Behaviour of self-organized large-scale vegetation patterns

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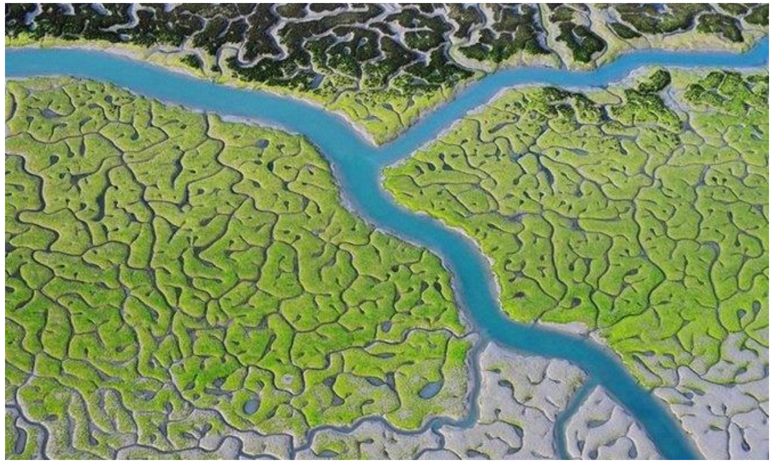
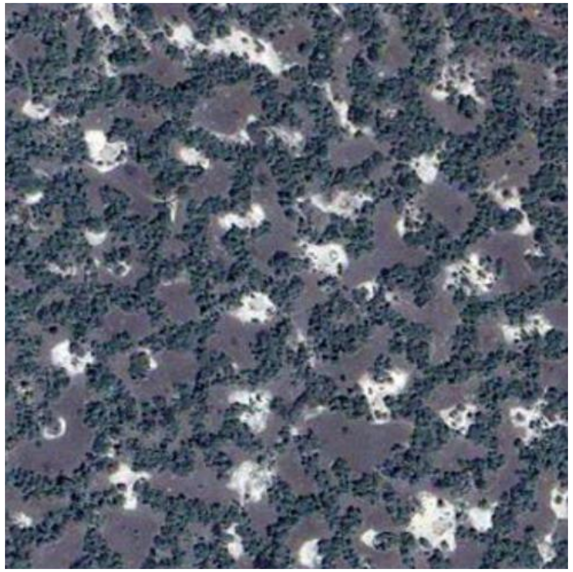
Examples of spatial patterning

mussel beds

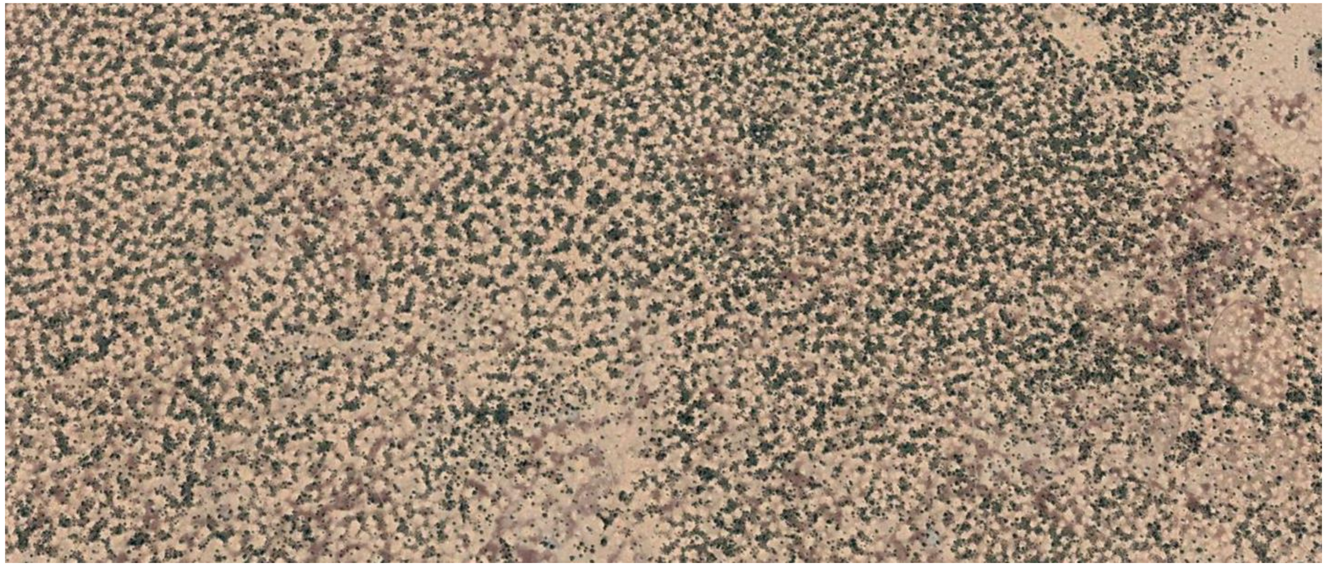
vegetation in coastal systems



marsh formation



savannas

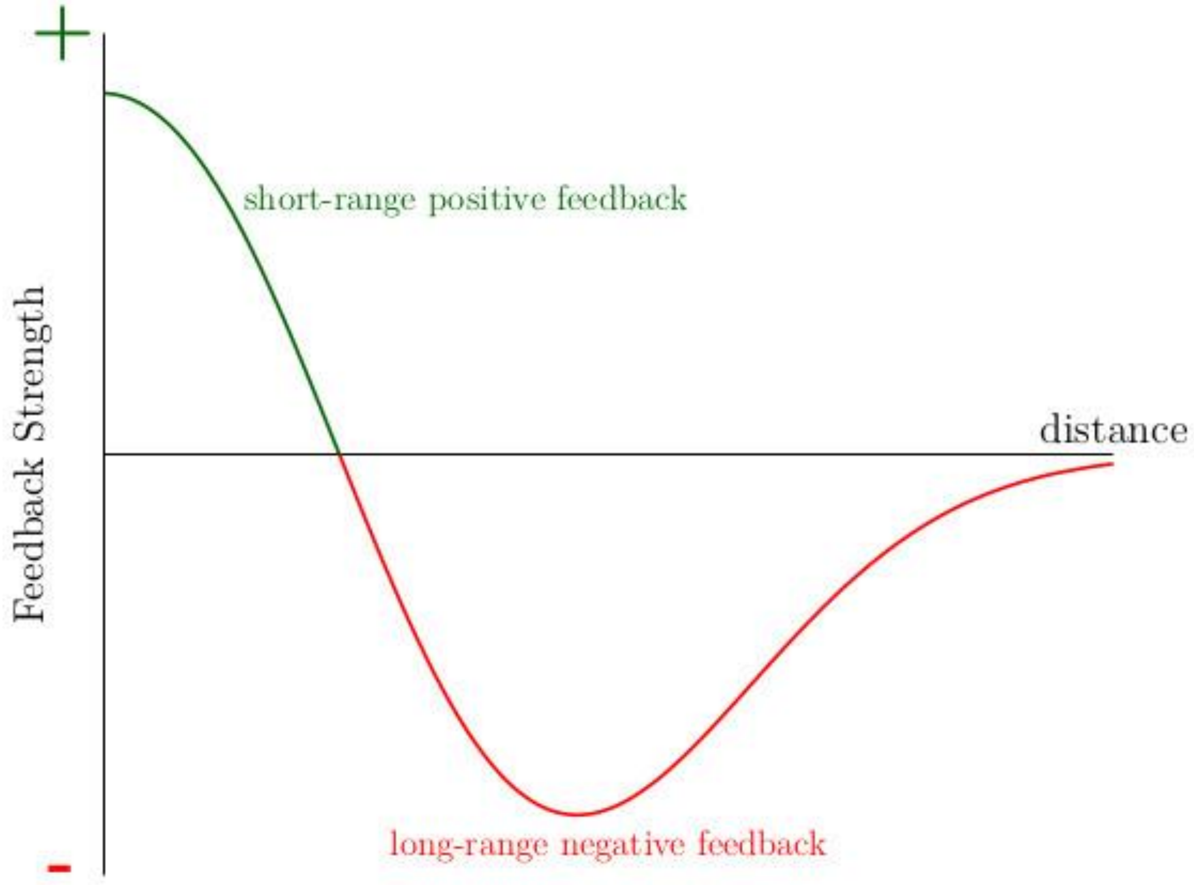


drylands

tropical forests

Self-organised patterns

- NO driving inhomogeneity
- BUT e.g. scale-dependent feedback



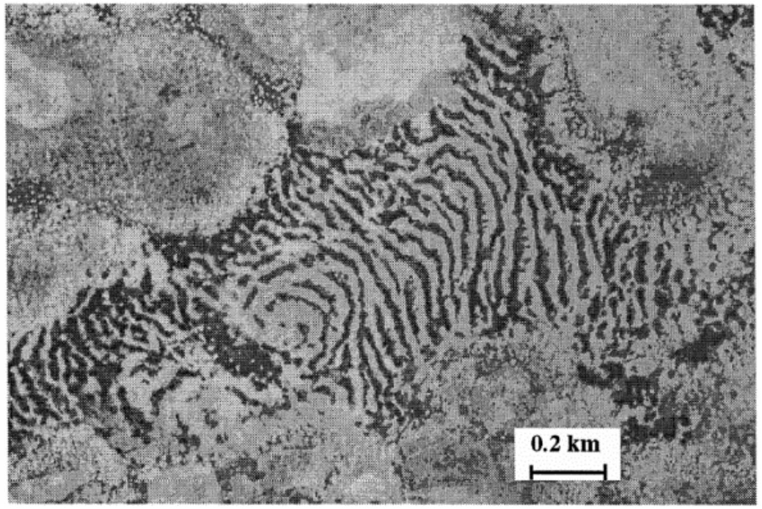
Ecology review: [Rietkerk & Van de Koppel, 2007]

Pattern adaptation (SLOW)

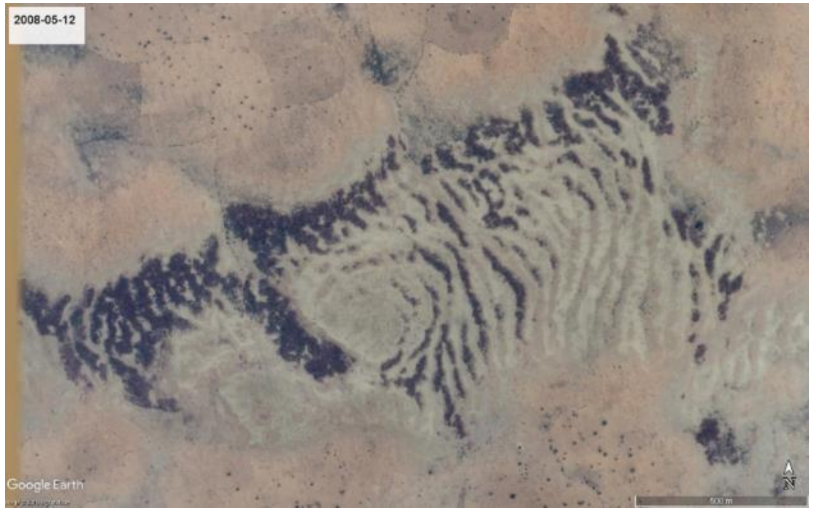
Somaliland, 1948 [Macfadyen, 1950]

Somaliland, 2008

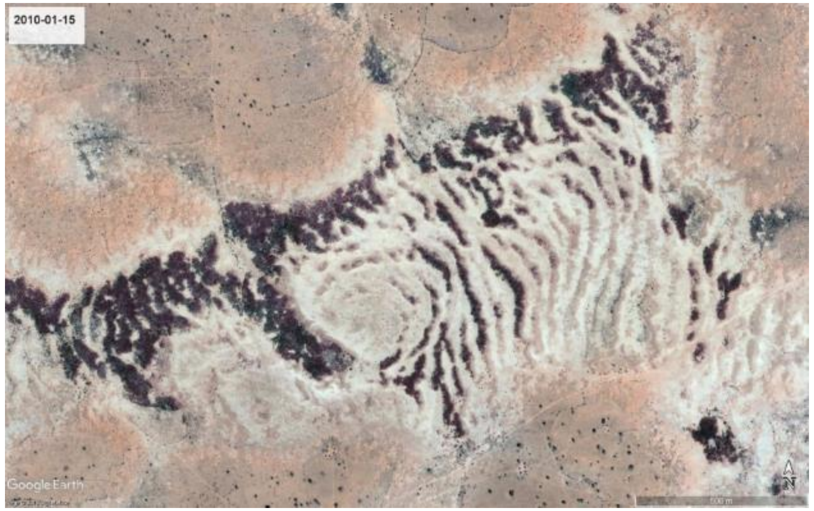
Pattern degredation (FAST)



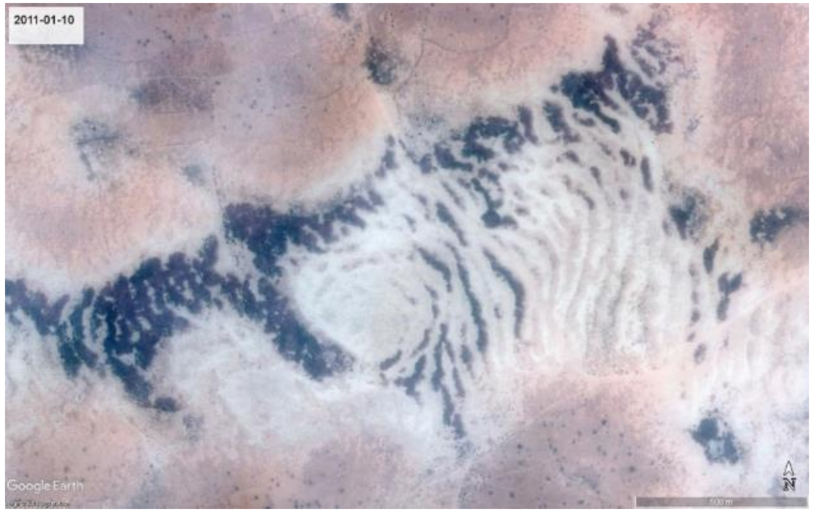
Niger, 1950 [Valentin, 1999]



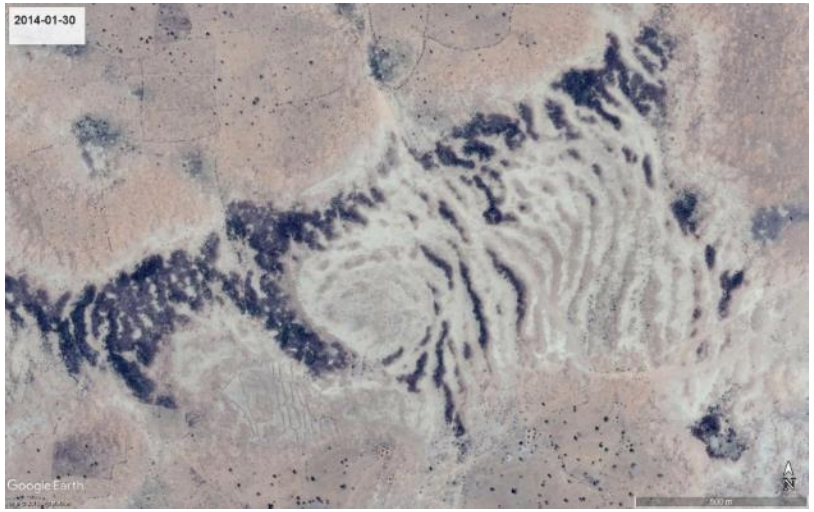
Niger, 2008



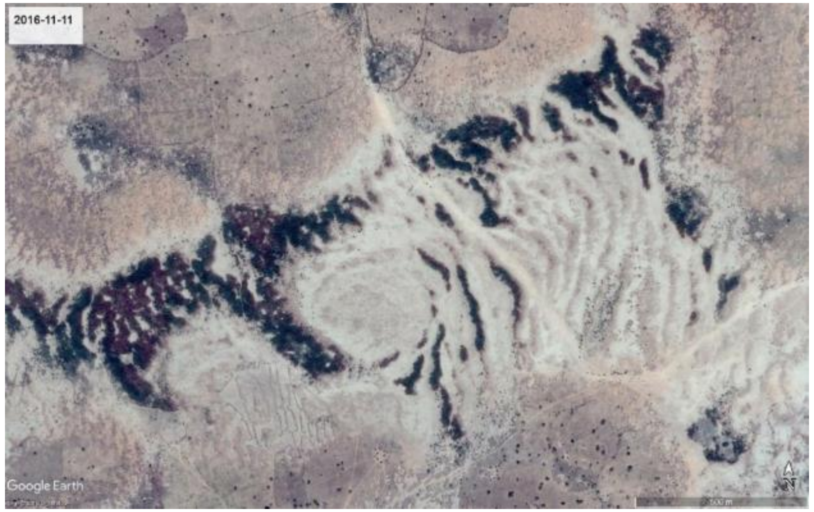
Niger, 2010



Niger, 2011



Niger, 2014

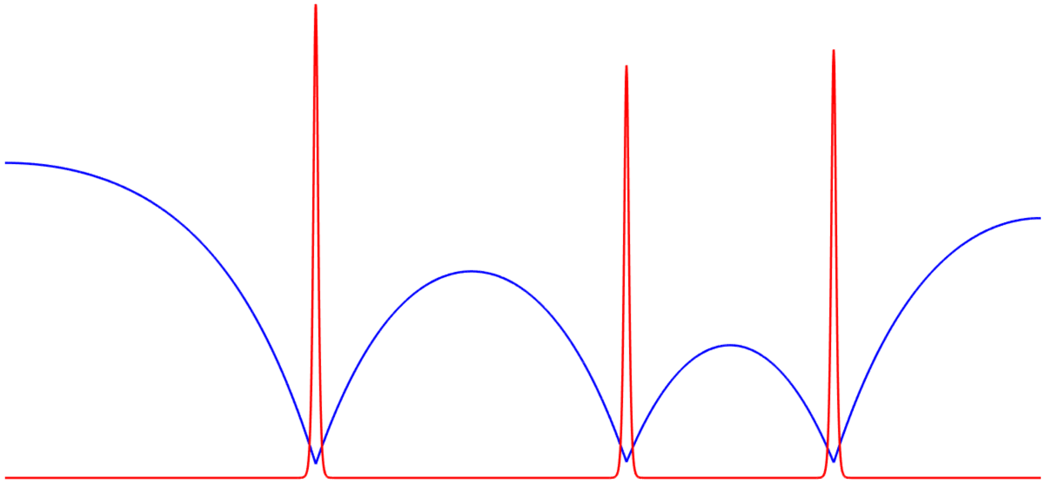


Niger, 2016

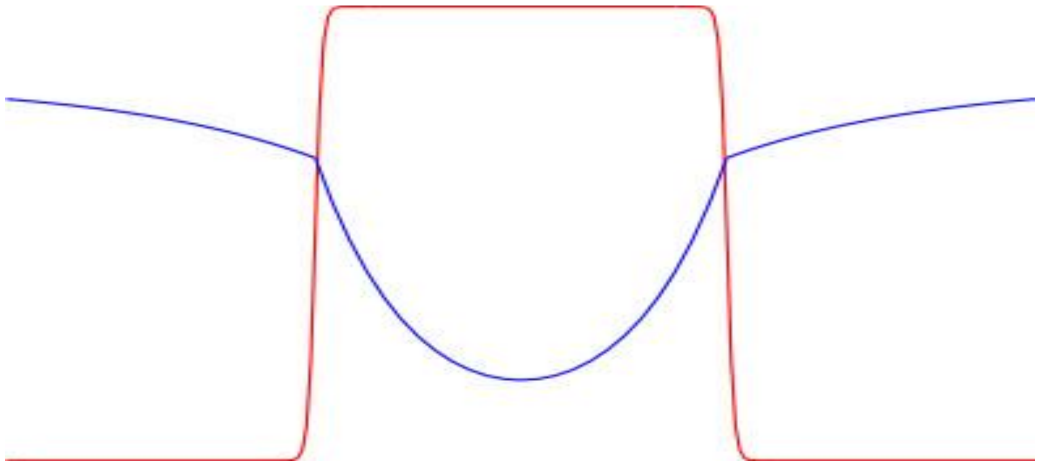
Mathematical treatment

Localized patterns ↔ localized structures

Seperation of scales ↔ small parameter

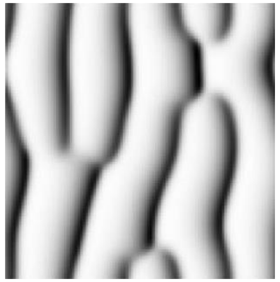


pulses

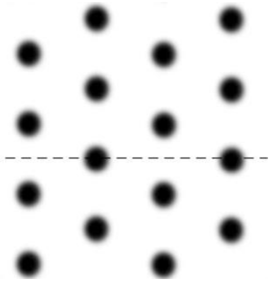


fronts

2D Reaction-diffusion models:

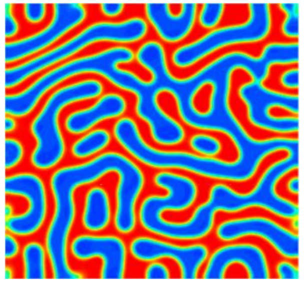


[Klausmeier, 1999]

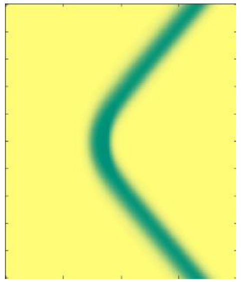


[Gilad et al, 2004]

[Rietkerk et al, 2002]



[Liu et al, 2013]



[Bastiaansen et al, 2019]

Archetypical ecosystem model

Extended-Klausmeier model

$$w_t = w_{xx} + (h(\mathbf{x})_x w)_x - w + a(\mathbf{t}) - wv^2$$
$$v_t = D^2 v_{xx} - mv + wv^2$$

w : water

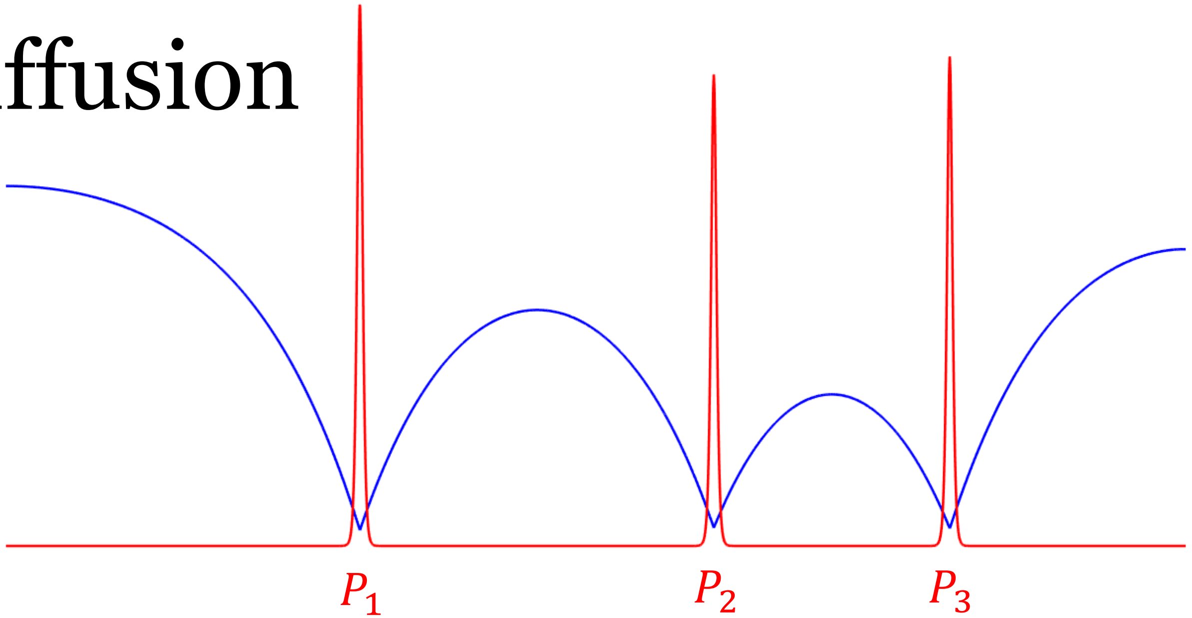
v : vegetation

h : height

D : ratio of diffusion

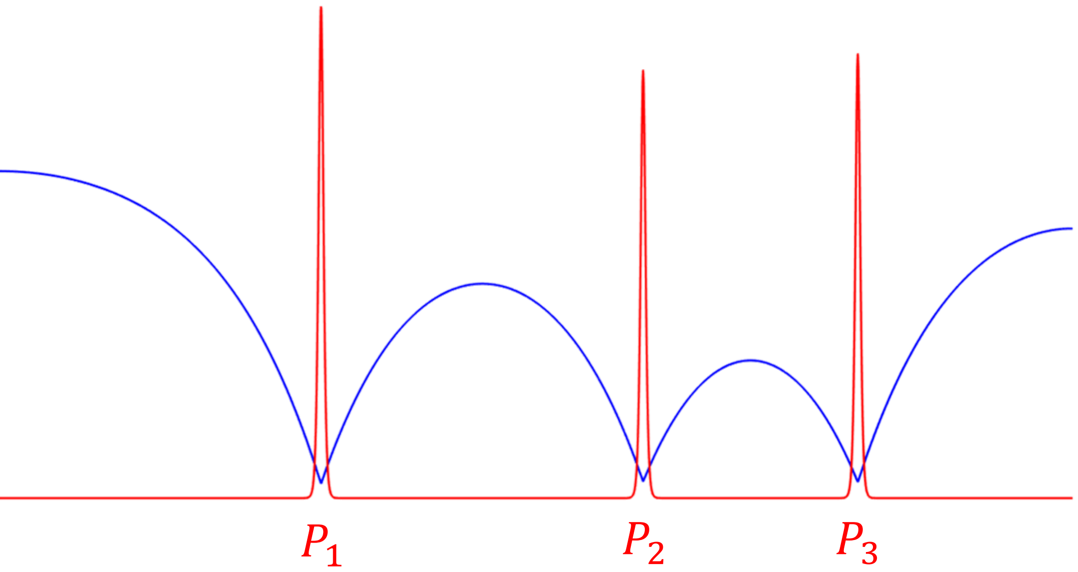
a : rainfall

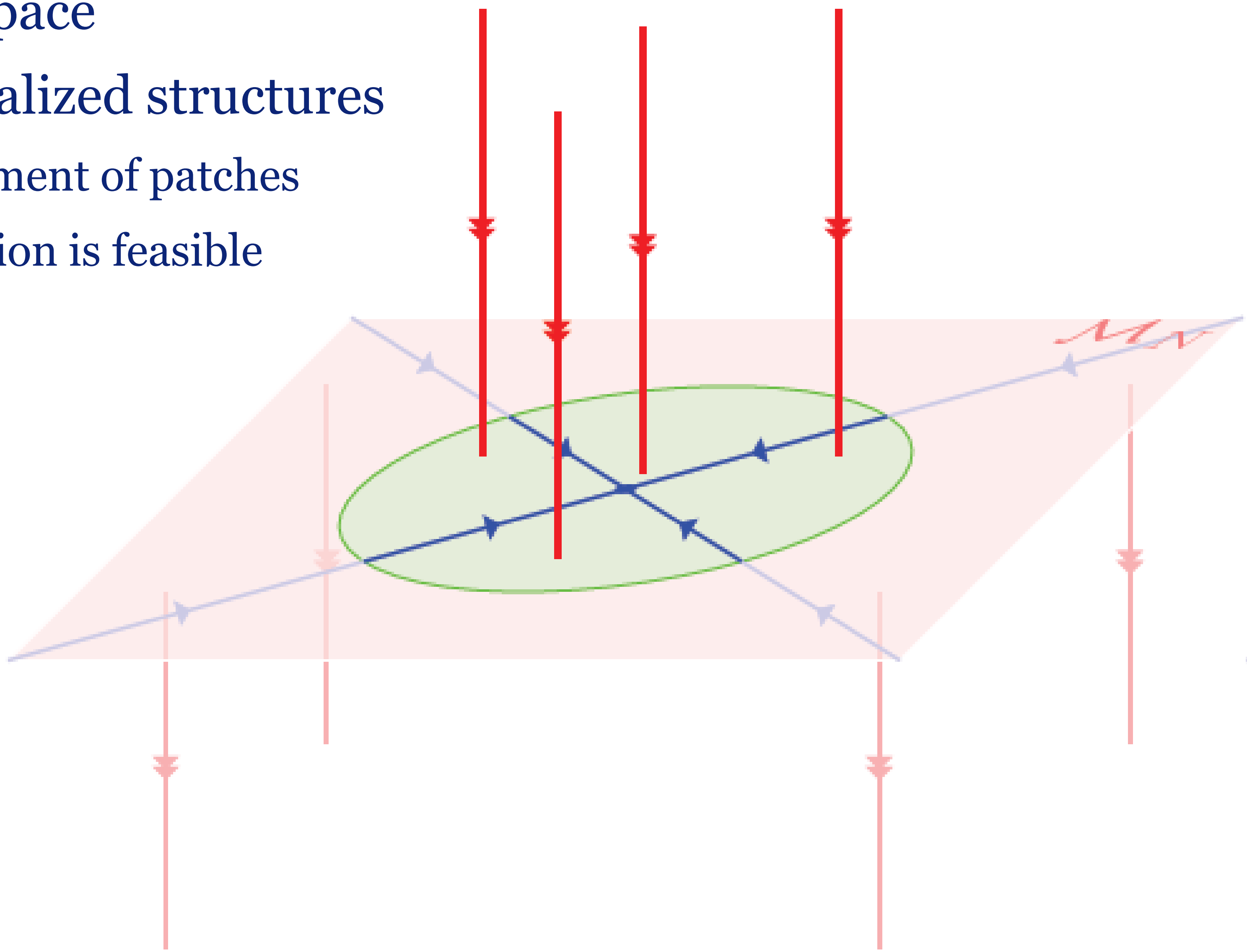
m : mortality



Understanding patches in the model

- PDE: infinite-dimensional state space
- Reduction possible because of localized structures
  1. Patch-location ODE: describe movement of patches
  2. Stability criterium: test if configuration is feasible

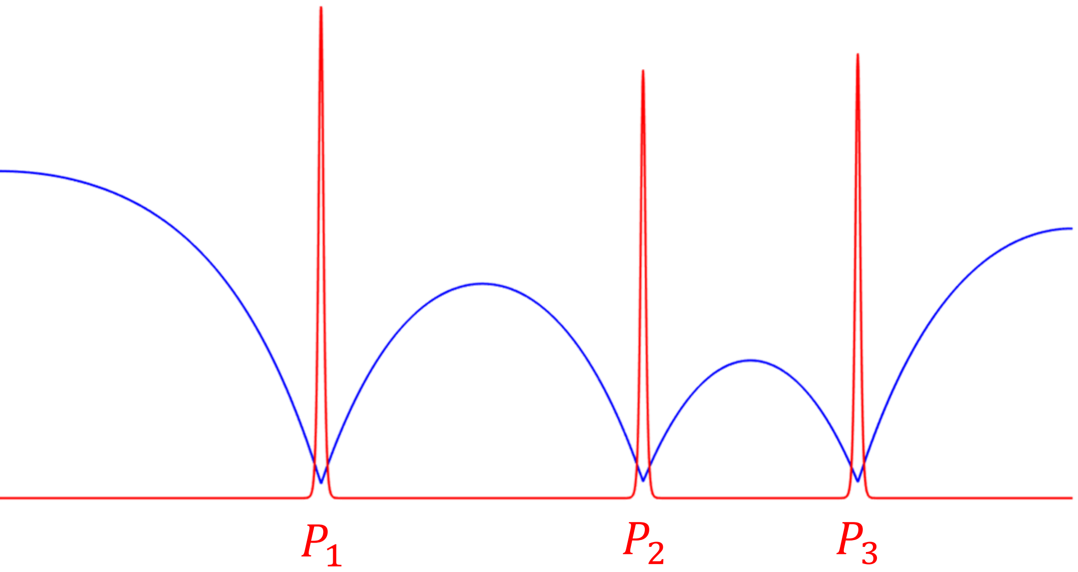


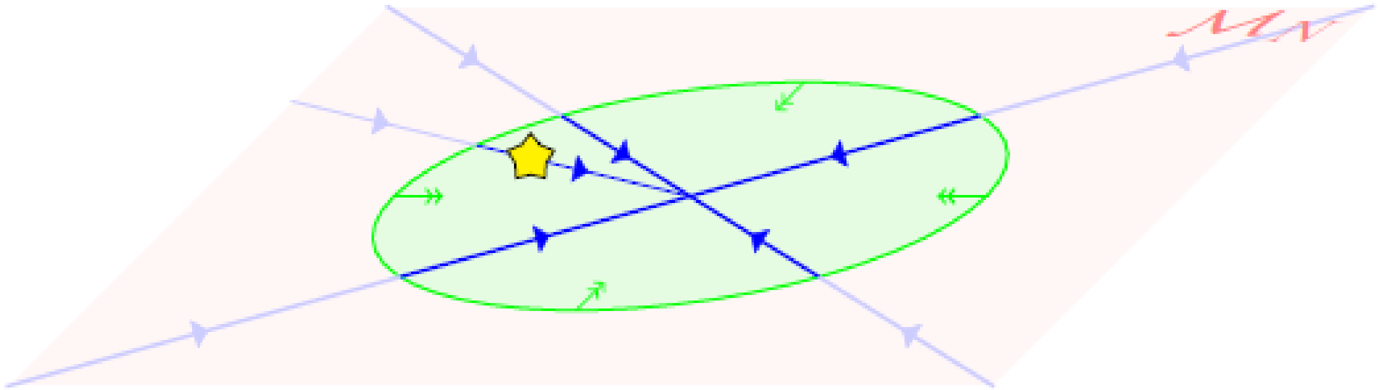


Patch-location ODE

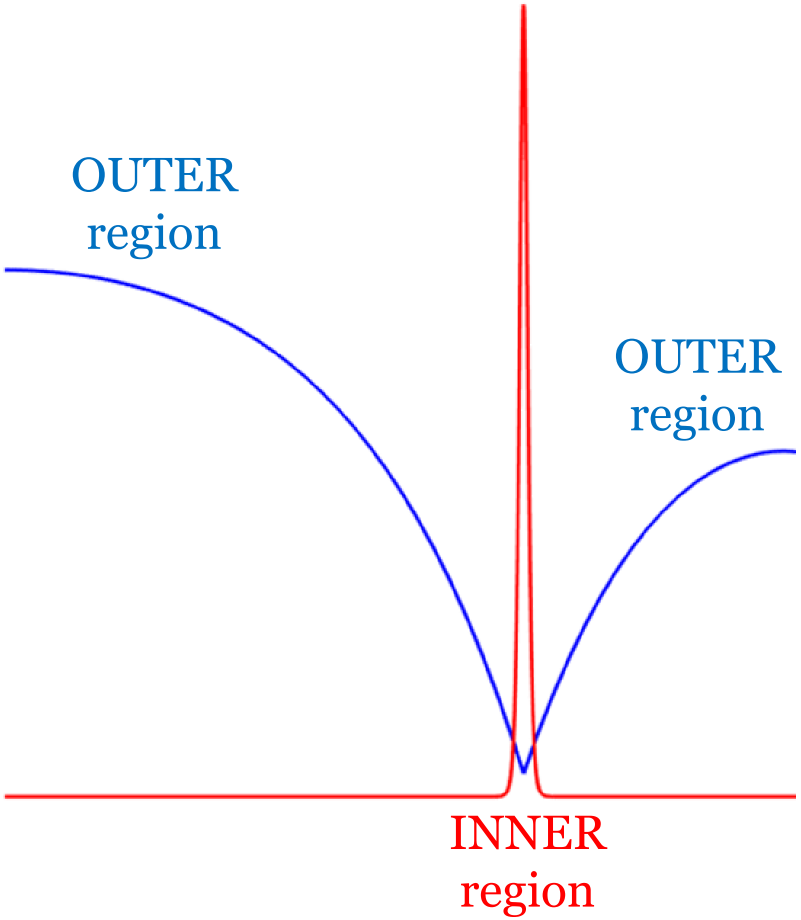
$$\frac{dP_j}{dt} = \frac{Da^2}{m\sqrt{m}}\left[w_x({P_j}^+)^2 - w_x({P_j}^-)^2\right]$$

Resource availability dictates patch movement





How to derive the ODE?



$$w_t = w_{xx} + (h(\mathbf{x})_x w)_x - w + a(\mathbf{t}) - wv^2$$
$$v_t = D^2 v_{xx} - mv + wv^2$$

INNER regions:

$$0 = D^2 v_{xx} - mv + wv^2$$

$$\Rightarrow v_p(x - P_j(\mathbf{t}))$$

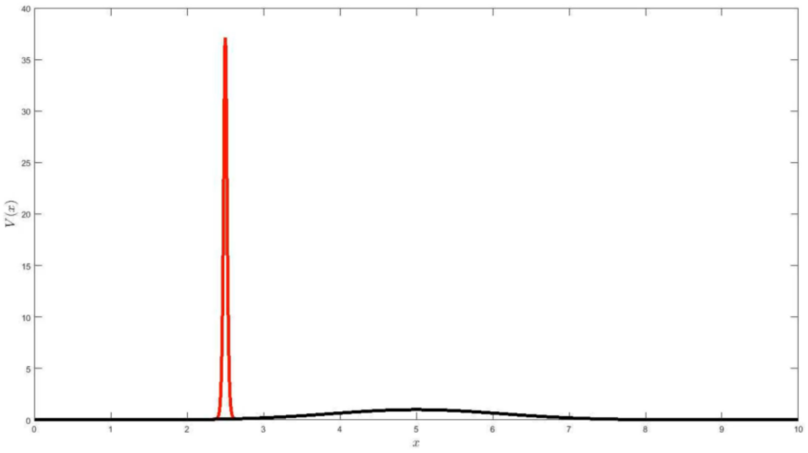
OUTER regions:

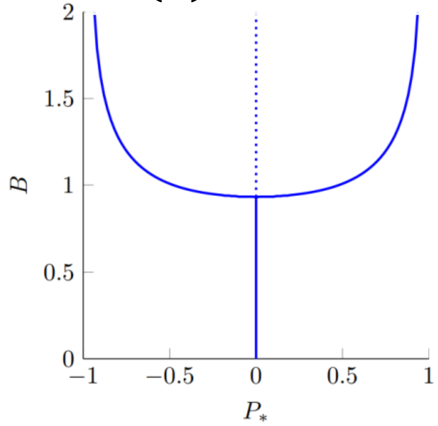
$$0 = w_{xx} + (h(\mathbf{x})_x w)_x - w + a(\mathbf{t})$$

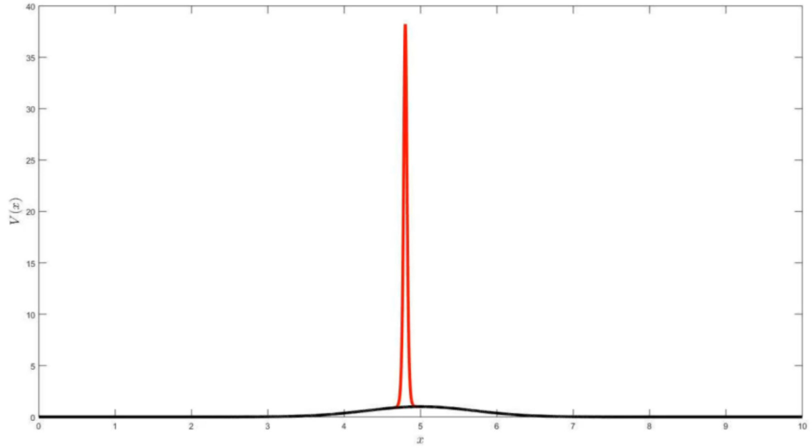
Match solutions at boundaries:

$$\Rightarrow \frac{dP_j}{dt} = \frac{Da^2}{m\sqrt{m}} \left[w_x(P_j^{+})^2 - w_x(P_j^{-})^2 \right]$$

Intermezzo: the effect of topography







Vegetation pulses can move uphill and downhill

flat terrain

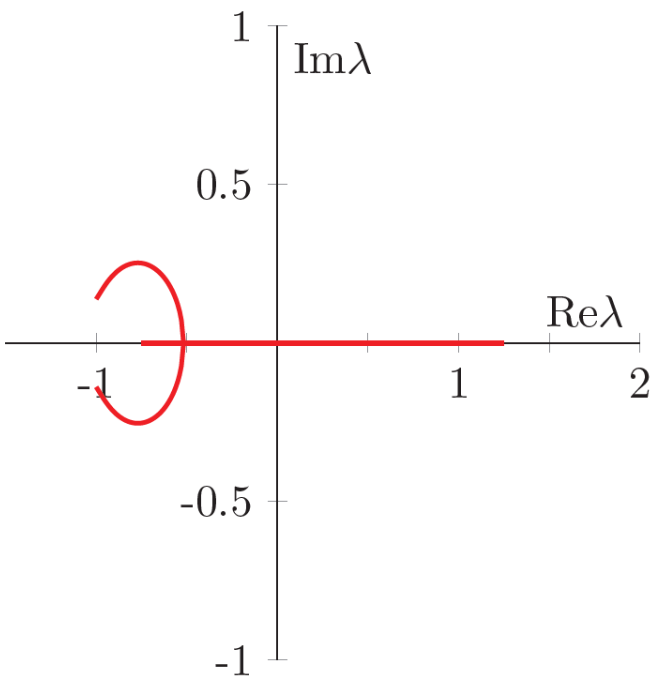
complexer topography

Stationary multi-pulse solutions do exist

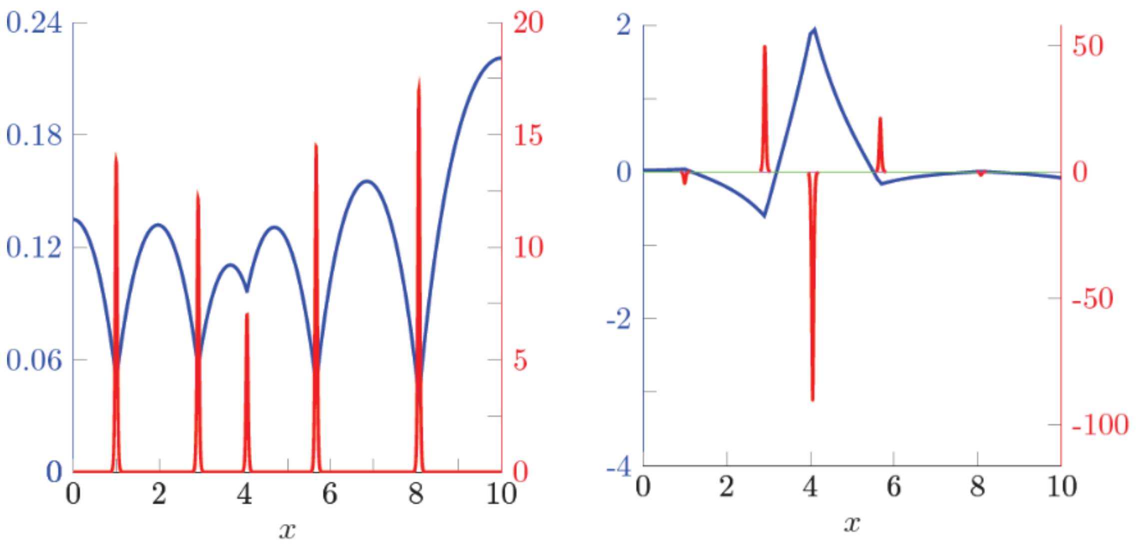
More detailed and rigorous treatment:
Bastiaansen, Chirilus-Bruckner, Doelman (2020),
'Pulse Solutions for an Extended Klausmeier Model with Spatially Varying Coefficients'

Stability criterium

- Freeze solution in time
- Study (quasi-steady) eigenvalues & eigenfunctions



point spectrum skeleton



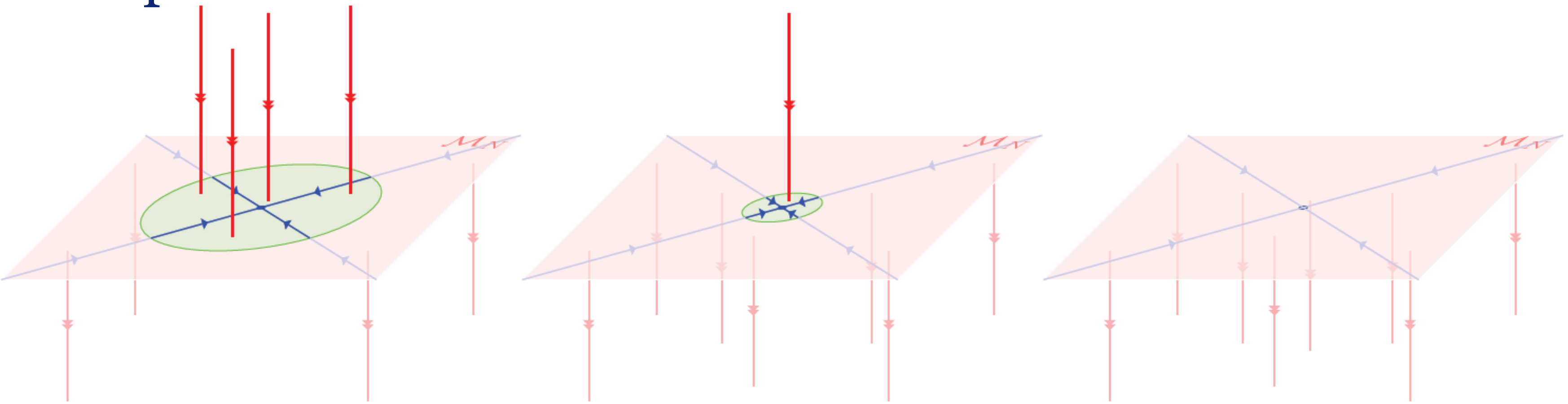
solution   eigenfunction

Nonlinear prediction based on linear analysis

Stability criterium

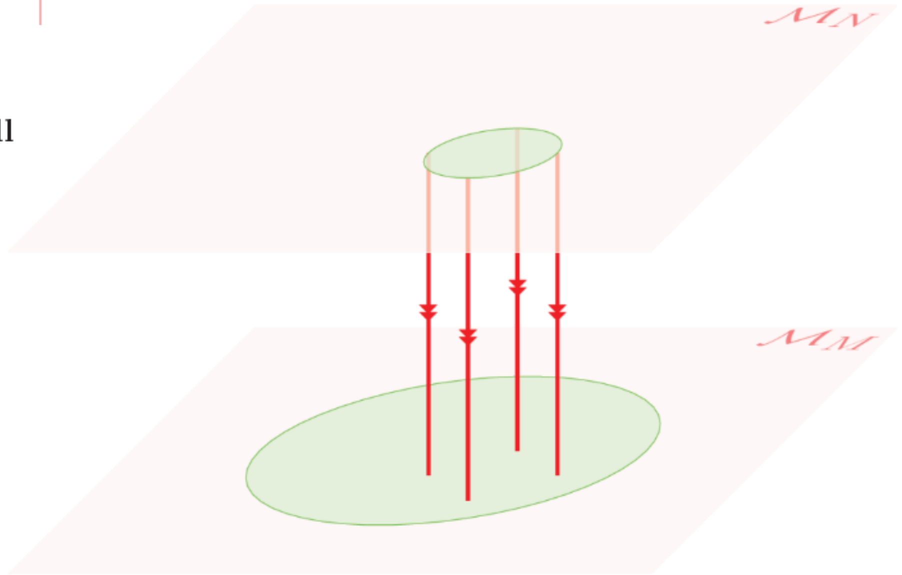
Enough resources to sustain all vegetation patches?

Depends on **amount of rainfall** and **distance between patches**



high rainfall | medium rainfall | low rainfall

What happens when outside feasible region?



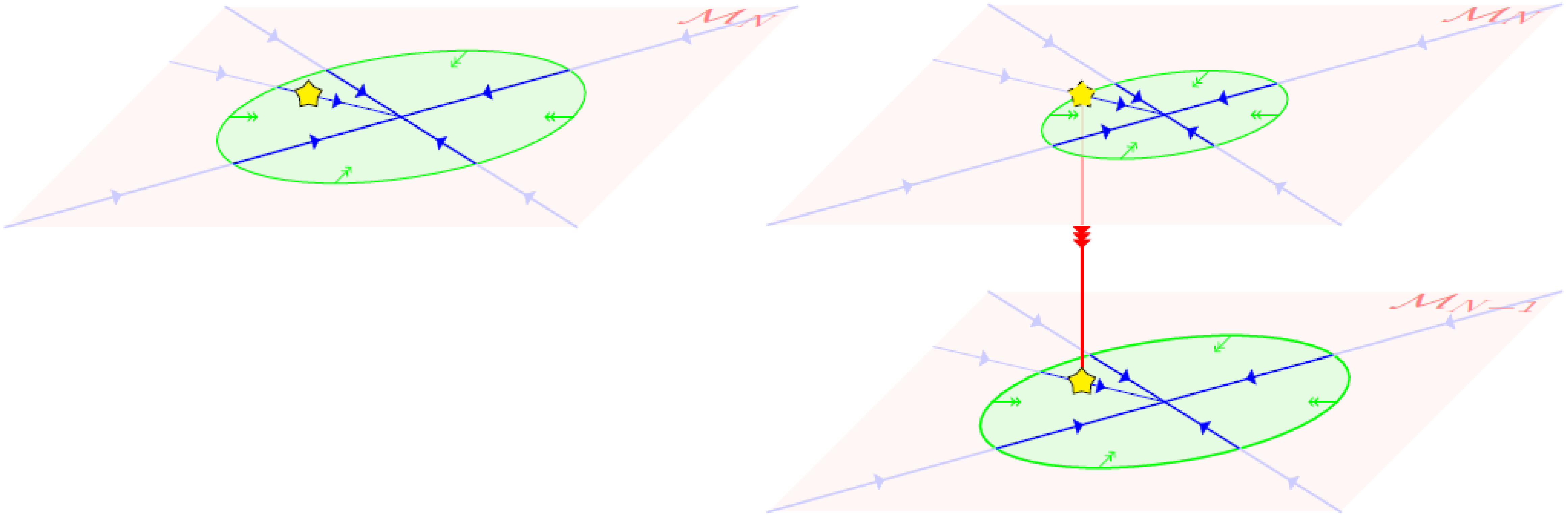
irregular configuration:	**One** patch disappears (least amount of biomass)
regular configuration:	**Half** of the patches disappears (wavelength doubling)

Patches during climate change (1)

Competition of two effects:

1. Patch rearrangement
2. Shrinking of feasible region

fast climate change



(a) initial configuration

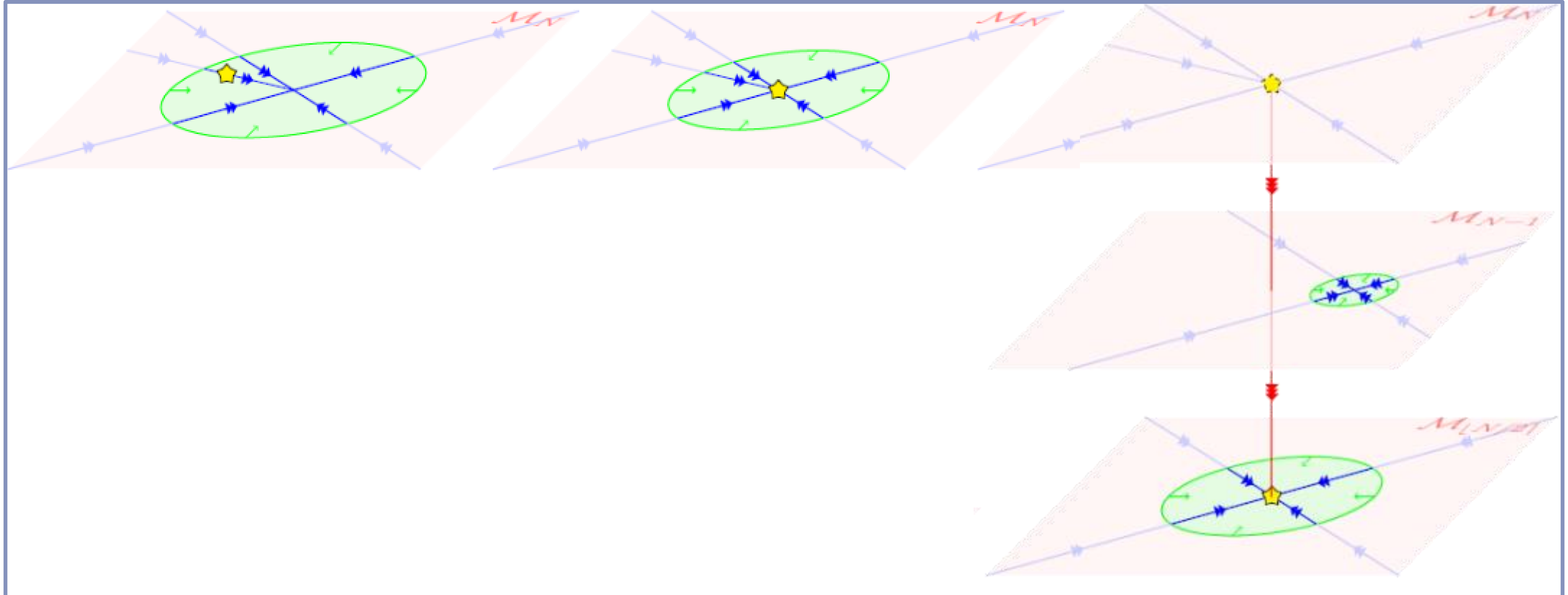
(b) at/after destabilization

Patches during climate change (2)

Competition of two effects:

1. Patch rearrangement
2. Shrinking of feasible region

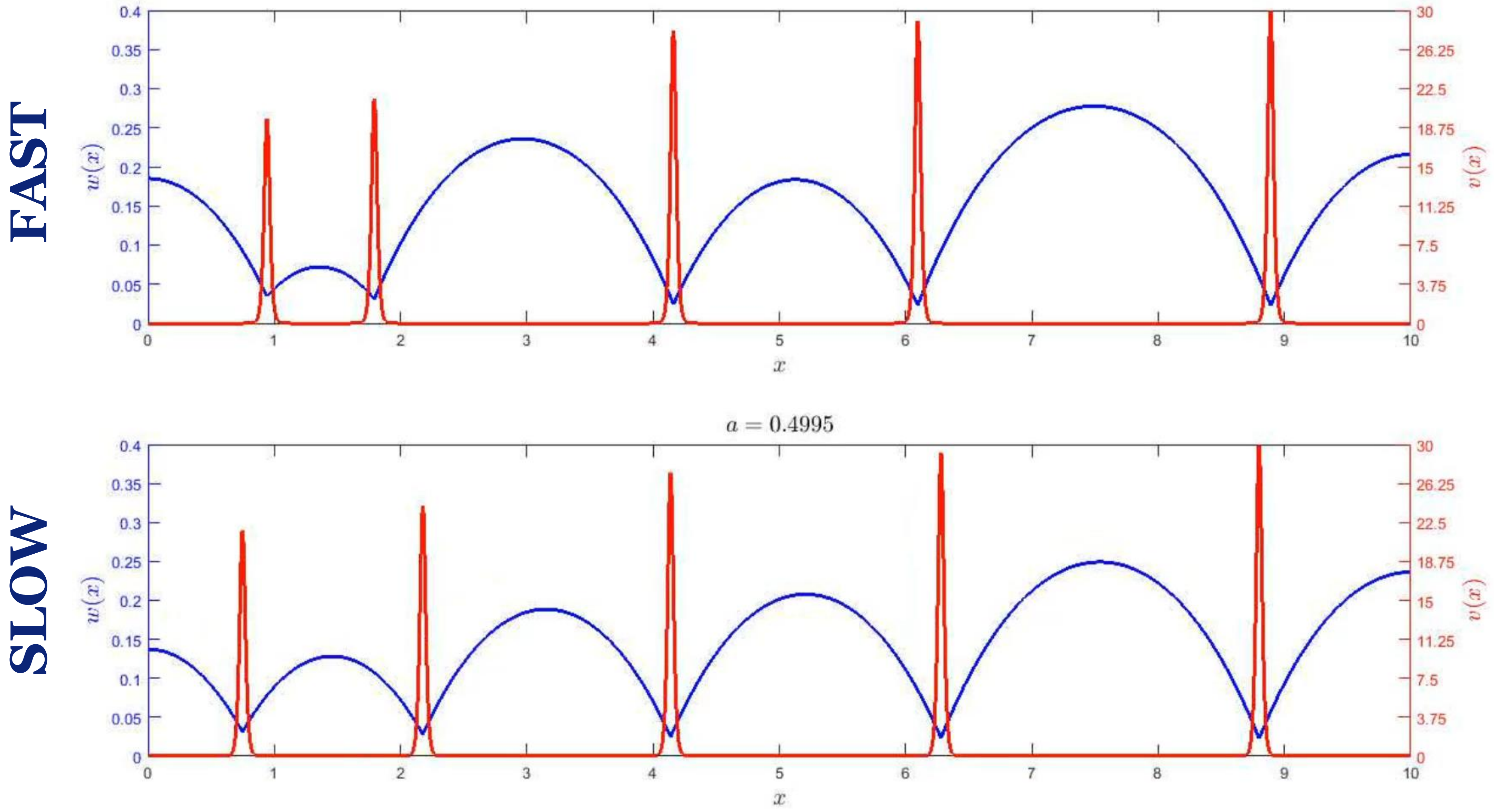
slow climate change



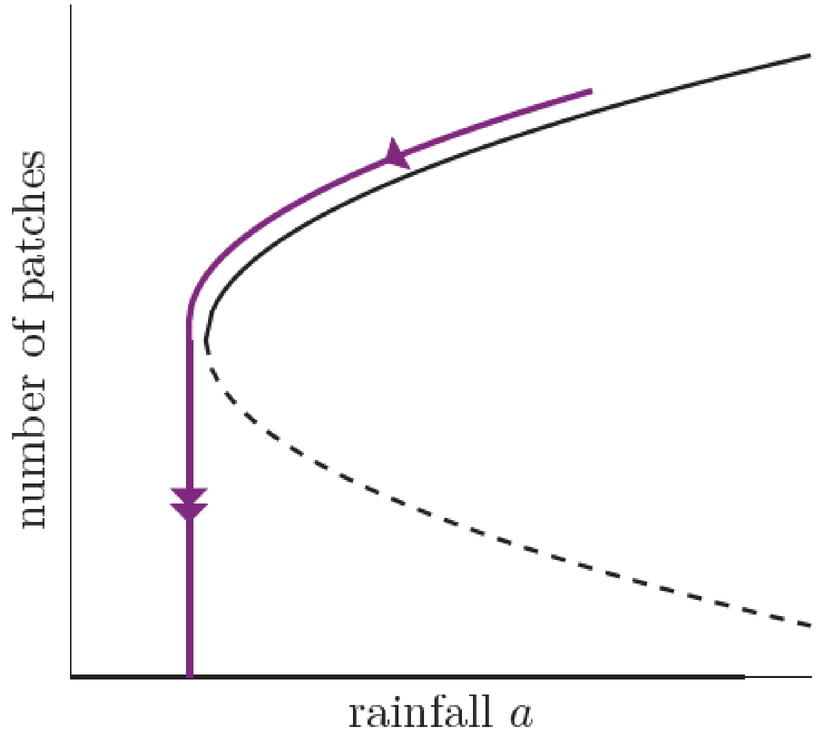
(a) initial configuration (b) regular pattern achieved (c) at/after destabilization

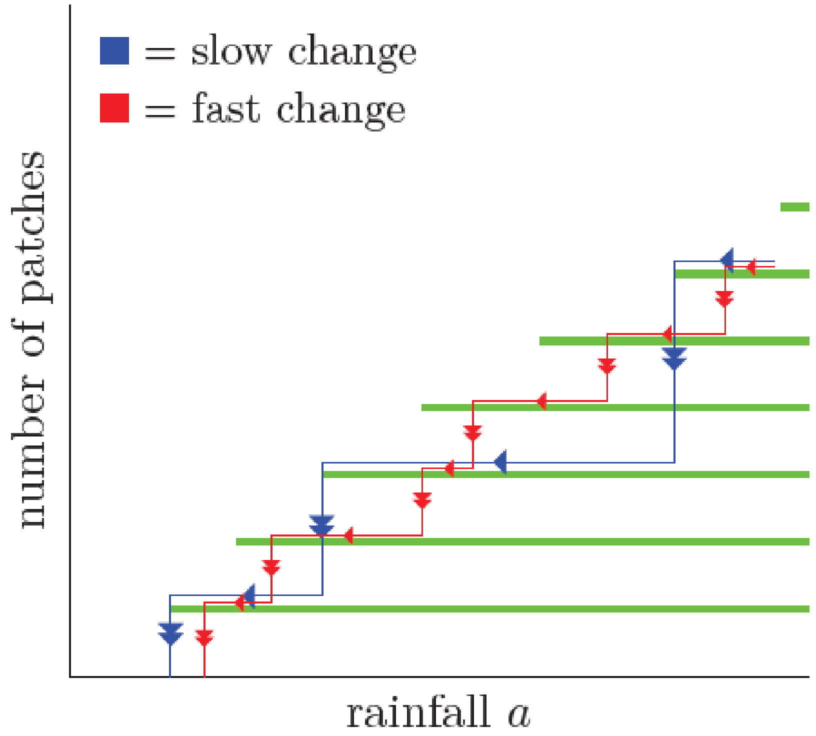
Patches during climate change (3)

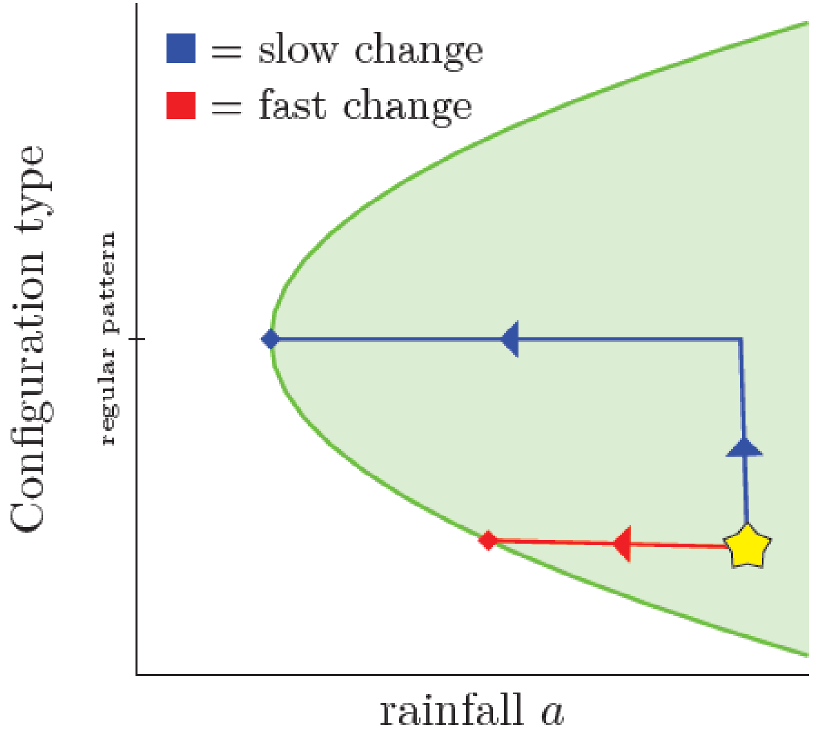
Rate of climate change



Ecosystem degradation pathways







critical transition

smaller pattern transitions

(zoom-in on a line)

Classical view - fold

Multistable systems

Summary

Behaviour of self-organised patterns

I. Patch rearrangement
II. Pattern to pattern transitions

PDE to ODE reduction

reveals

Topographic effects

downhill movement

stationary multi-pulse solutions

Rate of change effects

fast: multiple smaller ecosystem shifts

slow: few larger ecosystem shifts

Mathematical paper:
Bastiaansen & Doelman (2019)
'The dynamics of disappearing pulses in a singularly perturbed reaction–diffusion system with parameters that vary in time and space'
doi.org/10.1016/j.physd.2018.09.003

Ecology paper:
Bastiaansen, Doelman, Eppinga & Rietkerk (2020)
'The effect of climate change on the resilience of ecosystems with adaptive spatial pattern formation'
doi.org/10.1111/ele.13449