TIPPING POINTS IN THE EARTH SYSTEM

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Multivariate estimations of Equilibrium Climate Sensitivity

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Current work

- Postdoc @ Utrecht University

  (with Anna von der Heydt & Henk Dijkstra)

- Within H2020 project TiPES: Tipping Points in the Earth System

- Work on **Climate Sensitivity**:

  If we increase the atmospheric CO_2 concentration,
  how much warmer does the Earth get?

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Climate response and sensivitity metrics

Climate response is the change (response) in an observable due to increase in forcing

Two common metrics:

- **Equilibrium Climate Sensitivity**: change in equilibrium temperature due to (instantaenous) doubling of atmospheric CO2

- **Transient Climate Response**: change in temperature after 100 years with 1% CO2 increase per year (until doubling)

Equilibrium Climate Sensitivity

- Derived from dedicated experiments with climate models
  - Start from equilibrium with pre-industrial levels of CO2
  - Instantaneous increase in CO2
  - Monitor change in observables compared to a control run
- However: equilibrating climate models takes very, very long
- Need for techniques to estimate equilibrium temperature
- Mathematical context: $y' = f(y;\ \mu)$

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Idea behind warming estimation techniques

Warming is due to net positive radiative imbalance

$$N = N_{SW,\downarrow} - N_{SW,\uparrow} - N_{LW,\uparrow}$$

incoming sunlight

reflected sunlight

Planck radiation

When $N = 0$ no more warming:

→ equilibrium warming $\Delta T_* = T_* - T_0$

Basic idea of Gregory method

Express imbalance as function of system state

$$N(t) = N(y(t))$$

Close to equilibrium y_*, Taylor expansion gives approximation

$$N(t) = N(y_*) + \sum_{j \in \mathcal{F}} \frac{\partial N}{\partial y_j}(y_*) \left[y_j(t) - y_{j*}\right]$$

Close to equilibrium, state variables $y_j(t) = y_j\big(T(t)\big)$

Thus, another Taylor expansion yields

$$N(t) = \left\{\frac{\partial N}{\partial T} + \sum_{j \in \mathcal{F}} \frac{\partial N}{\partial y_j} \frac{\partial y_j}{\partial T}\right\} [T(t) - T_*]$$

Rewriting $T = T_0 + \Delta T$ gives:

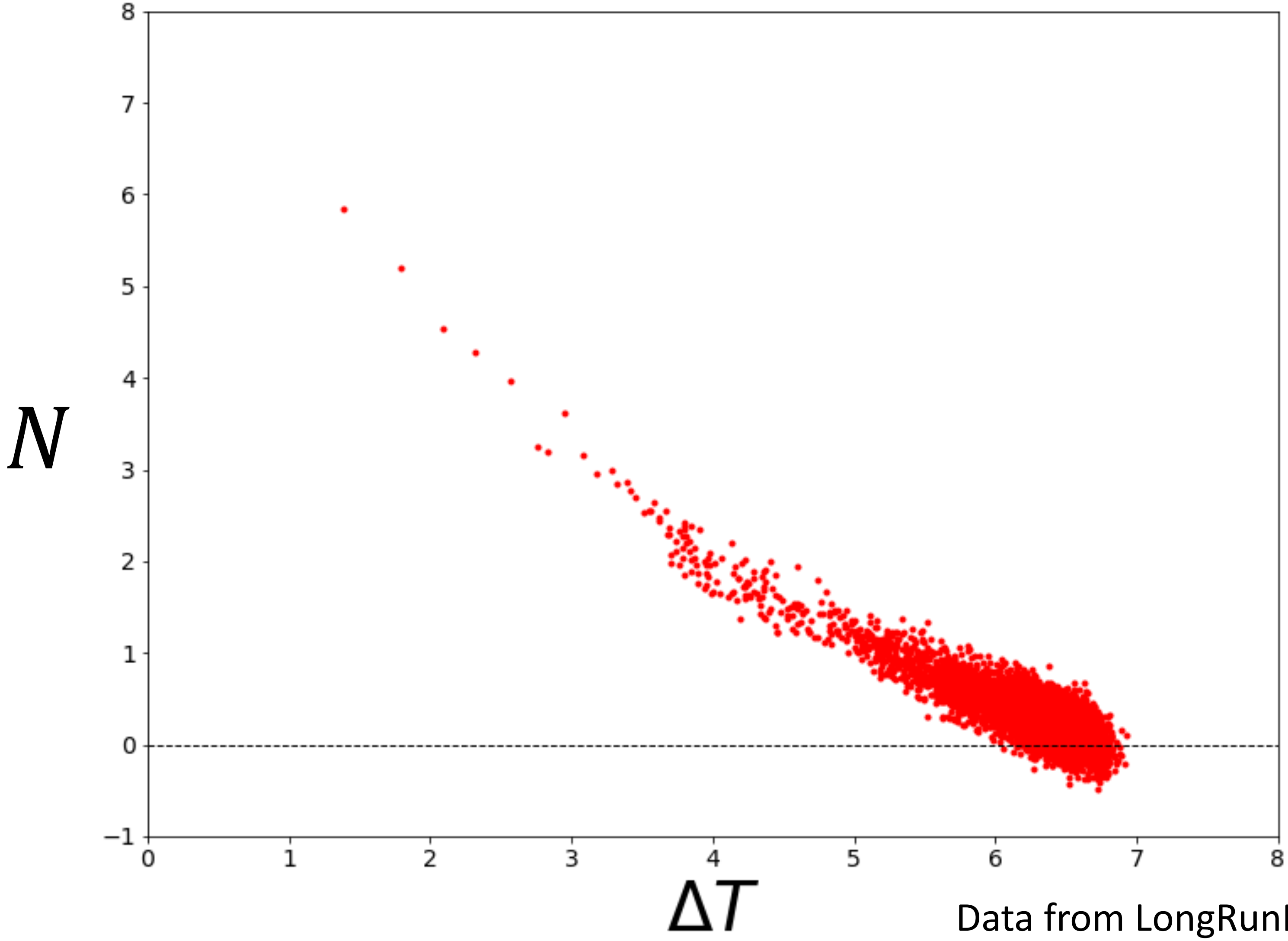
$$N(t) = a\,\Delta T(t) - a\,\Delta T_*$$

Linear regression on data:

$$N(t) = a\,\Delta T(t) + f \;\rightarrow\; \Delta T_*^{est} = -\,a^{-1}\,f$$

[Gregory et al (2004)]

Gregory Plot for CESM 1.0.4



Data from LongRunMIP; [Rugenstein et al (2019)]

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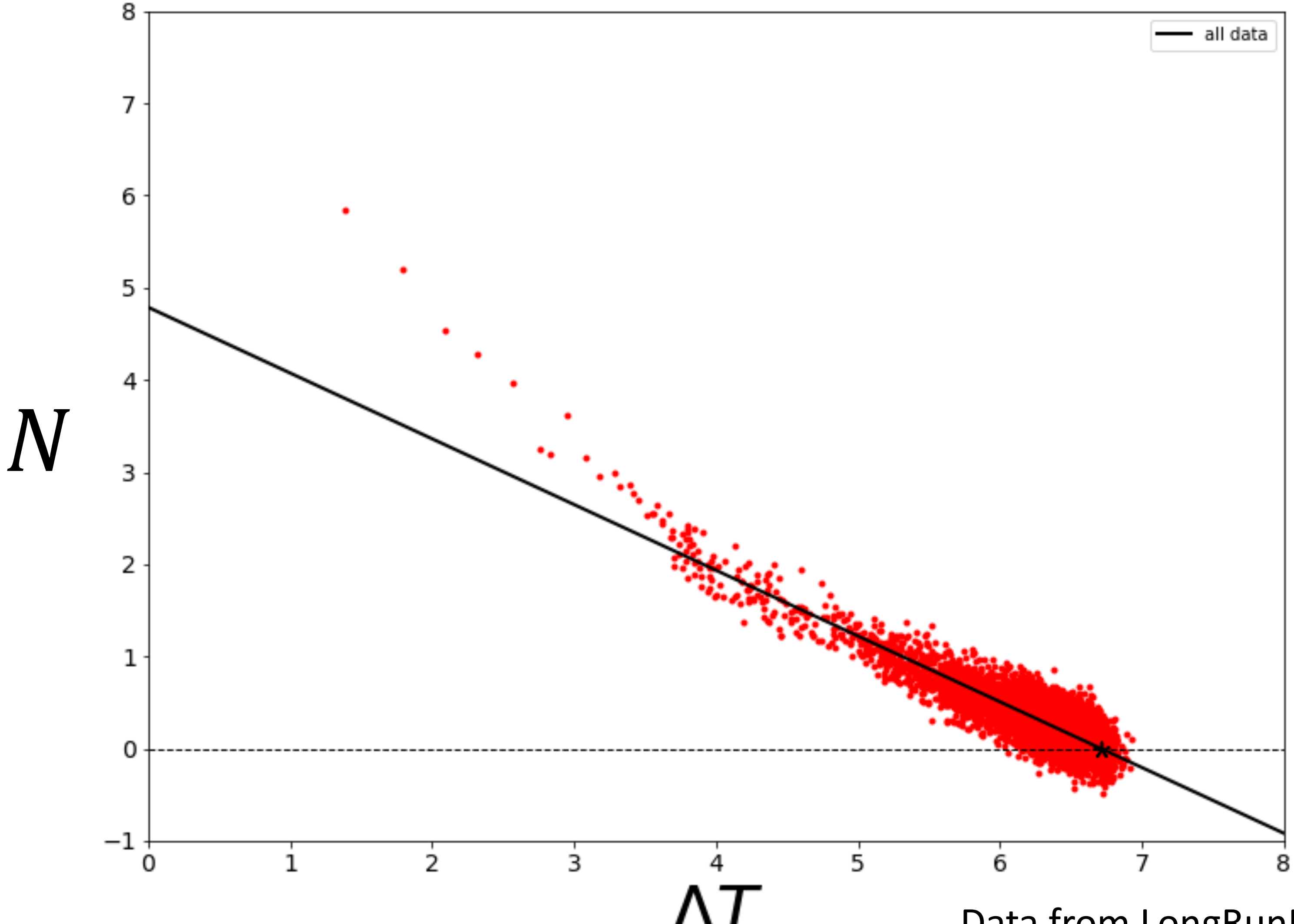

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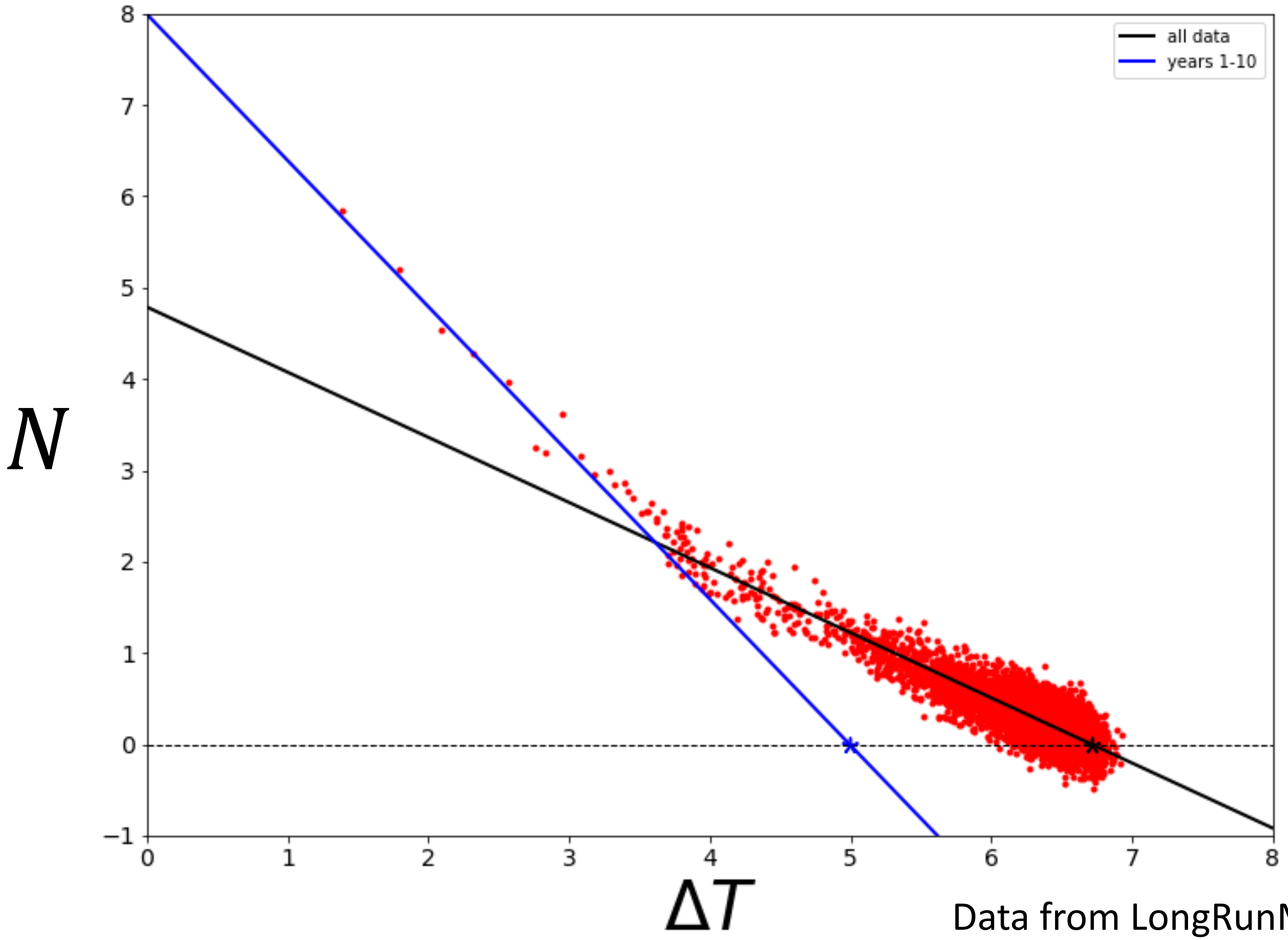

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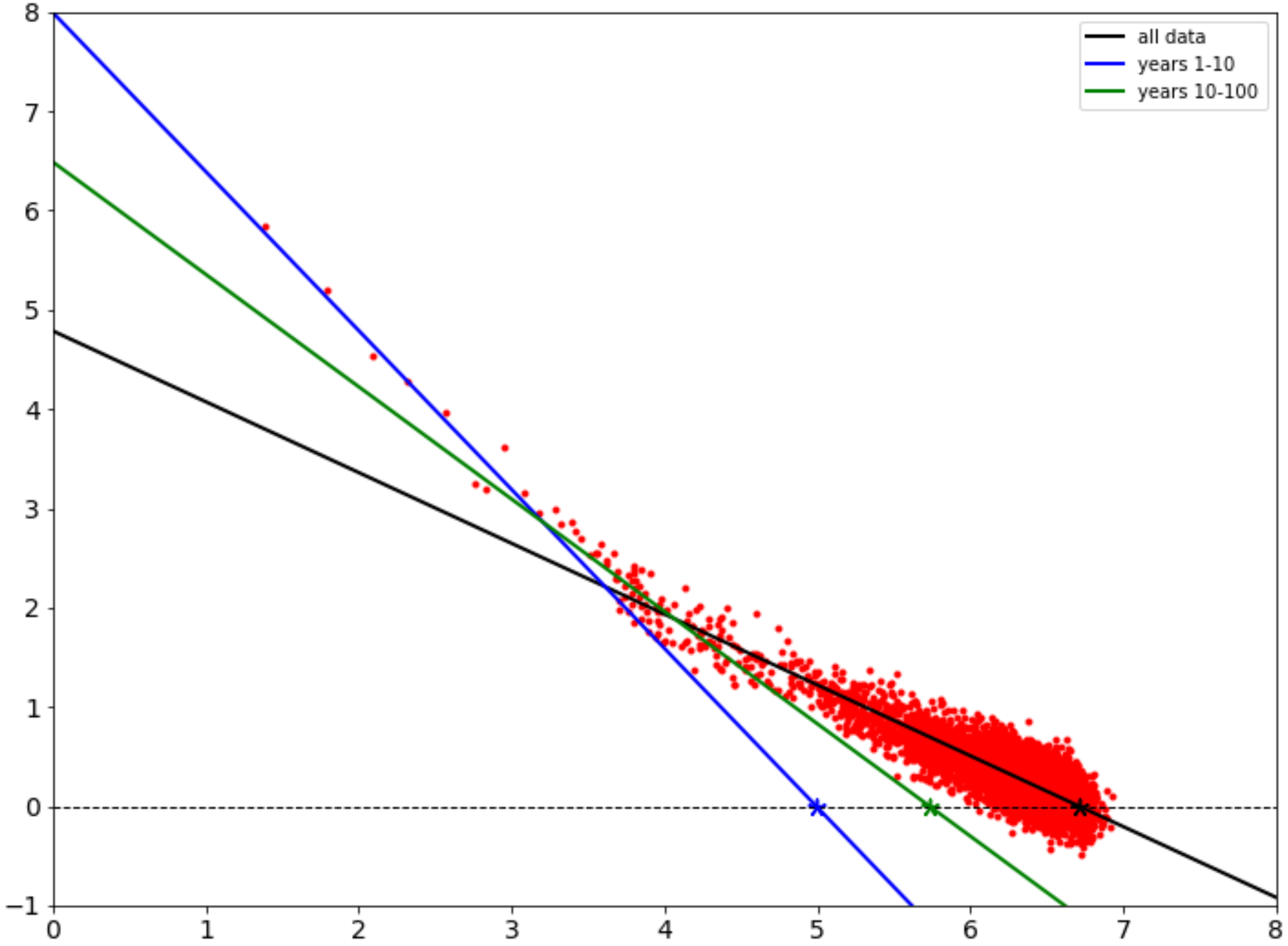
N



ΔT

Data from LongRunMIP; [Rugenstein et al (2019)]

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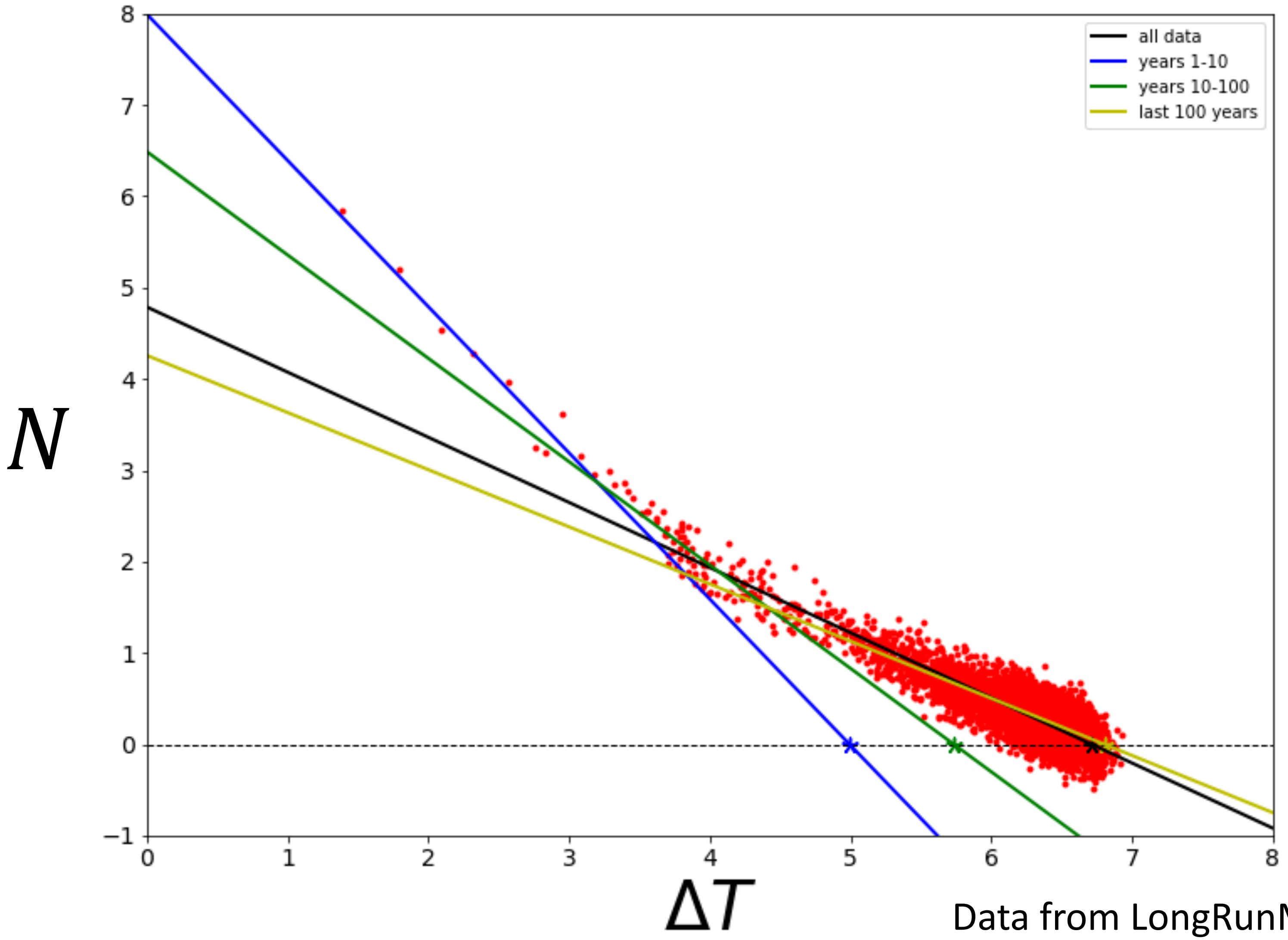

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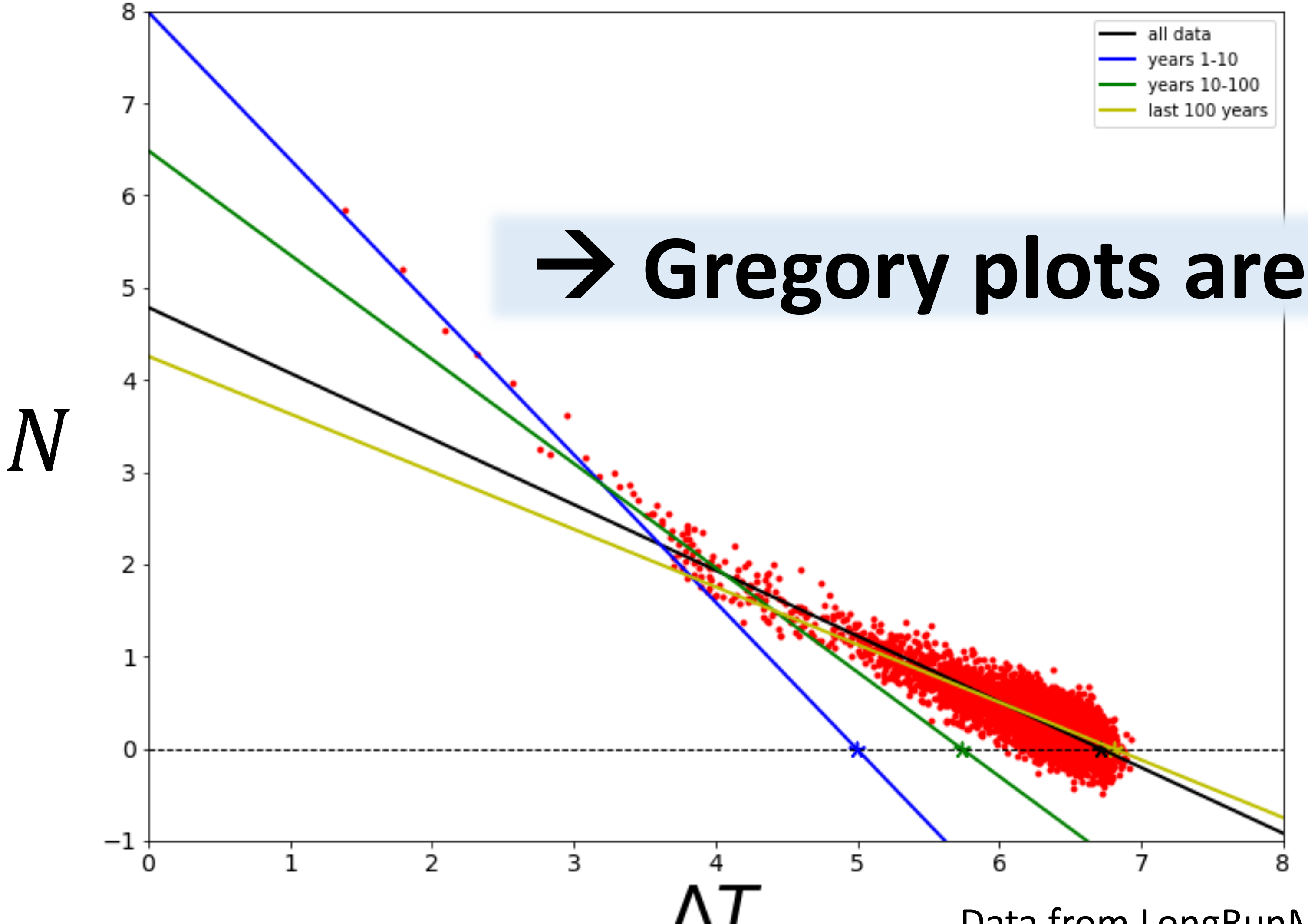


→ Gregory plots are non-linear!

Data from LongRunMIP; [Rugenstein et al (2019)]

Step back: dynamical system point of view

$y' = f(y)$

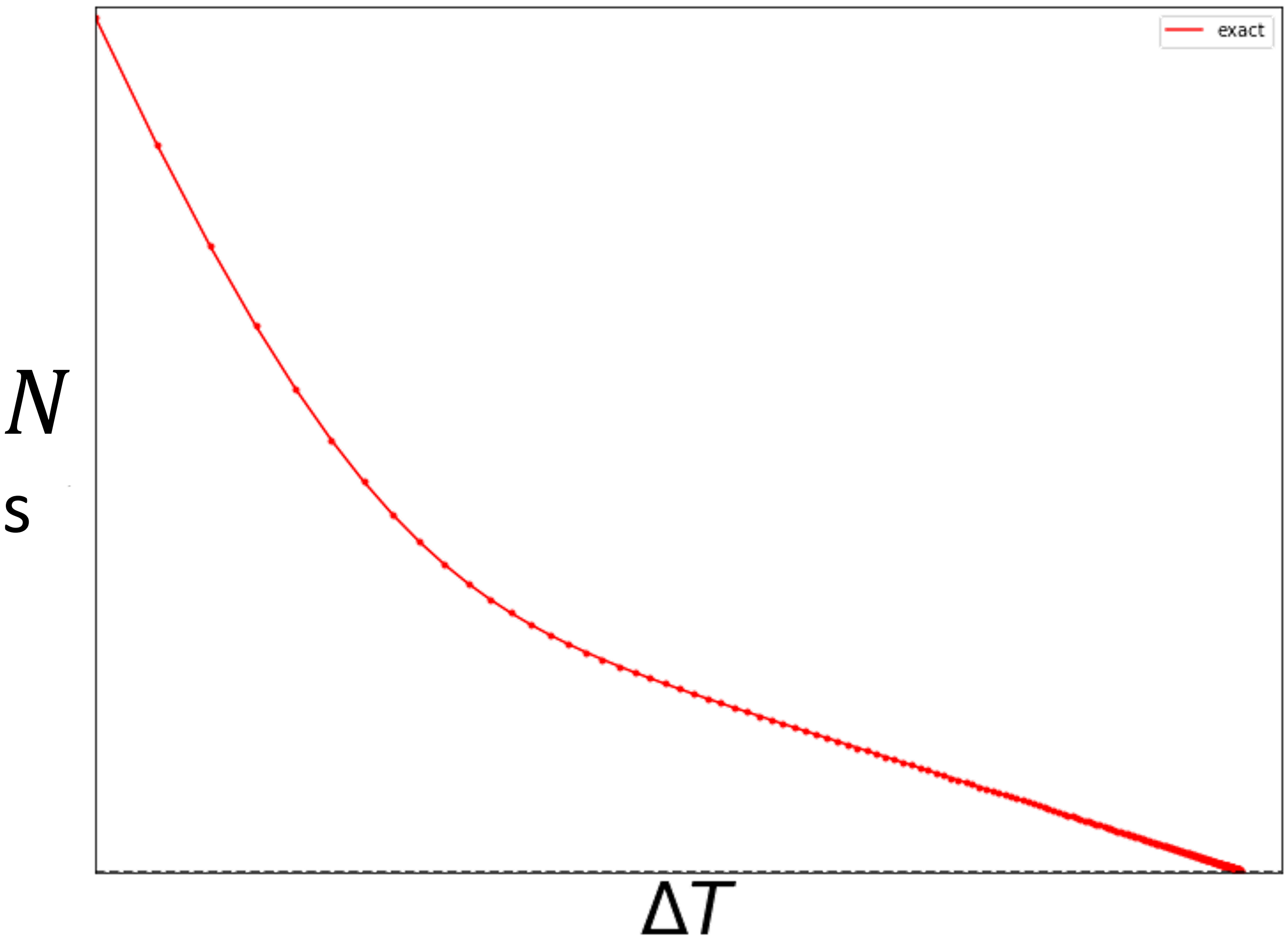
Close to equilibrium:

$\Delta y' = Df(y_*)\,\Delta y + F$

Solutions sum of exponentials

$$\Delta y(t) - \Delta y_* = \sum_{j=1}^{n} c_j v_j e^{\lambda t}$$

(v_j eigenvectors, λ_j eigenvalues of A)



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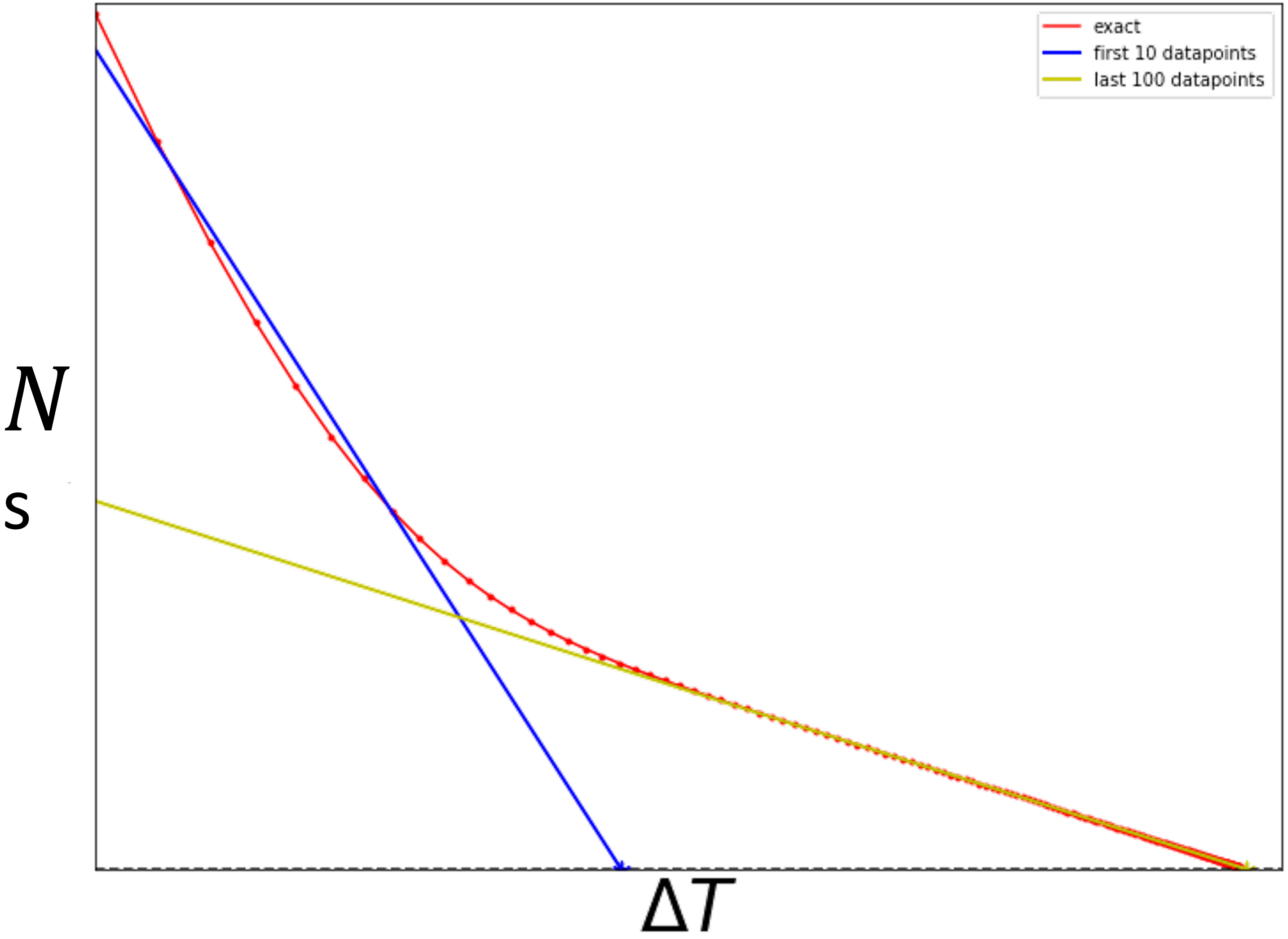

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Basic idea of Gregory method

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This is fine

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This is fine

This is not

We need as many observables as relevant eigenmodes

$$y_j(t) = y_j(T(t), \ldots)$$

The problem with classical method

In linear regime of decay to equilibrium:

$$N(t) - N_* = \sum_m \beta_m^{[N]} \, e^{\lambda_m t}$$

$$\Delta T(t) - \Delta T_* = \sum_m \beta_m^{[T]} \, e^{\lambda_m t}$$

If only one eigenmode present:

$$[N(t) - N_*] = {\beta_1^{[N]}}/{\beta_1^{[T]}} \, [\Delta T(t) - \Delta T_*]$$

Since $N_* = 0$ this leads to Gregory method:

$$N(t) = a \, \Delta T(t) + f$$

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Idea:

A Multi-Component Linear Regression (MC-LR):

$$Y = A\,X + F \rightarrow X_*^{est} = -A^{-1}\,F$$

Y:
M observables that tend to 0 in equilibrium

X:
M observables that are estimated in equilibrium

Example:

$$\begin{bmatrix} N \\ \Delta\alpha' \\ \Delta\varepsilon' \end{bmatrix} = \mathrm{A} \begin{bmatrix} \Delta\mathrm{T} \\ \Delta\alpha \\ \Delta\varepsilon \end{bmatrix} + F$$

α : effective top-of-atmosphere short-wave albedo

$$\alpha = {}^{N_{SW,\uparrow}}\!/_{N_{SW,\downarrow}}$$

ε : effective top-of-atmosphere long-wave emissivity

$$\varepsilon = {}^{N_{LW,\uparrow}}\!/_{T^4}$$

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Toy system – Model Equations

$$\begin{cases} C_T \dfrac{dT}{dt} = Q_0\,(1 - \alpha) - \varepsilon\,\sigma\,T^4 + \mu \end{cases}$$

incoming sunlight

reflected sunlight

Planck radiation

effect of CO2

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Toy system – Model Equations

$$
\begin{cases}
C_T \dfrac{dT}{dt} = Q_0\,(1 - \alpha) - \varepsilon\,\sigma\,T^4 + \mu \\
\dfrac{d\alpha}{dt} = -\delta_\alpha\,[\alpha - \alpha_0(T)]; \\
\dfrac{d\varepsilon}{dt} = -\,\delta_\varepsilon\,[\varepsilon - \varepsilon_0(T)].
\end{cases}
$$

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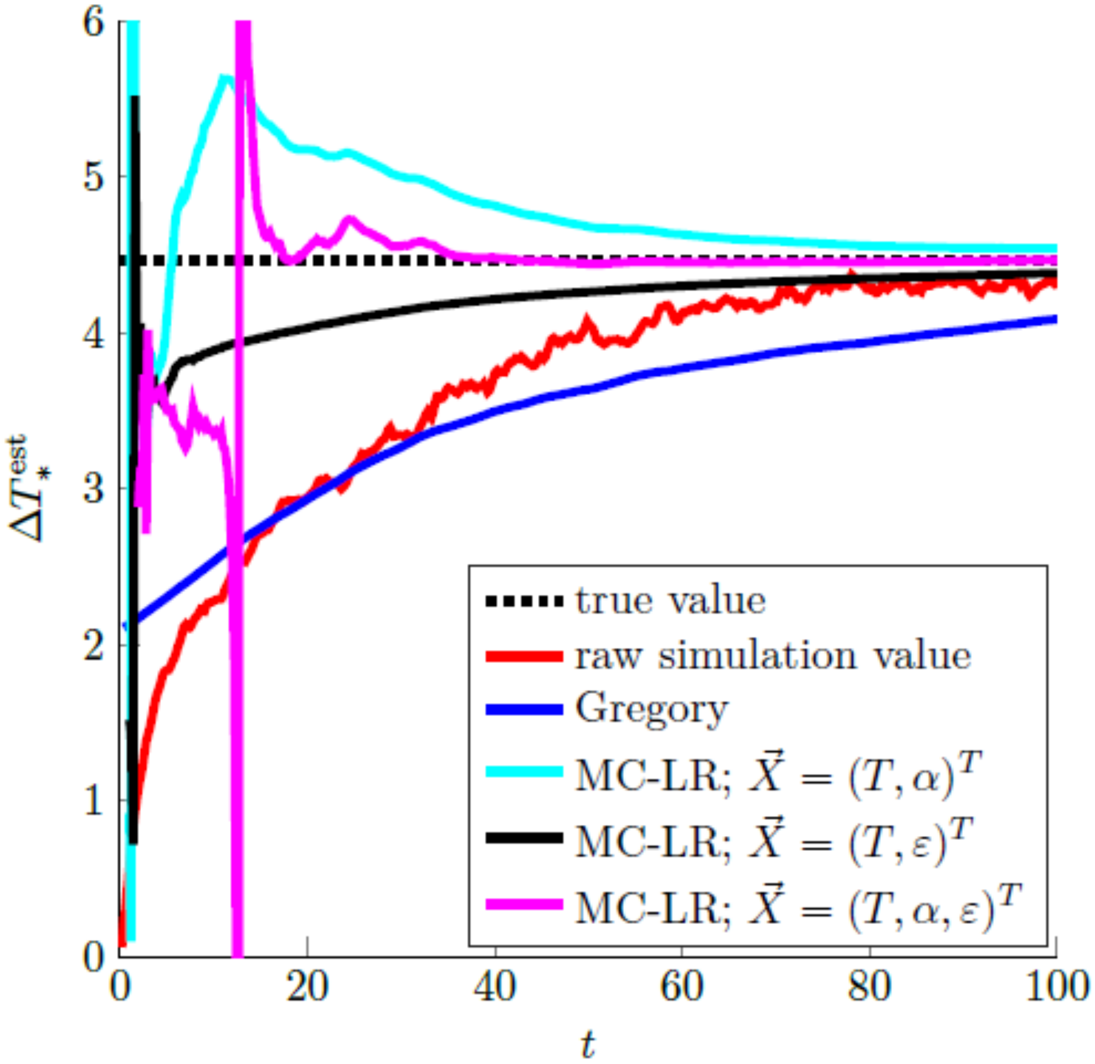
Toy system – Model Equations

$$\begin{cases} C_T \dfrac{dT}{dt} = Q_0\,(1 - \alpha) - \varepsilon\,\sigma\,T^4 + \mu + \nu\xi(t); \\ \dfrac{d\alpha}{dt} = -\delta_\alpha\,[\alpha - \alpha_0(T)]; \\ \dfrac{d\varepsilon}{dt} = -\,\delta_\varepsilon\,[\varepsilon - \varepsilon_0(T)]. \end{cases}$$

Toy system – Results



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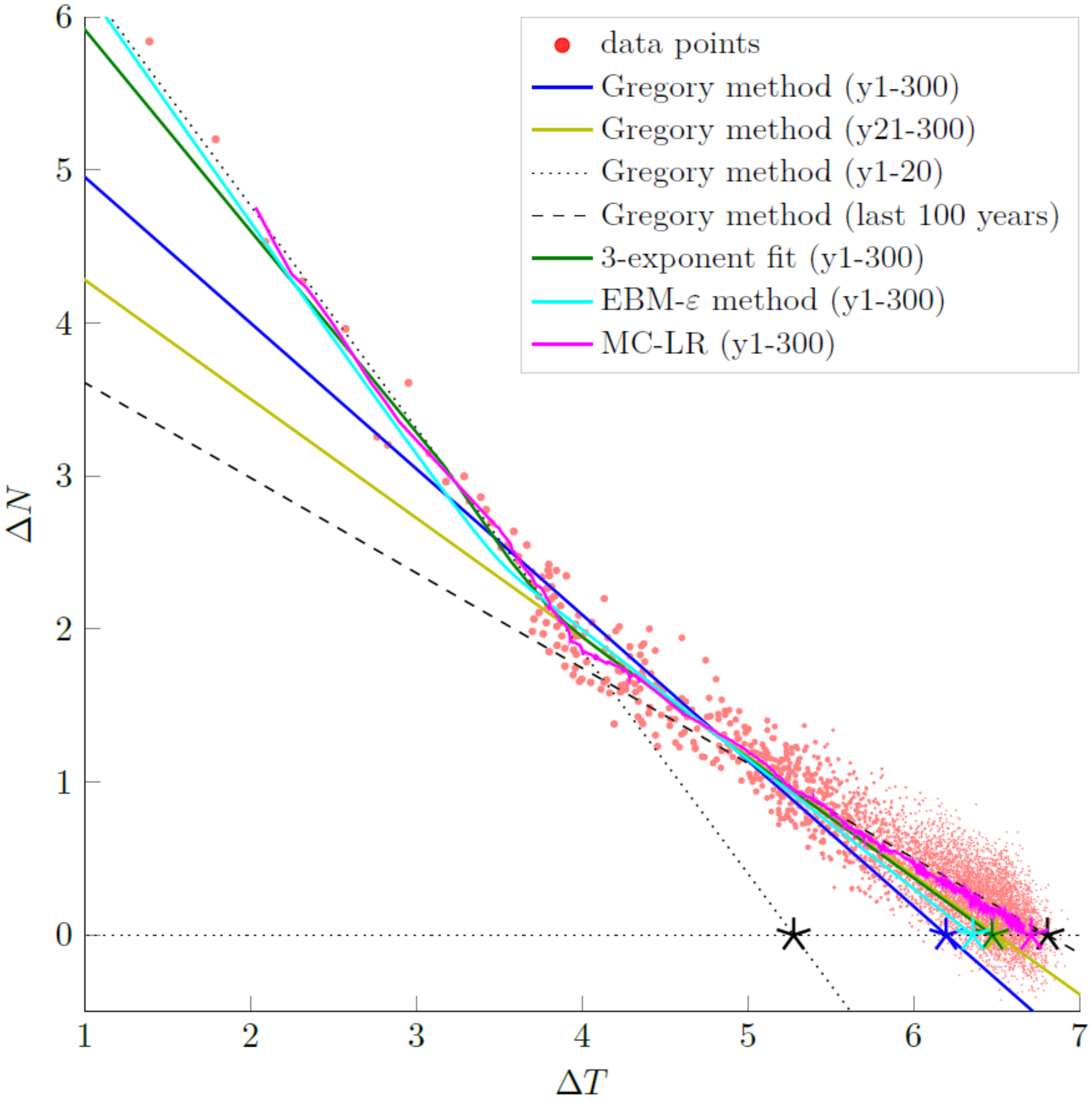
Testing on LongRunMIP data

- Models run to 'equilibrium' (practice: runs of at least 1,000 years)
- Work with 'abrupt-4xCO2' forcing experiments

Experiment

- Run estimation technique with data up to time t
- Compare with 'equilibrium value'
- Determine effectiveness of techniques for time frame



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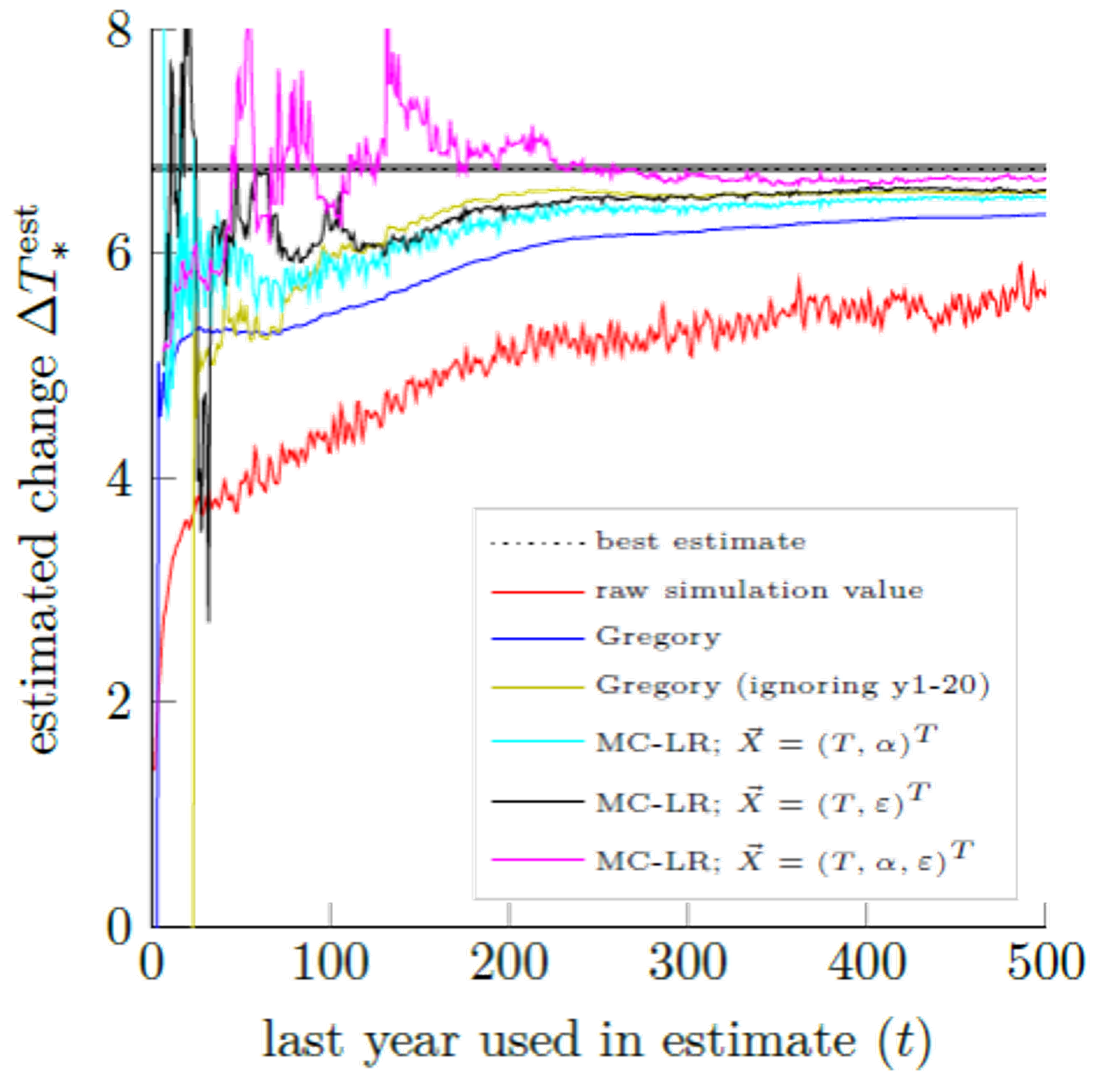
Results for CESM 1.0.4



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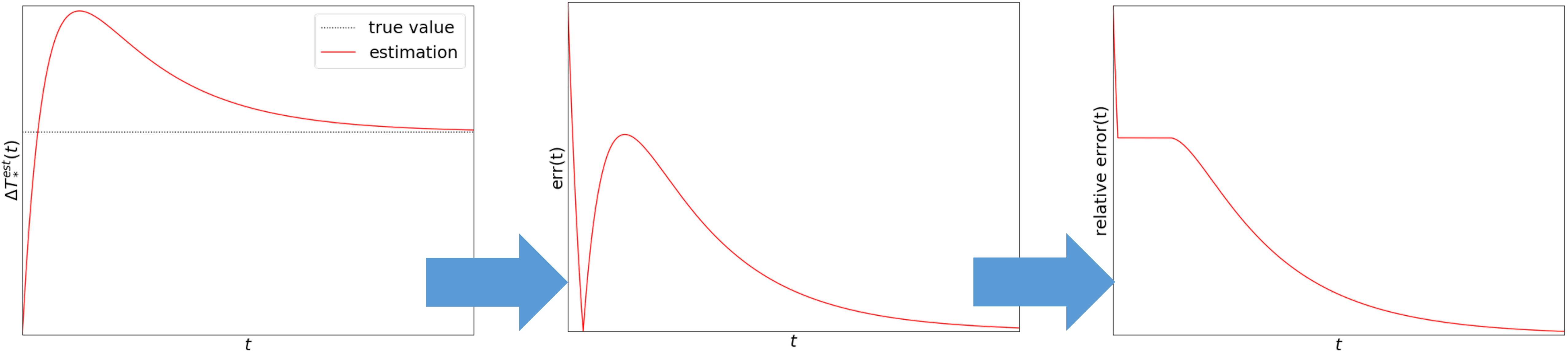
Measure for effectiveness

- Denote ‘equilibrium’ warming by ΔT_*
- Measure for maximum of relative error one ought to expect

$$\text{relative error } (t) := \max_{s\,\geq t} \left| \frac{\Delta T_*^{est}(s) - \Delta T_*}{\Delta T_*^{est}(s)} \right|$$



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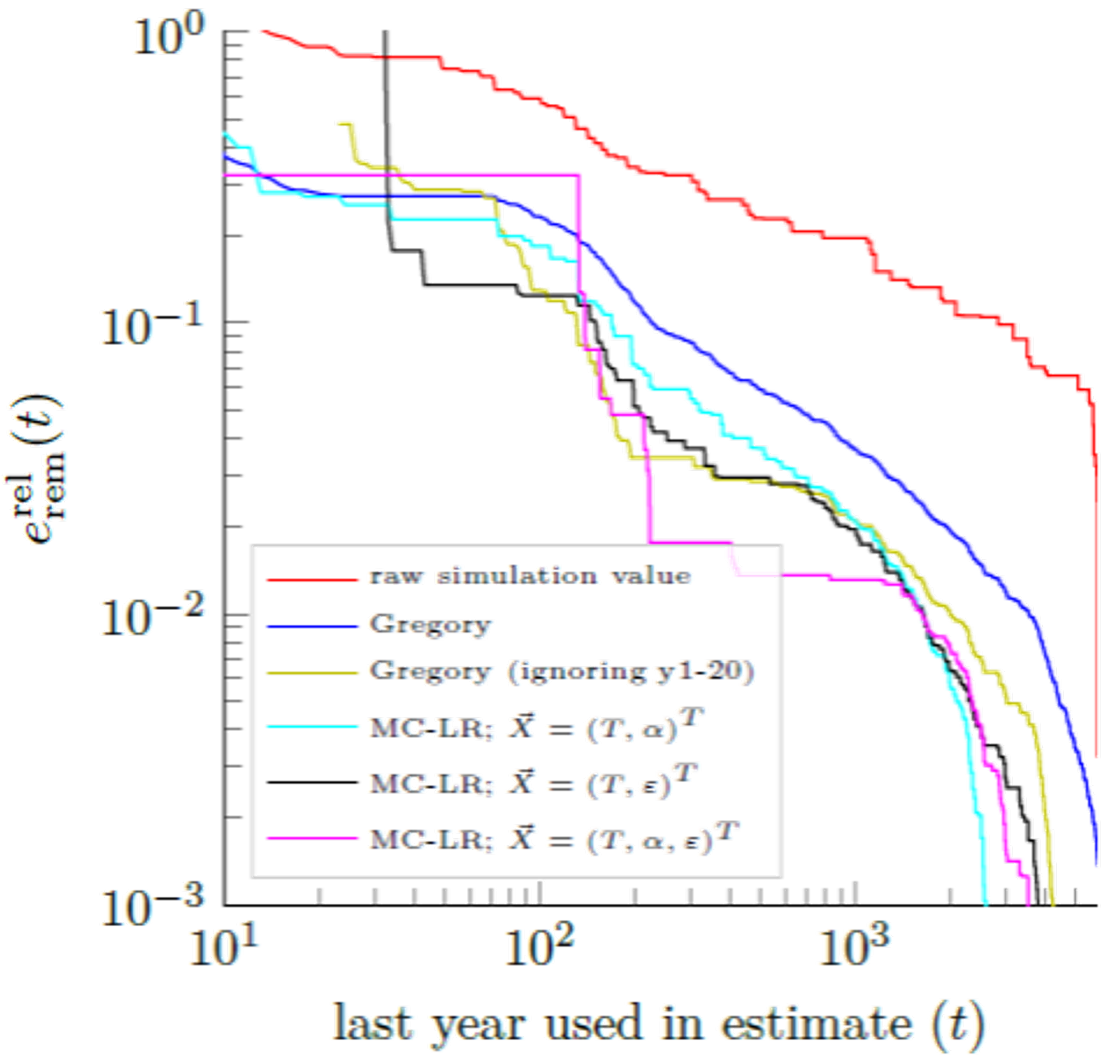

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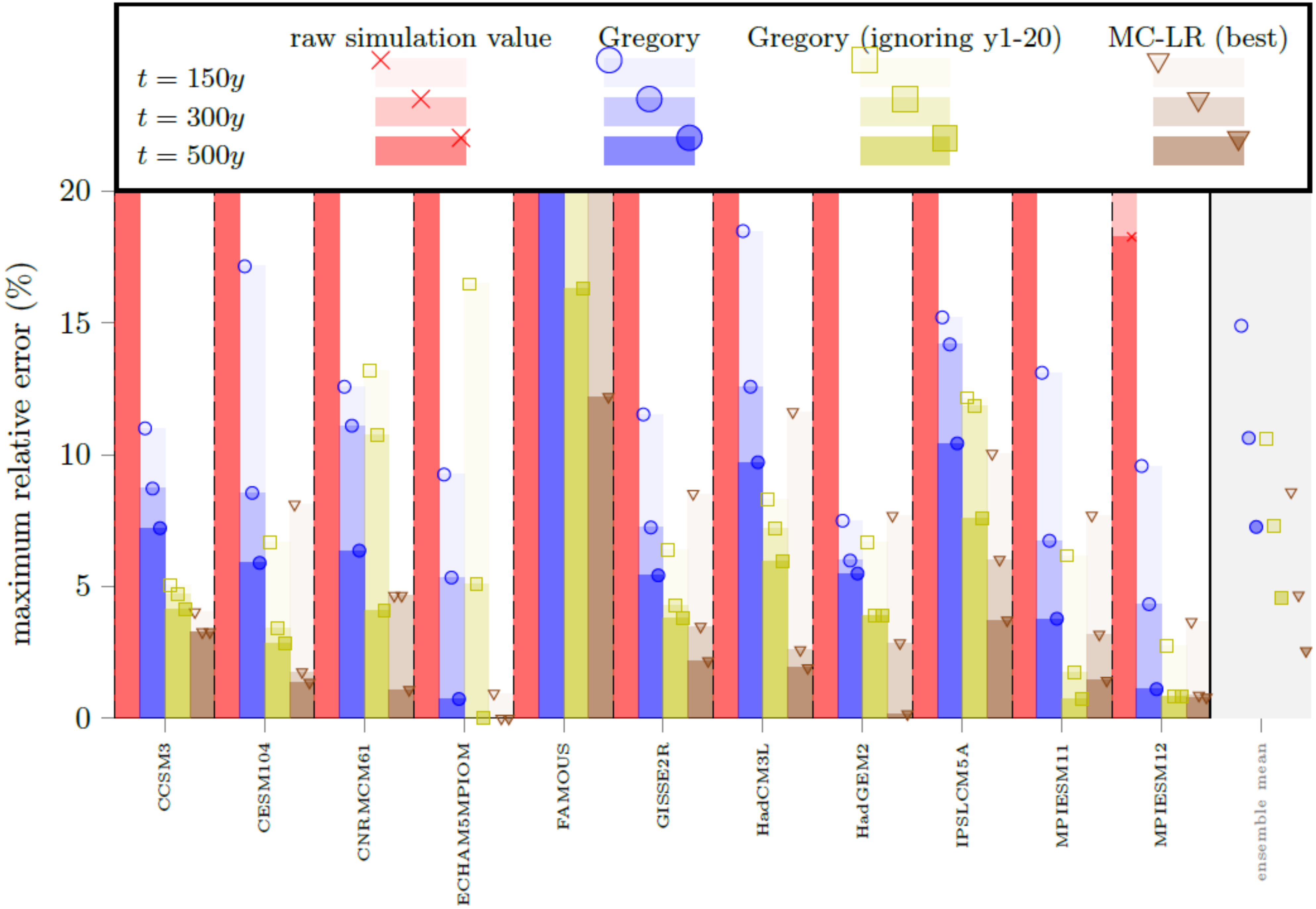


Conclusions

- Multi-component regression $Y = A\,X + F$ yields better results
    - especially for t > 150 years
    - depends on model characteristics

- Multivariate estimate $X_*^{est} = -A^{-1}F$ contains more than temperature
    - useful for estimating projections of climate subsystems

- Potential improvements:
    - more curated observables
    - dedicated ensemble of simulations

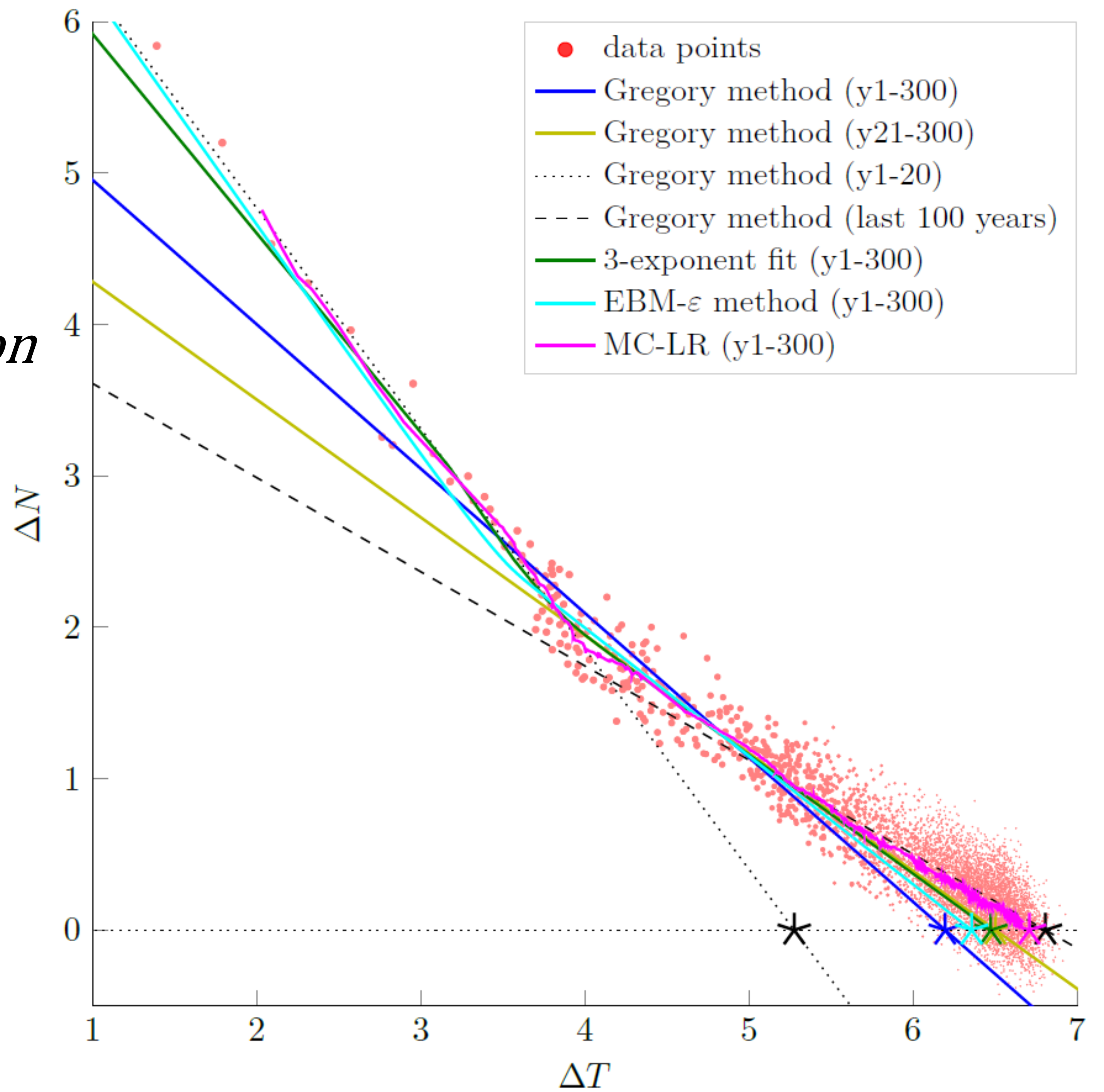
Multi-Component Linear Regression

$$Y = A\,X + F$$

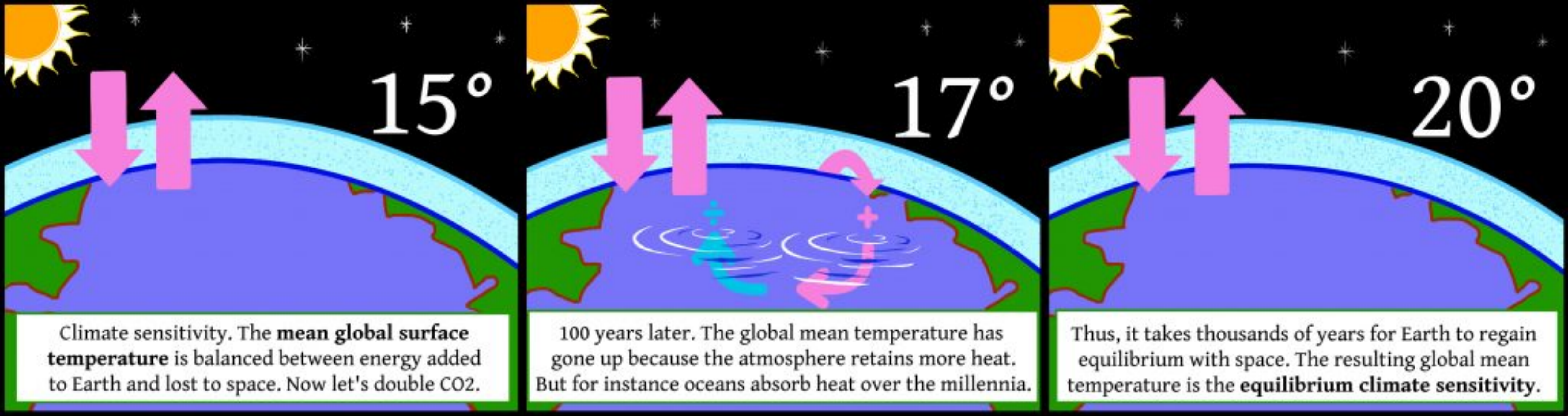
leads to

Multivariate Estimate

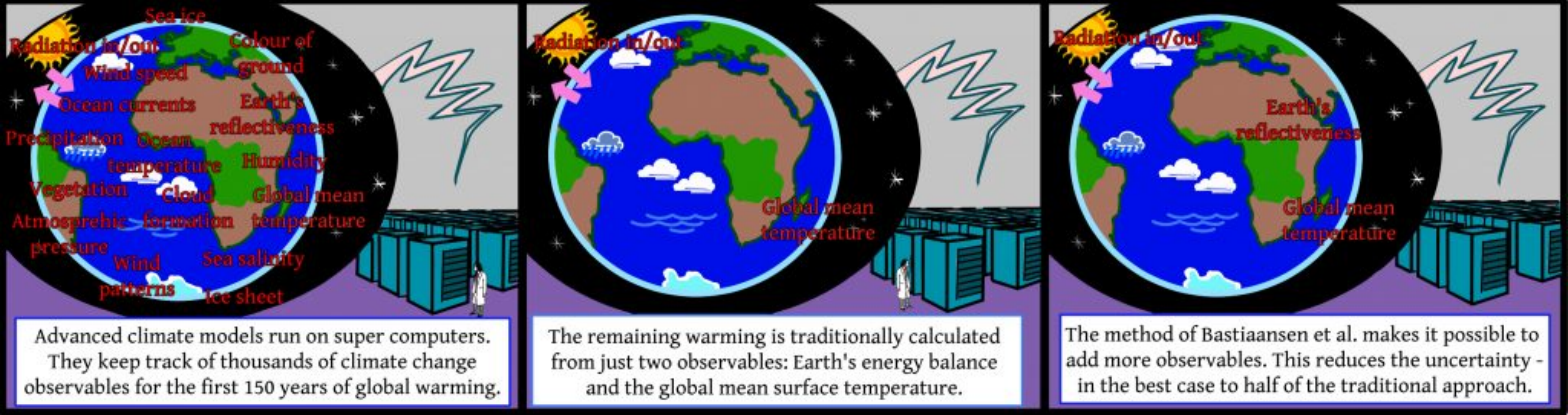
$$X_*^{est} = -A^{-1}F$$



[Bastiaansen et al (2021)]



[Bastiaansen, Dijkstra, Von der Heydt (GRL, 2021). DOI: doi.org/10.1029/2020GL091090]



ADDITIONAL SLIDES

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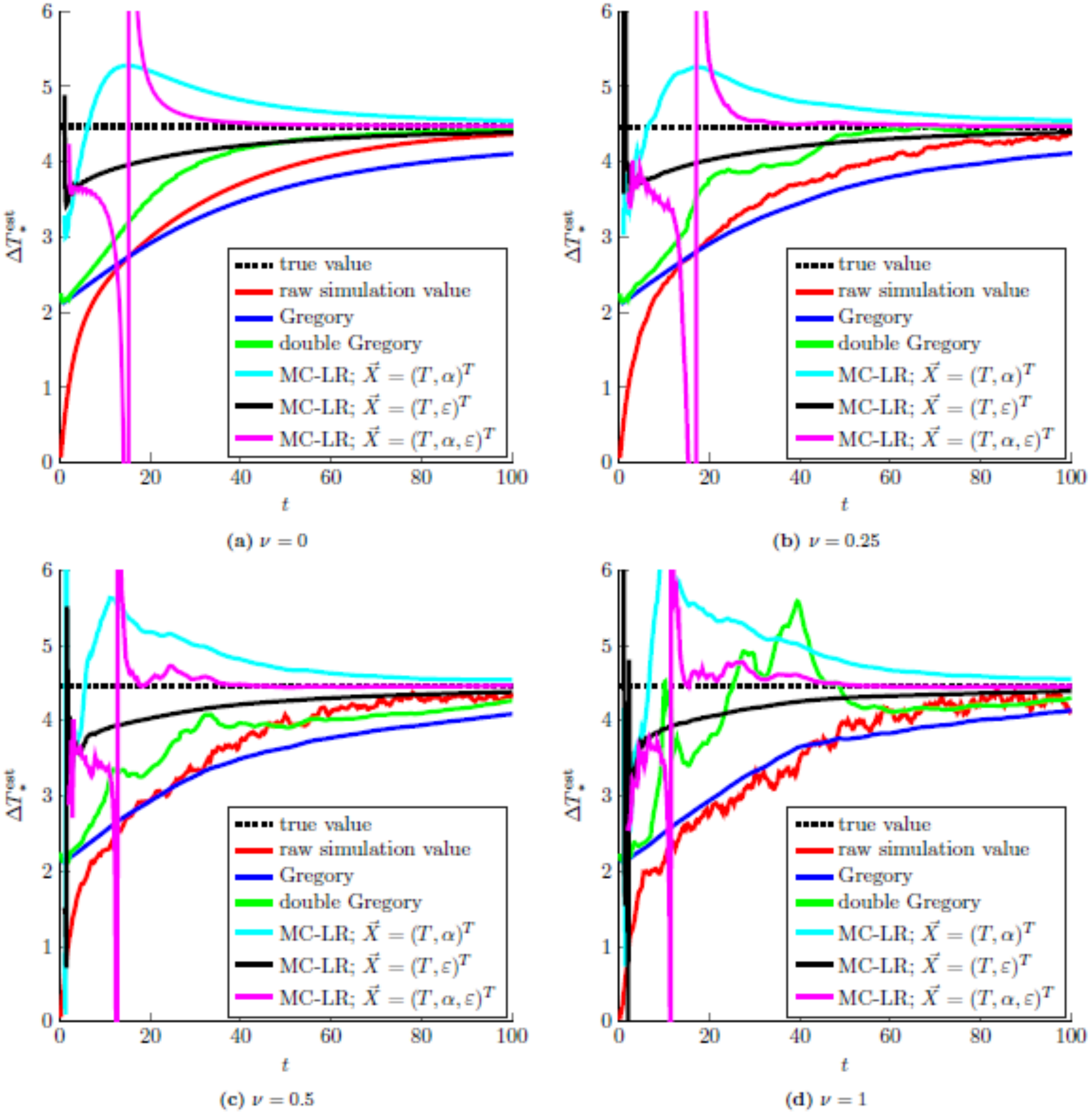

(a) $\nu = 0$

(b) $\nu = 0.25$

(c) $\nu = 0.5$

(d) $\nu = 1$

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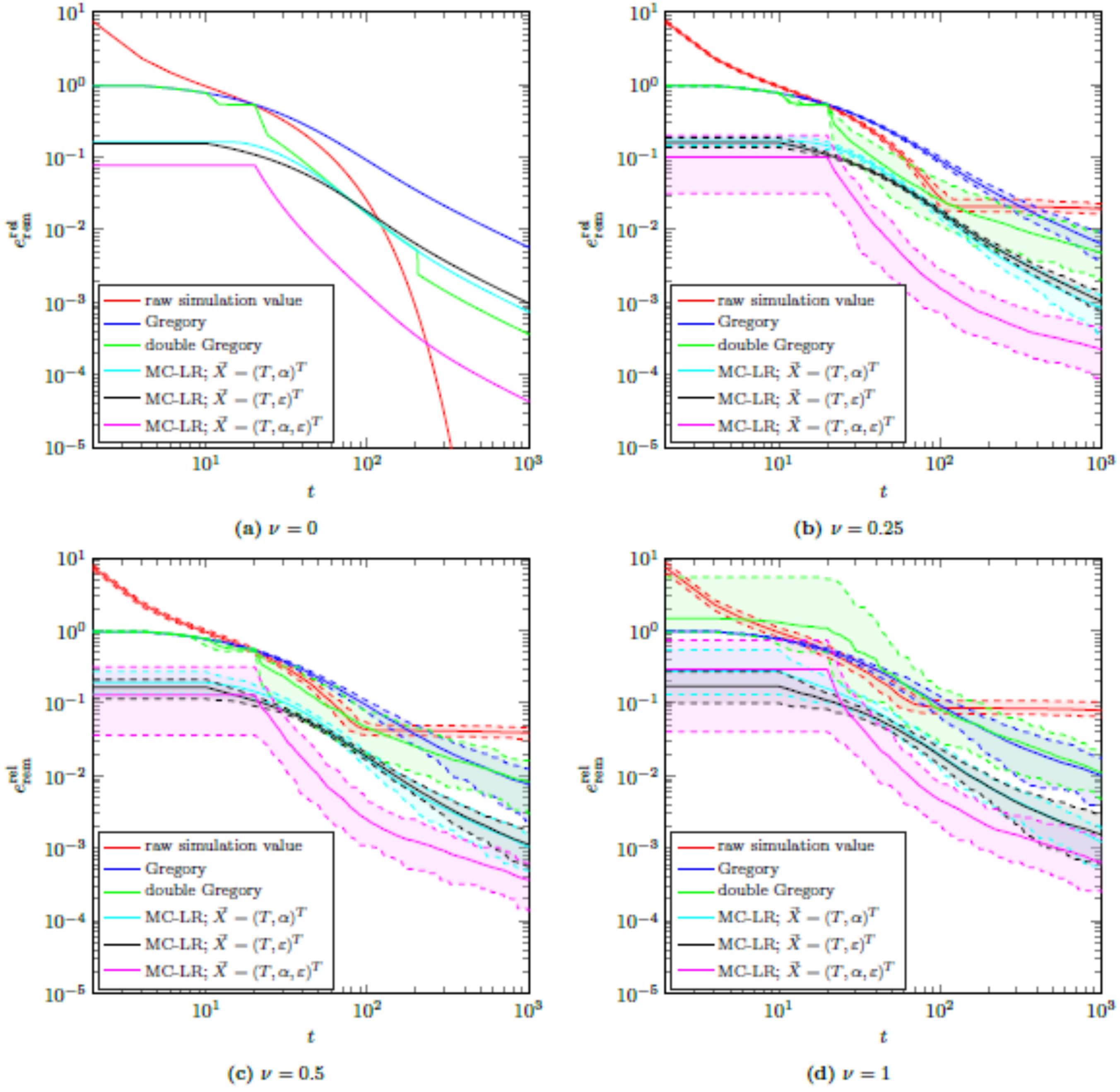
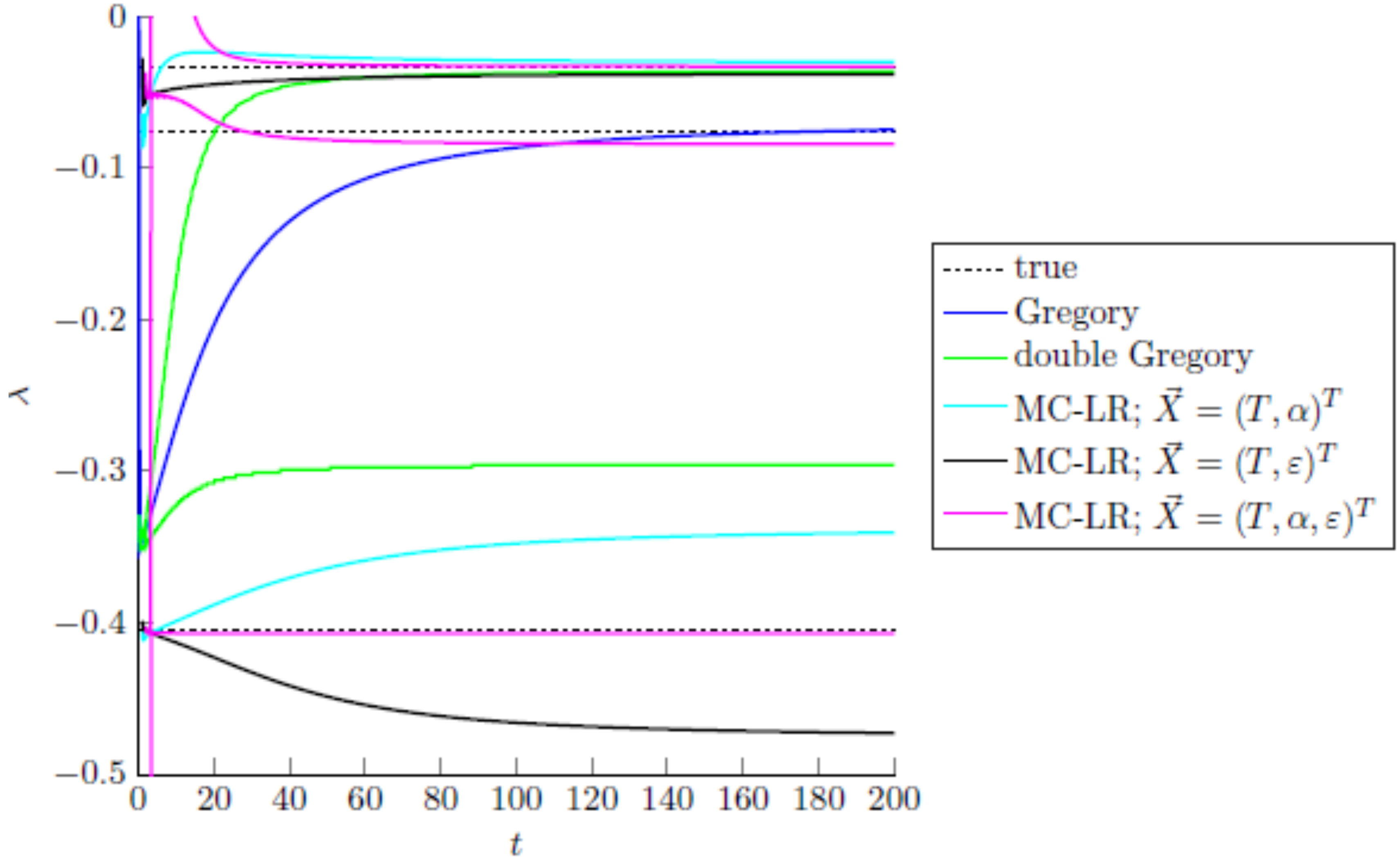
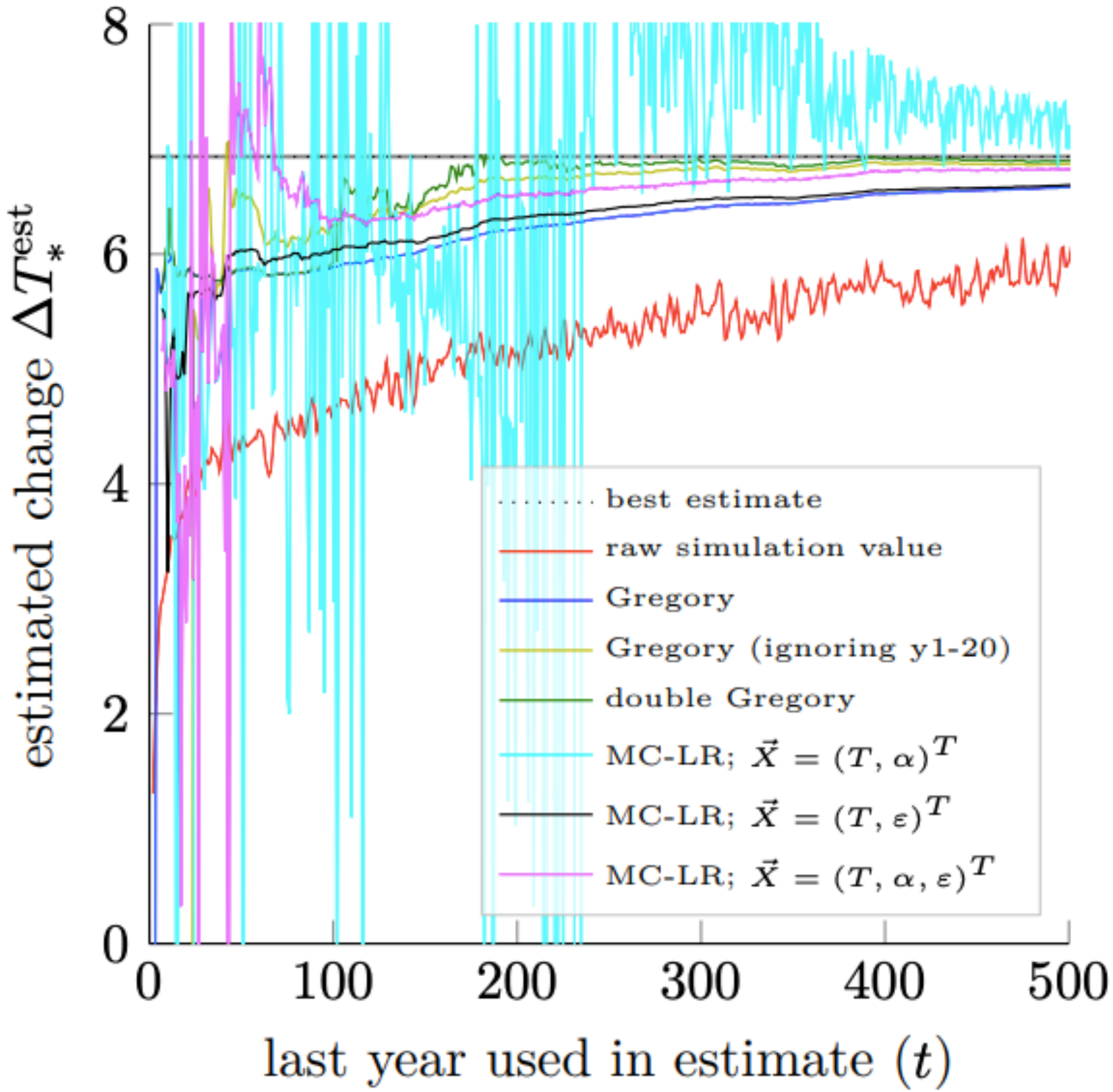


Toy model: Fitted eigenvalues



MPI-ESM 1.1



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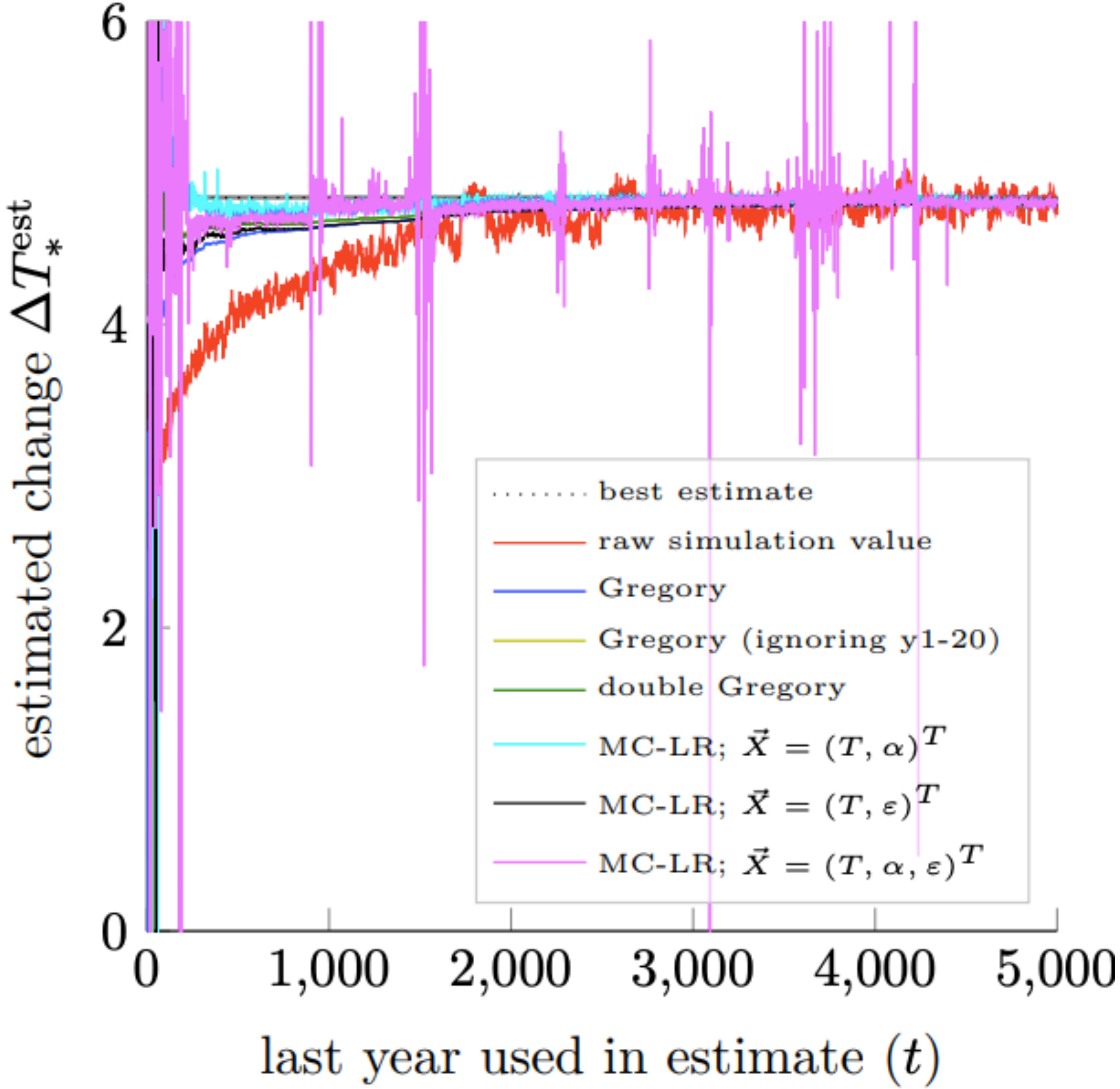
GISS-E2-R



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