible via Linear Response Theory
  - Tests presented are promising

Summary

Assumption 1:
approximately a linear system
$$O'(t) = \mathcal{L}O + g(t)$$

~~**Assumption 2:**
only one mode relevant~~
$$\Delta O = C^{[O]}\,\Delta T$$

- ☹ Univariate
- ☹ State dependence is ignored

MULTIVARIATE METHODS

A: Multivariate Estimation of ECS

$$\overrightarrow{\Delta Y} = \boldsymbol{A}\,\overrightarrow{\Delta X} + \vec{\boldsymbol{F}}$$

[Bastiaansen et al, 2021]

B: Projections of Climate Feedbacks

$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]}\,\mathcal{M}_m^{g}(t)$$

[Bastiaansen et al, *in progress*]

- 😍 More accurate long-term estimates
- 😍 Better view on *how* the climate will be different