Climate change in the Anthropocene
linear and nonlinear climate response

Robbin Bastiaansen

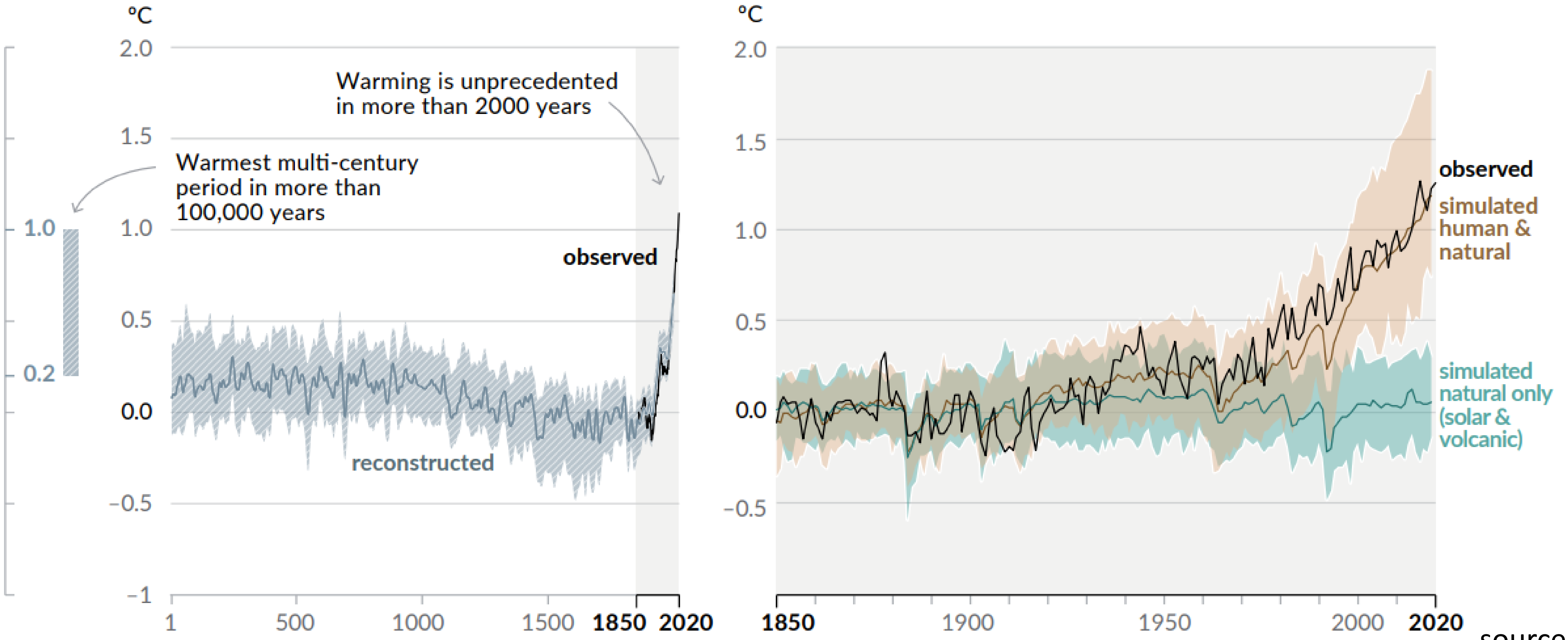
MI TALK - 17 November 2022

Human influence has warmed the climate at a rate that is unprecedented in at least the last 2000 years

Changes in global surface temperature relative to 1850–1900

(a) Change in global surface temperature (decadal average) as reconstructed (1–2000) and **observed** (1850–2020)

(b) Change in global surface temperature (annual average) as **observed** and simulated using human & natural and only natural factors (both 1850–2020)



source: IPCC AR6

Today's talk

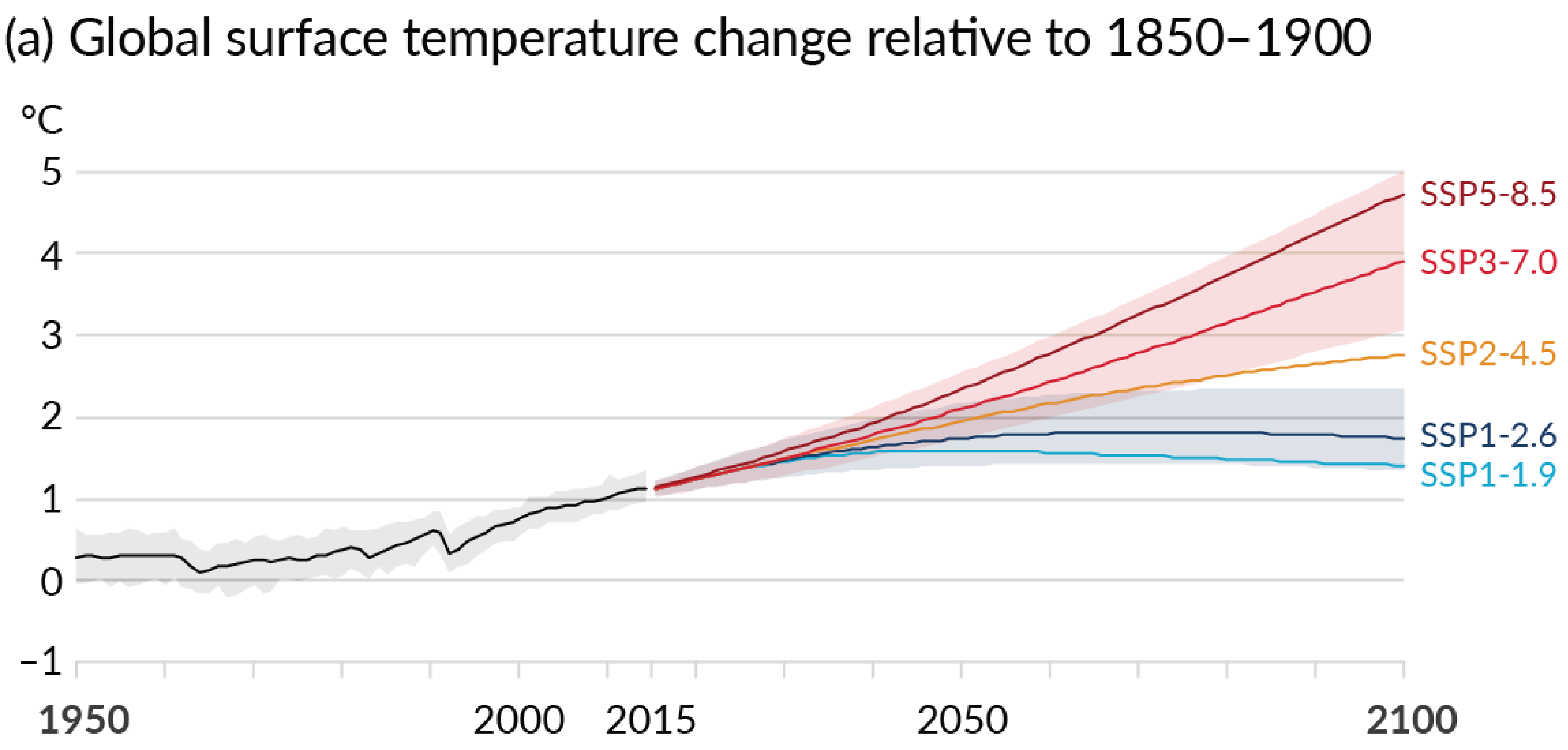
1. Future projections – *linear* response

2. Limitations of linear theory – *nonlinear* response

3. Beyond ODE theory – *nonlinear spatial* response

Part 1.
Future climate projections

Future climate projections



source: IPCC AR6

Experiments in Global Climate Models

- Much computing power
- Not long-term response
- Not many scenarios

Often, all other observables are assumed to be linearly related to the global mean surface temperature

Most used Climate Sensitivity Metrics

Equilibrium Climate Sensitivity (ECS)

change in equilibrium temperature
due to (instantaneous) doubling of CO_2

Transient Climate Response (TCR)

change in temperature after 100 years
with 1% CO_2 increase per year (until doubling)

Some Details

- Dedicated experiments with climate models
- Start from equilibrium with pre-industrial levels of CO_2
- Change compared to control run

Mathematical Context

$$\frac{dy}{dt} = f(\, y\, ;\, \mu(t)\,)$$

$y(t) \in \Omega$: state variable
$\mu(t) \in \mathbb{R}$: forcing parameter

Evolution of Observables

Observables

$\hat{O} : \Omega \to \mathbb{R}^N$

$O(t) := \hat{O}(\, y(t)\,)$

Linear Response Theory (& Koopman Theory):

Evolution

$$\frac{dO}{dt} = \mathcal{L}O + g(t)$$

$$\Delta O(t) = \left(G^{[O]} * g\right)(t) = \int_0^t G^{[O]}(s)\, g(t-s)\, ds$$

Green function

Evolution of Observables

Observables

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Approximation of Green Function:

$$G^{[O]}(t) = \sum_{m=1}^{M} \beta_m^{[O]}\, e^{-t/\tau_m}$$

So:

$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]}\, \mathcal{M}_m^{g}(t)$$

$$\mathcal{M}_m^{g}(t) = \int_0^t e^{-s/\tau_m}\, g(t-s) ds$$

all observable dependency

all forcing (and time) dependency

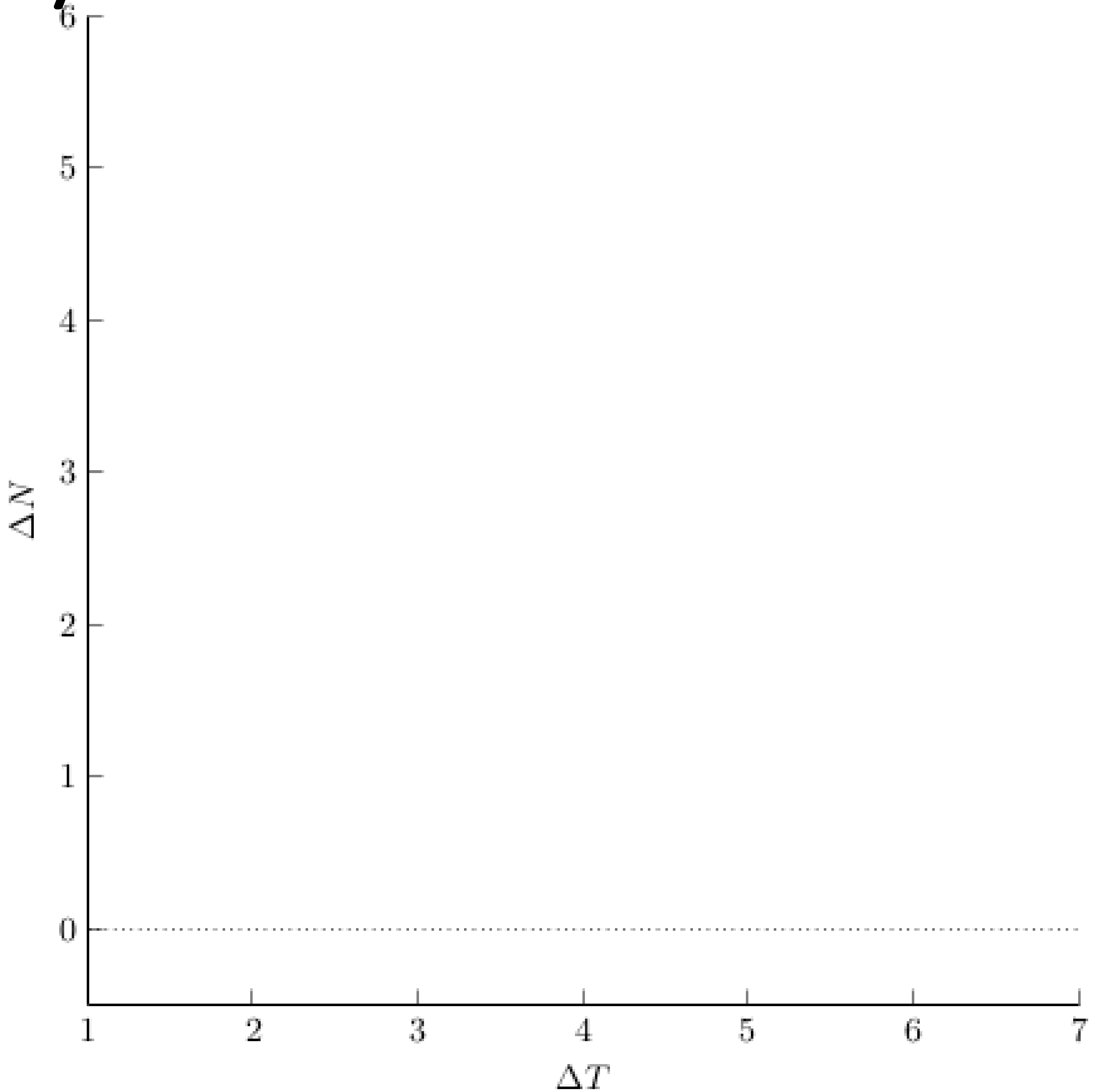
Gregory Method

Regress data to

$$\Delta N(t) = \boldsymbol{F} + \boldsymbol{\lambda}\, \Delta T(t)$$

Since $\Delta N_* = 0$ in equilibrium, ECS estimation is

$$\Delta T_*^{est} = -\,\boldsymbol{\lambda}^{-1}\, \boldsymbol{F}$$



[Gregory et al, 2014]

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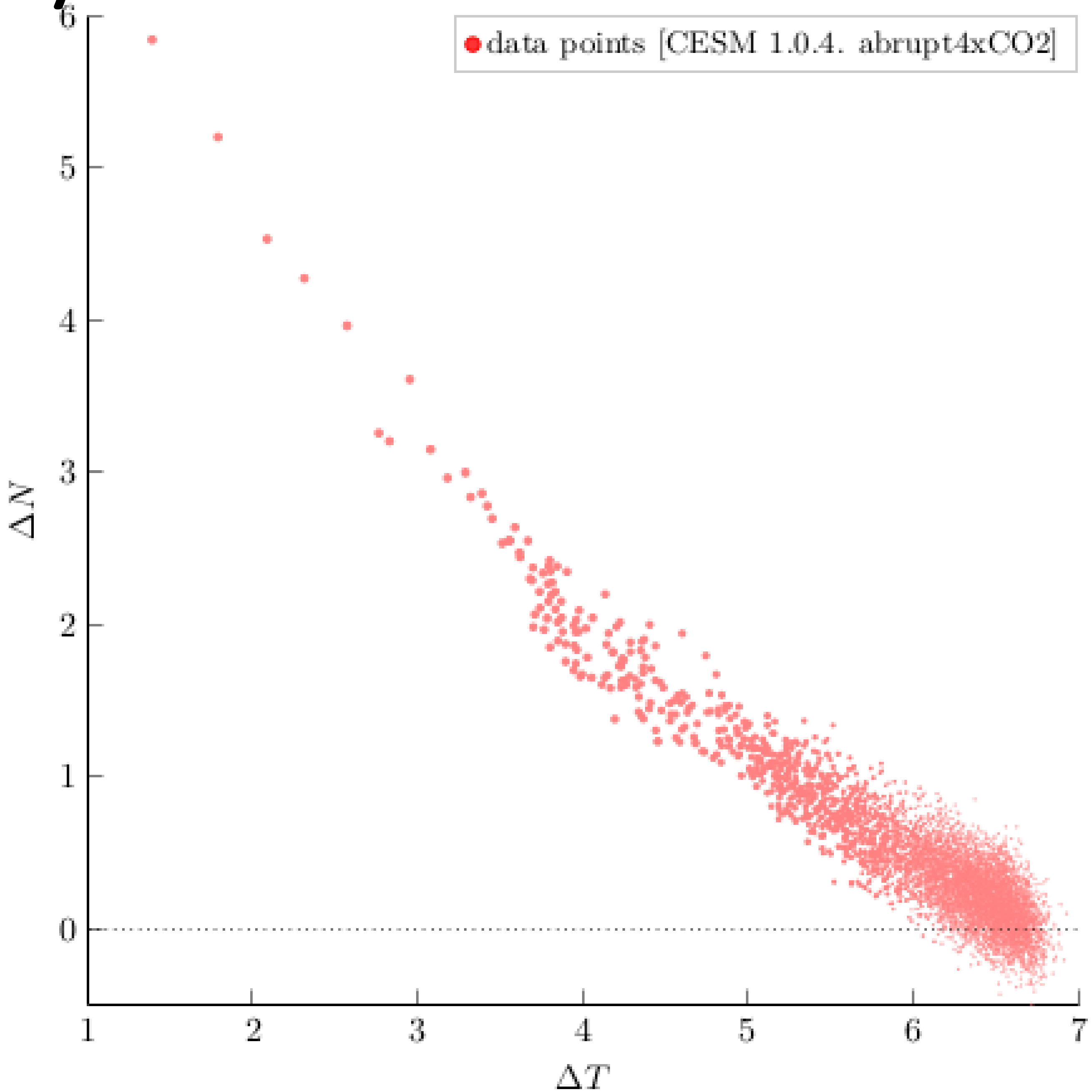

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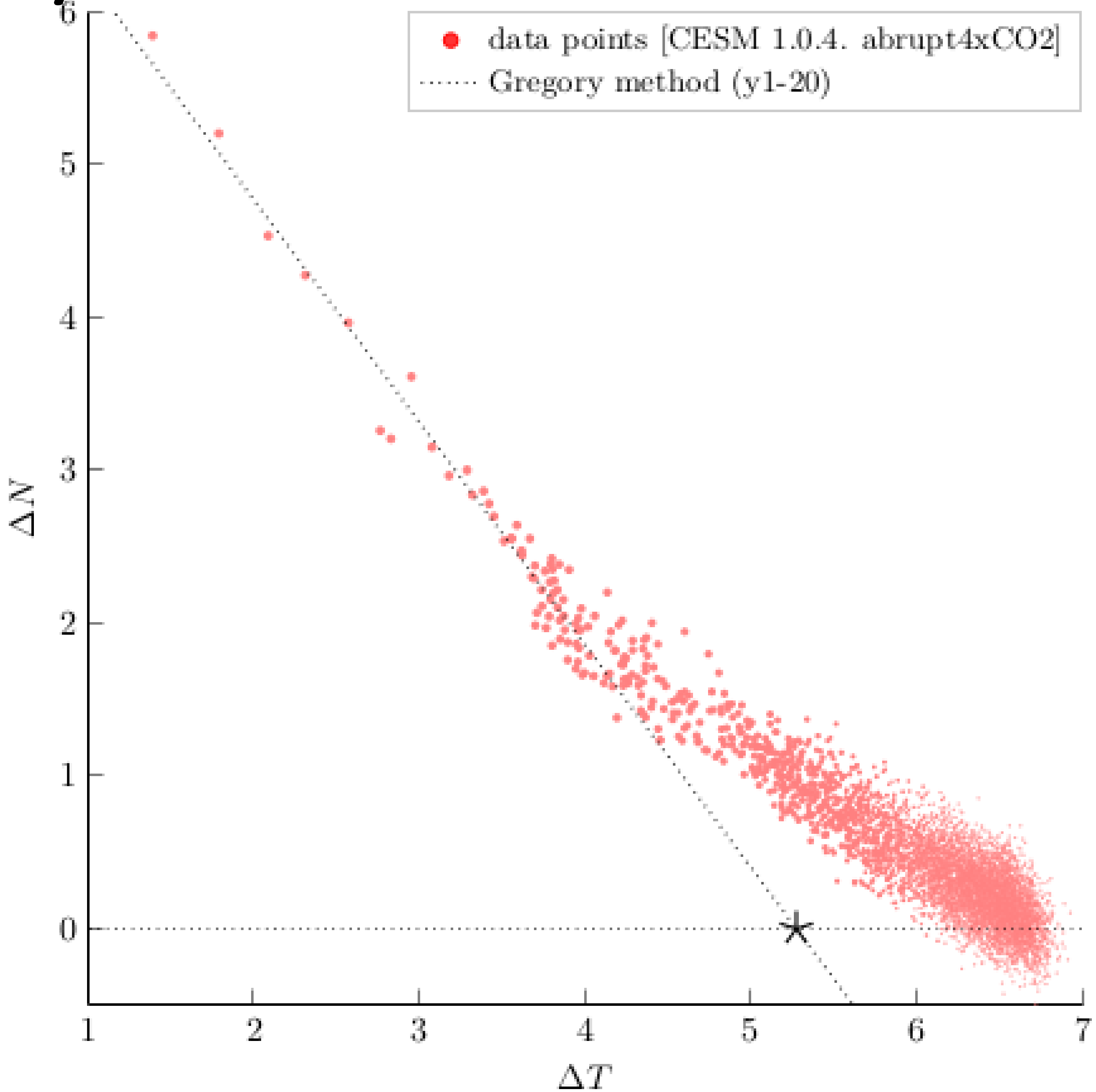

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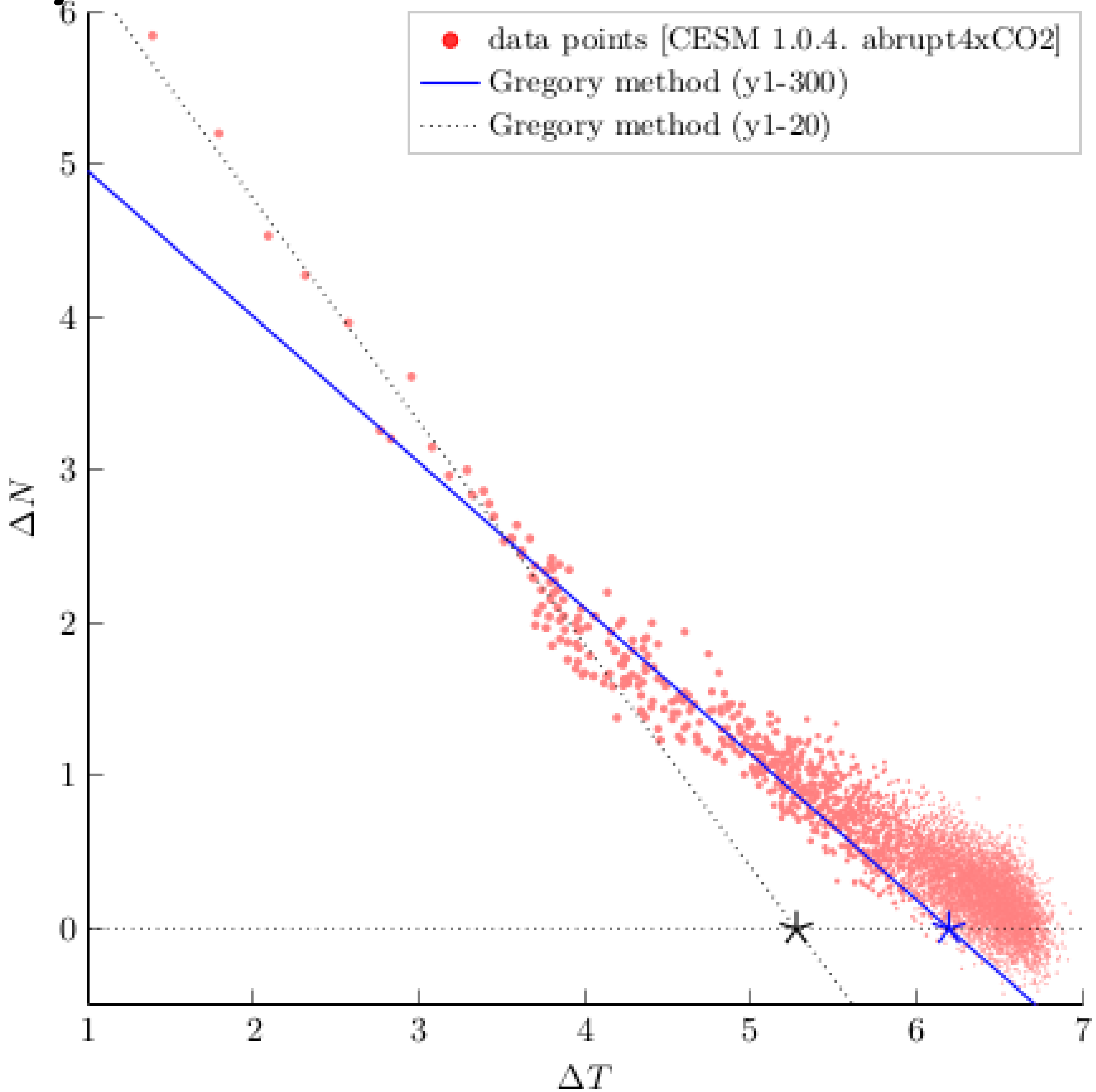

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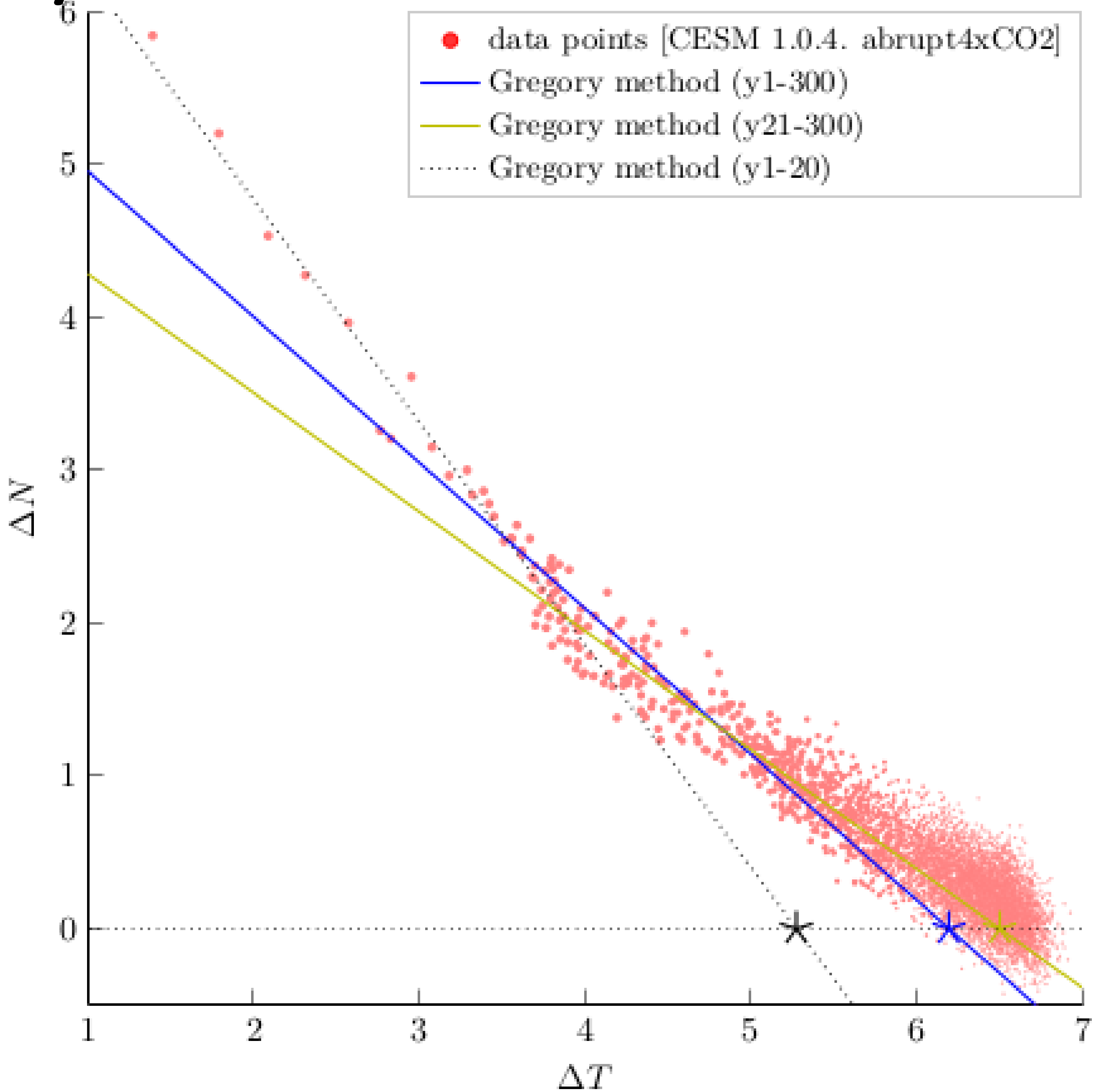

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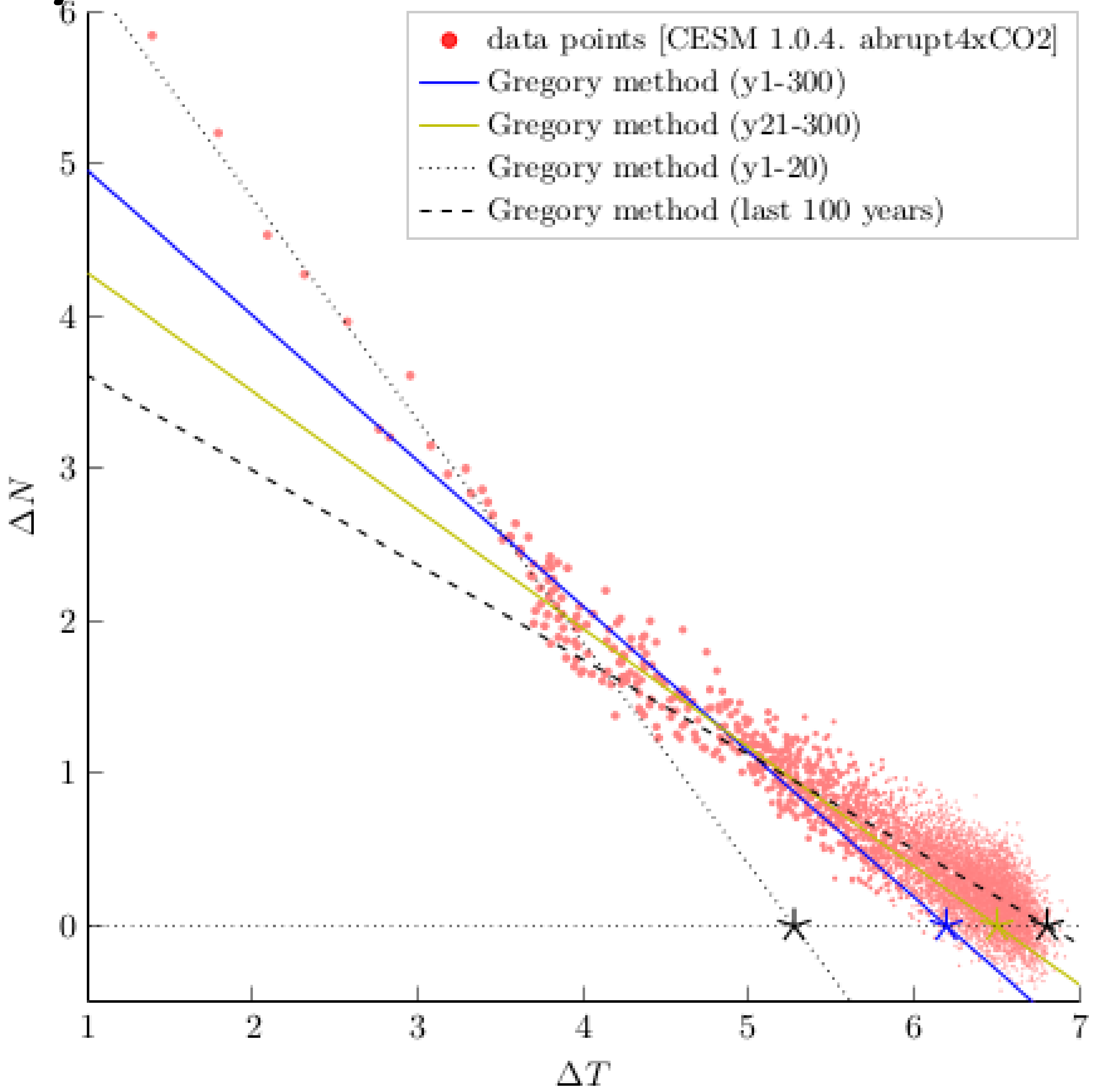


[Gregory et al, 2014]

New Multicomponent Linear Regression Method

Use additional observables!

Regress to:

$$\overrightarrow{\Delta Y} = \boldsymbol{A}\,\overrightarrow{\Delta X} + \vec{\boldsymbol{F}}$$

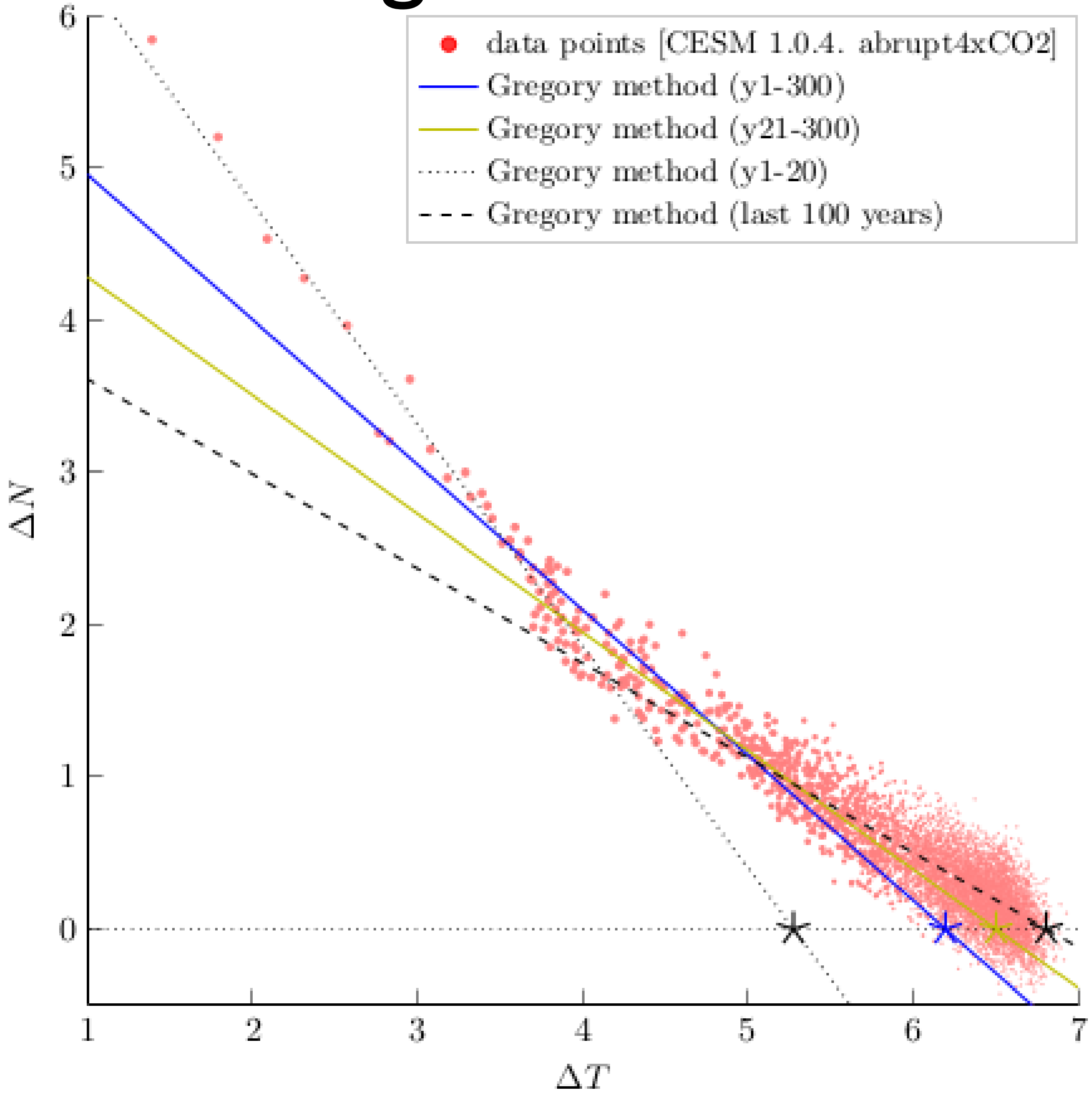
$\overrightarrow{\Delta Y}$: observables that **tend to 0** in equilibrium

$\overrightarrow{\Delta X}$: observables that **get estimated** in equilibrium

Multivariate ECS estimation is

$$\overrightarrow{\Delta X}_*^{est} = -\boldsymbol{A}^{-1}\,\vec{\boldsymbol{F}}$$

[Bastiaansen, Dijkstra, Von der Heydt, 2021]



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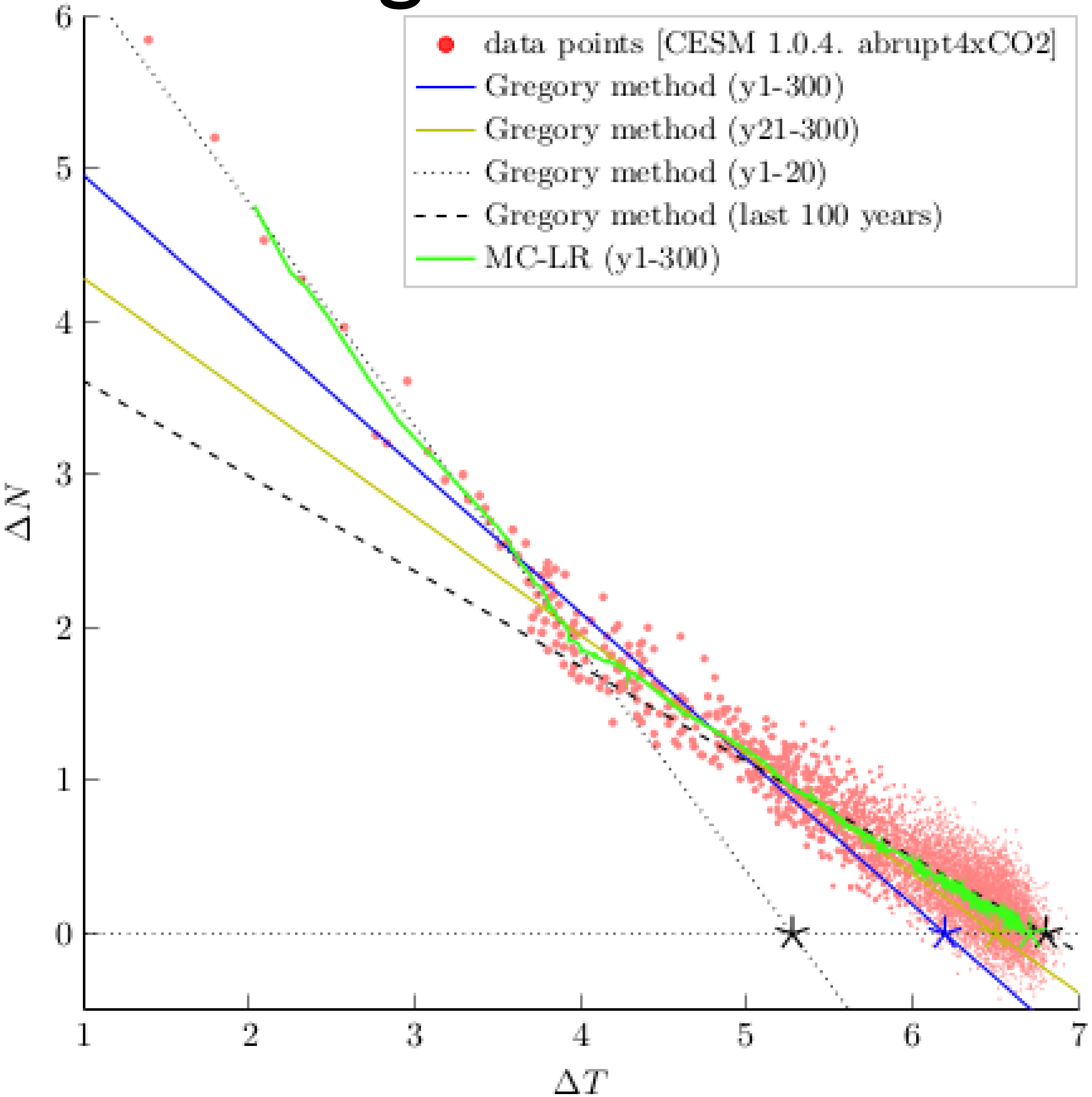


Projections for other forcings

$$\Delta O(t) = \sum_{m=1}^{M} \beta_m^{[O]} \, \mathcal{M}_m^{g}(t)$$

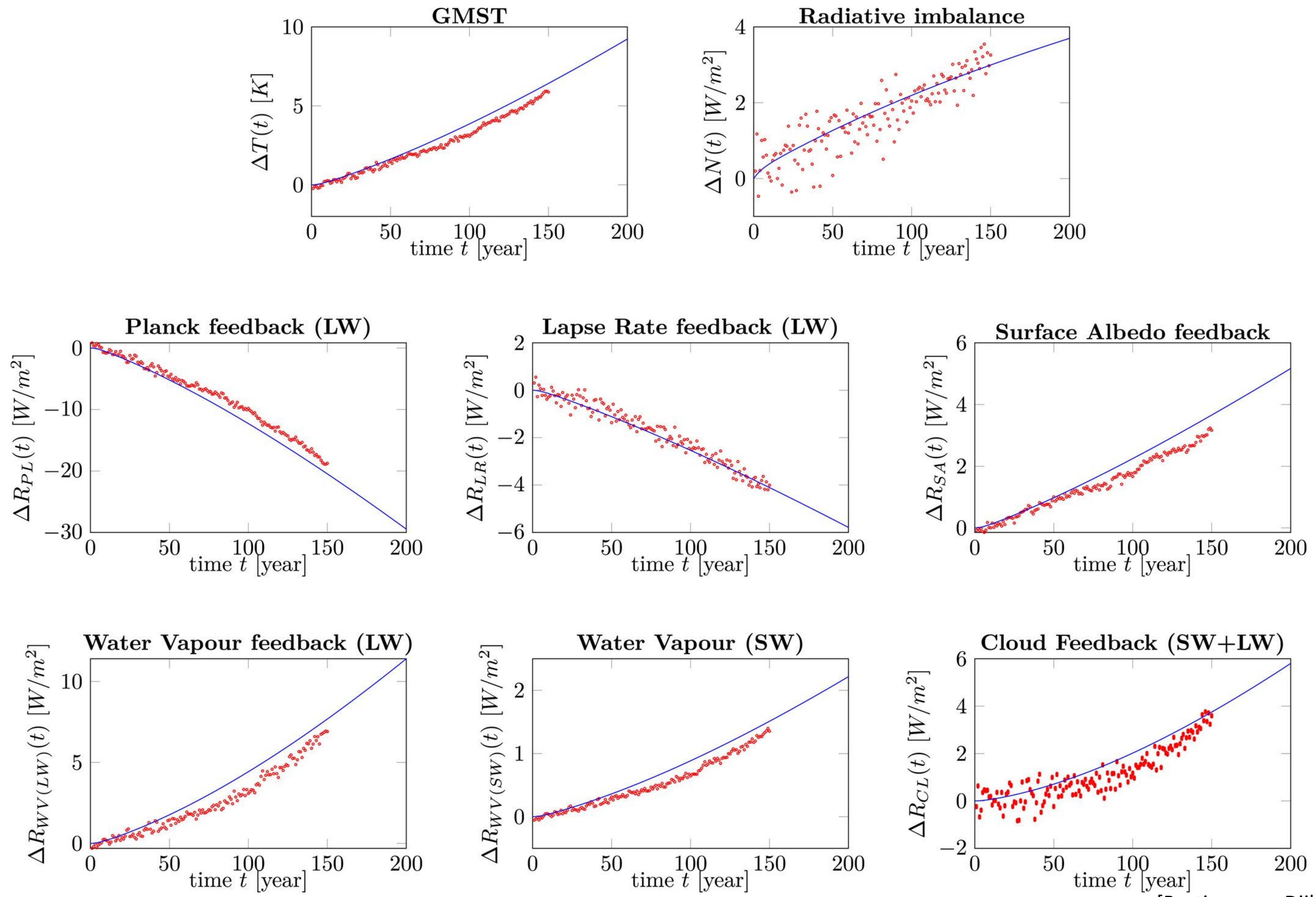
only this gets changed!

Linear Response Theory – CAVEATS:

i. forcings & responses should be ‘small enough’

ii. should look at ensemble means

Projections for CESM2's 1pctCO2 experiment



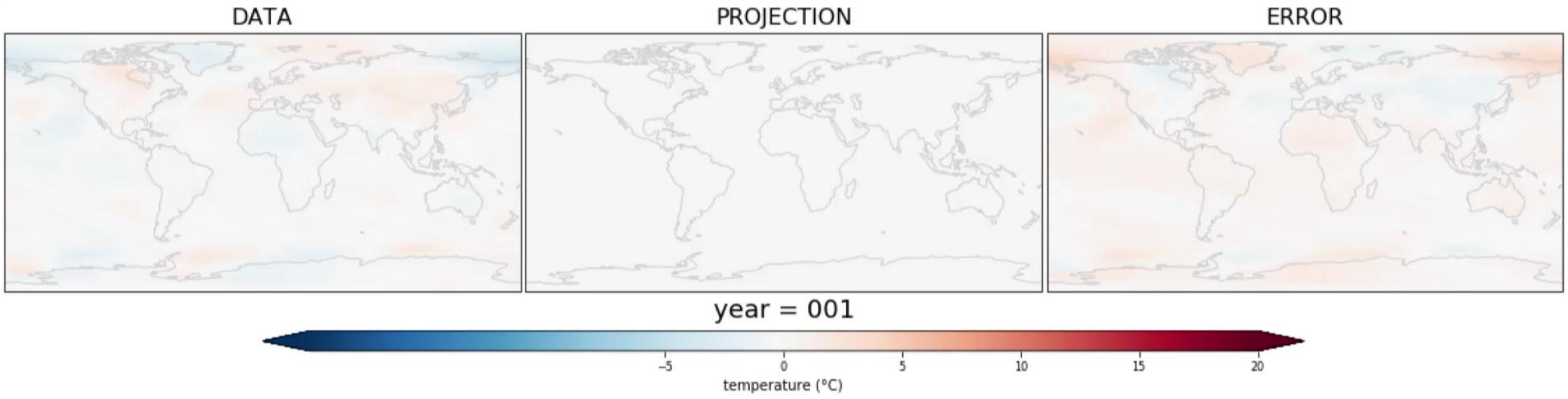
[Bastiaansen, Dijkstra, Von der Heydt, 2021]

Spatial projections for 1%CO2 experiment

Spatial Response

$$\Delta O(\boldsymbol{x},\mathrm{t}) = \sum_{m=1}^{M} \beta_m^{[O]}(\boldsymbol{x})\, \mathcal{M}_m^{g}(t)$$

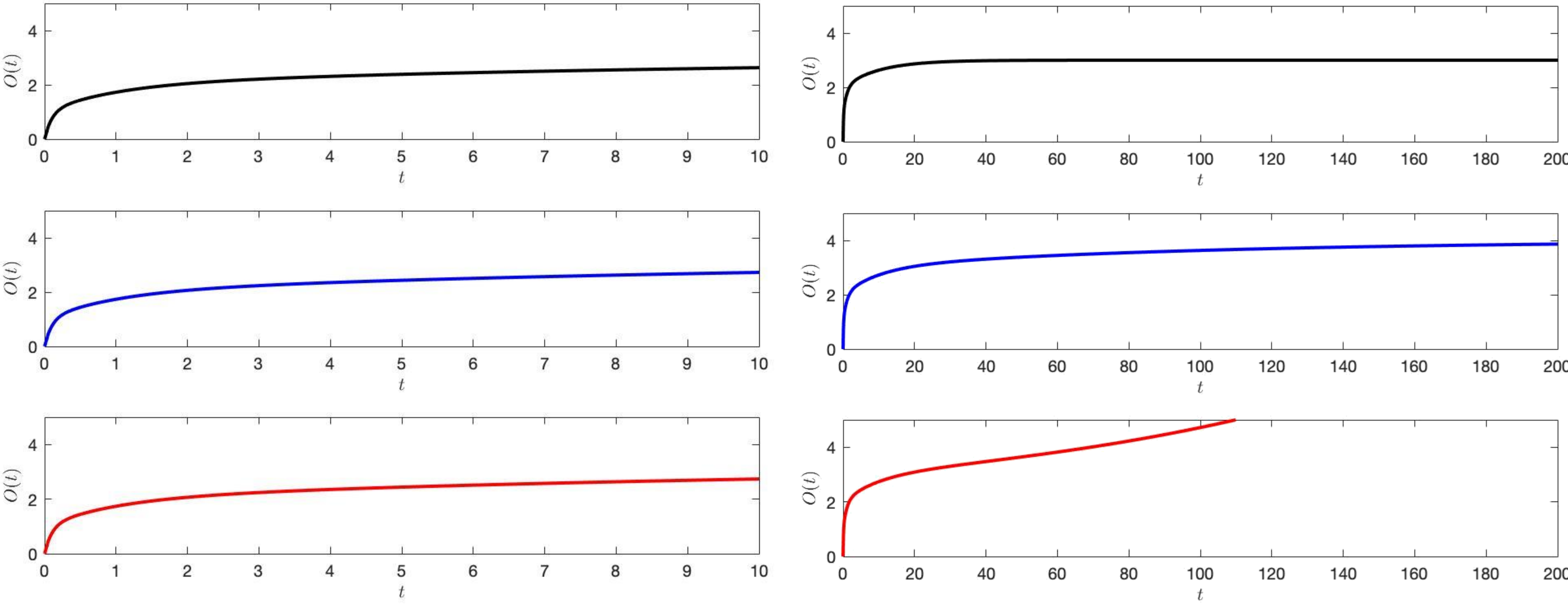
TEMPERATURE



[Bastiaansen, Dijkstra, Von der Heydt, 2021]

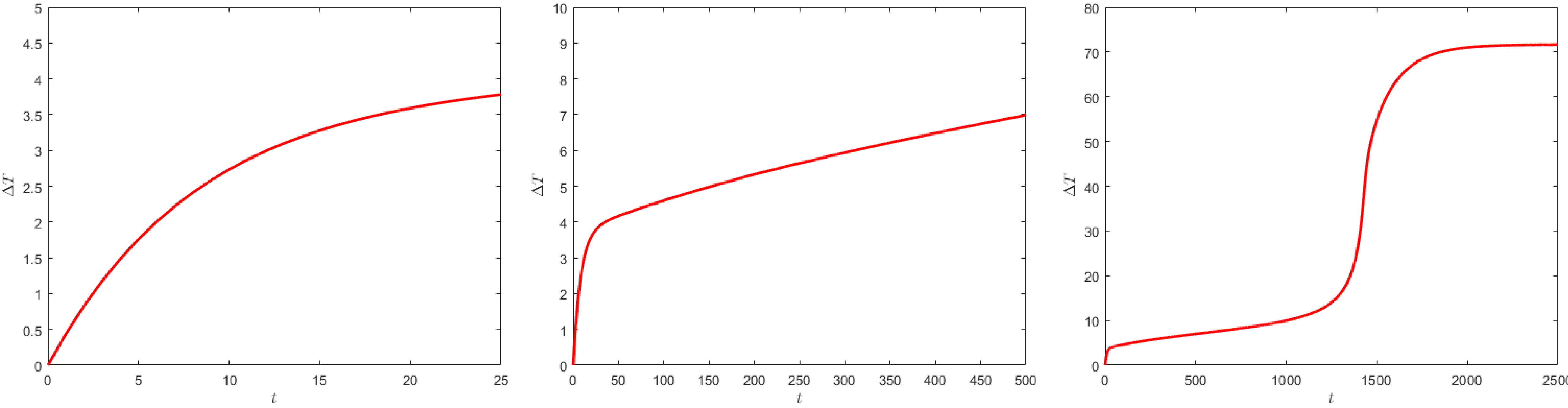
PART 2 LIMITATIONS OF LINEAR THEORY

Pitfalls and problems



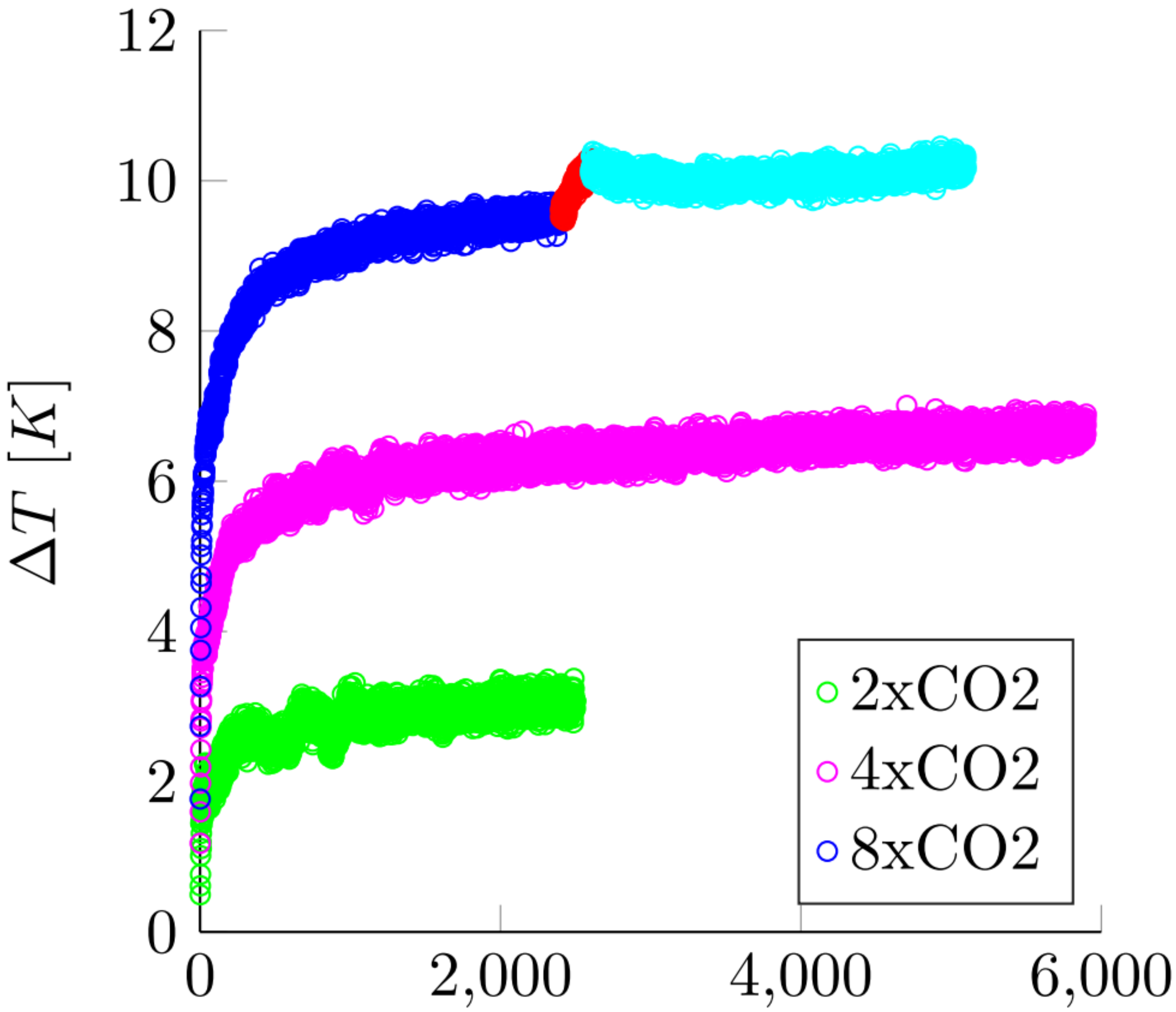
[Bastiaansen, Ashwin, Von der Heydt, 2022, *preprint*]

Nonlinear Response



[Bastiaansen, Ashwin, Von der Heydt, 2022, *preprint*]

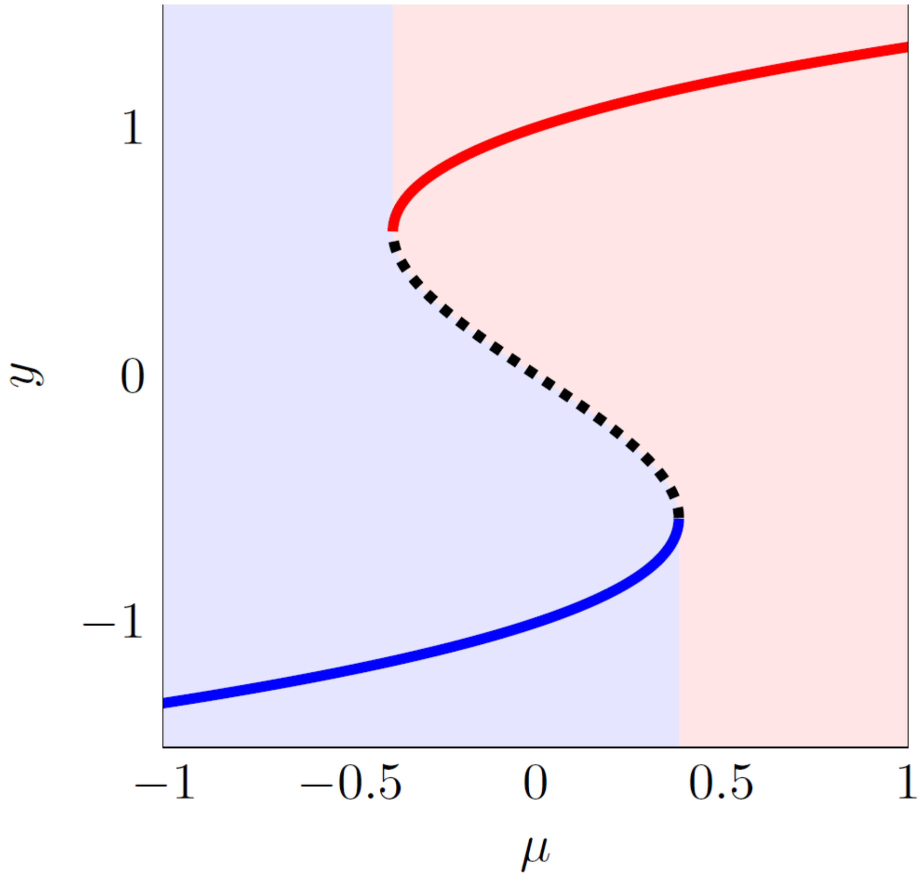
Nonlinear Response

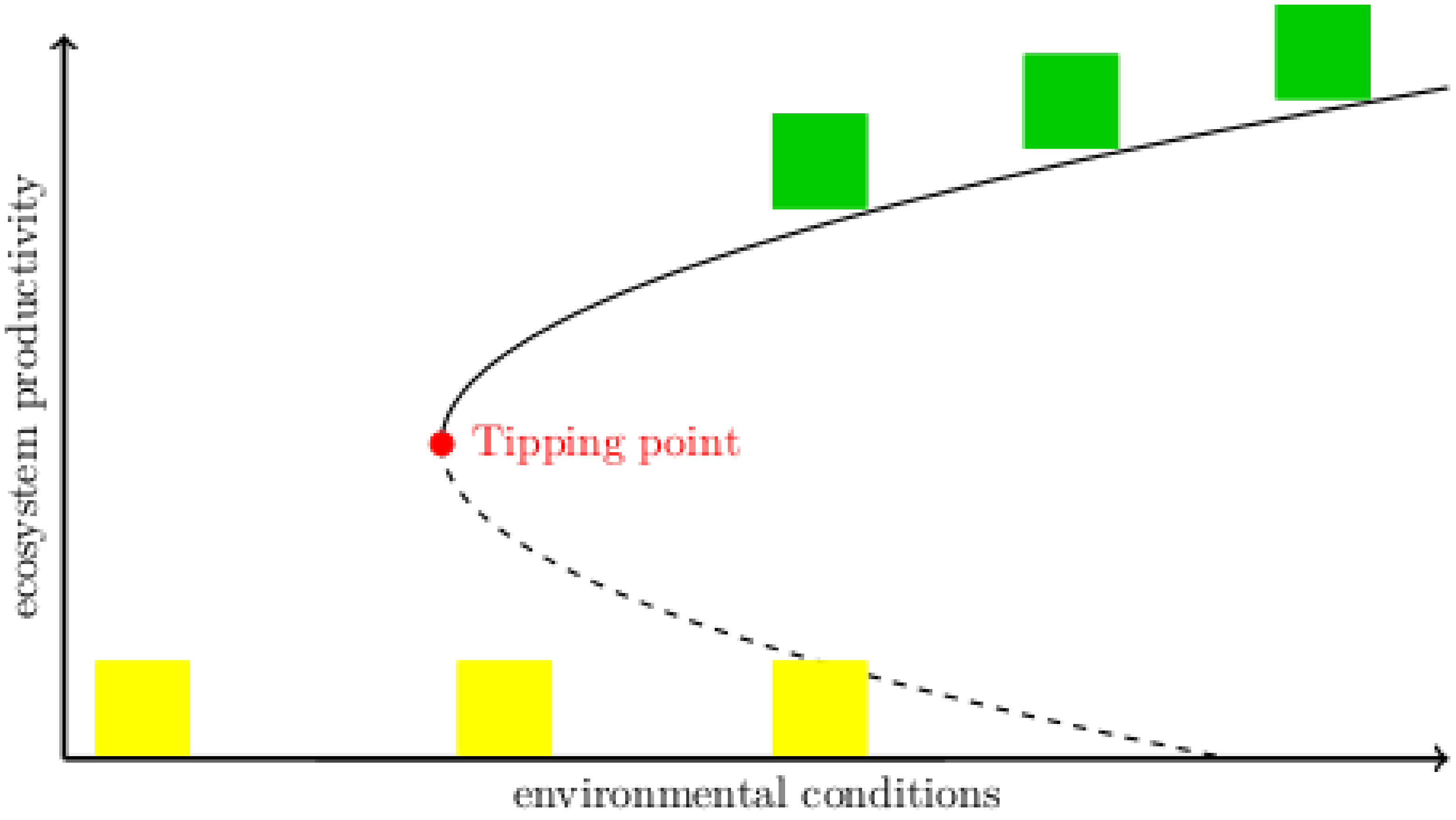


[Bastiaansen, Ashwin, Von der Heydt, 2022, *preprint*]

(Data from CESM1.0.4 runs in LongRunMIP)

Bifurcations / Tipping Points





Canonical example:

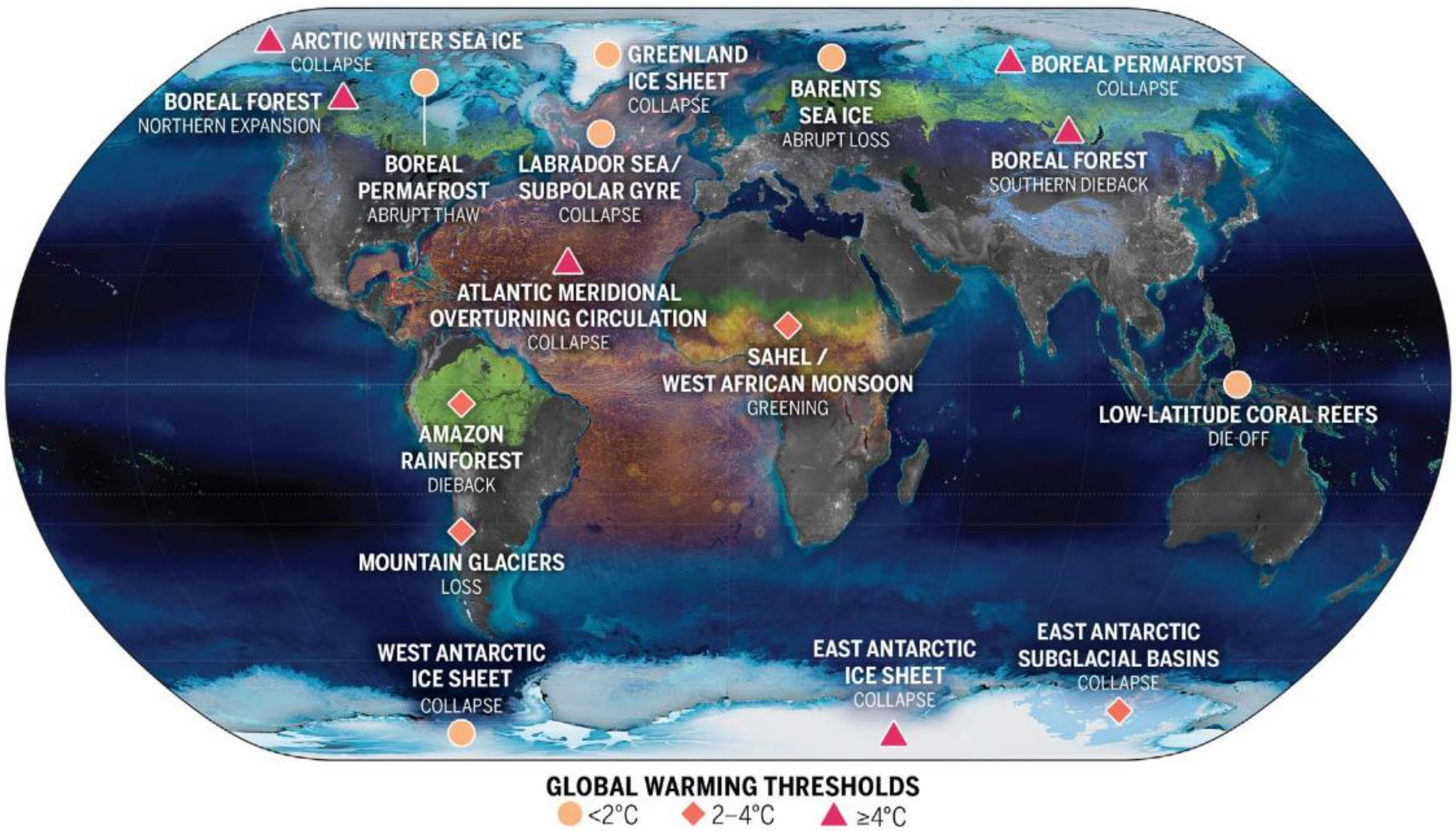
$$\frac{dy}{dt} = y(1 - y^2) + \mu$$

$$\frac{d\vec{y}}{dt} = f(\vec{y}; \mu)$$

$$\begin{cases} \frac{du}{dt} = f(u, v; \mu) \\ \frac{dv}{dt} = g(u, v; \mu) \end{cases}$$

Tipping Points

IPCC AR6 (2021) : "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly"

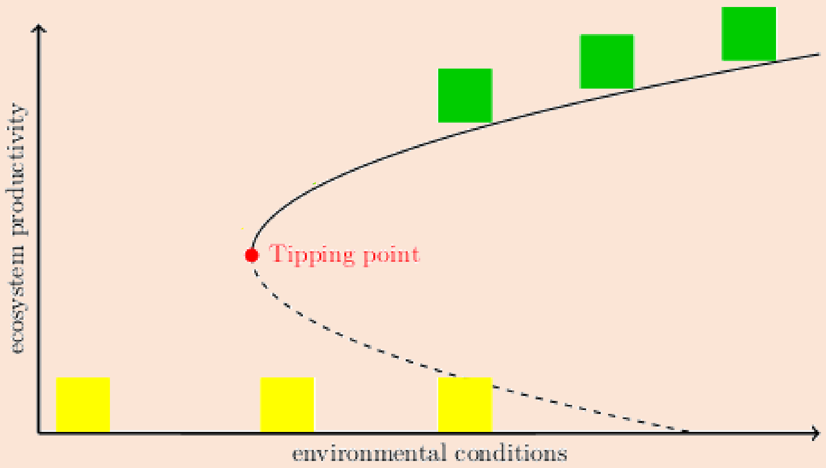


source: McKay et al, 2022

Mathematics

Tipping points ↔ Bifurcations

$$\frac{dy}{dt} = f(y, \mu)$$

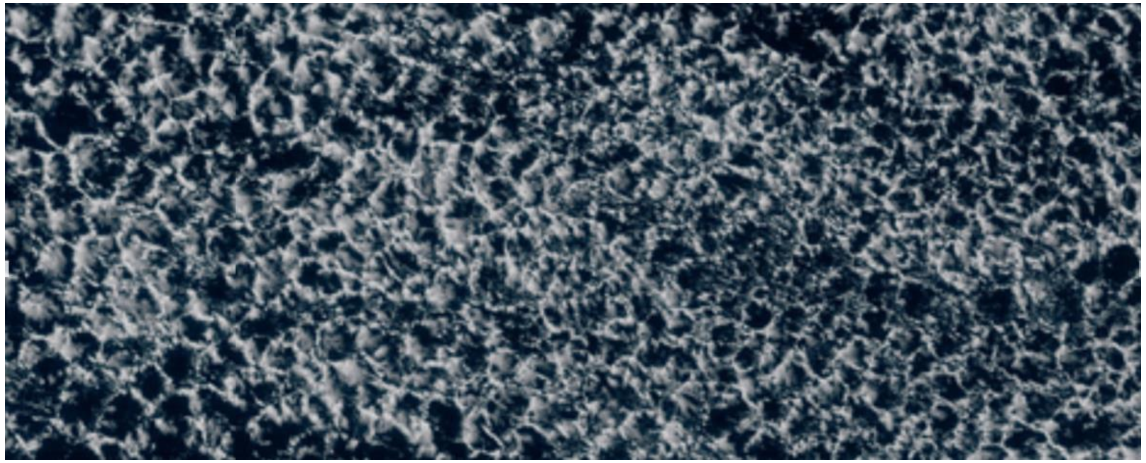


Part 3

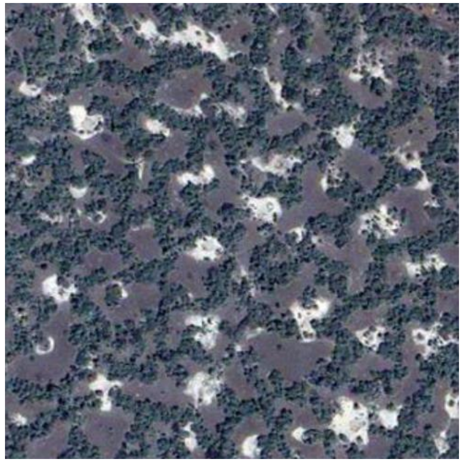
Tipping in spatially extended systems

Examples of spatial patterning – regular patterns

mussel beds

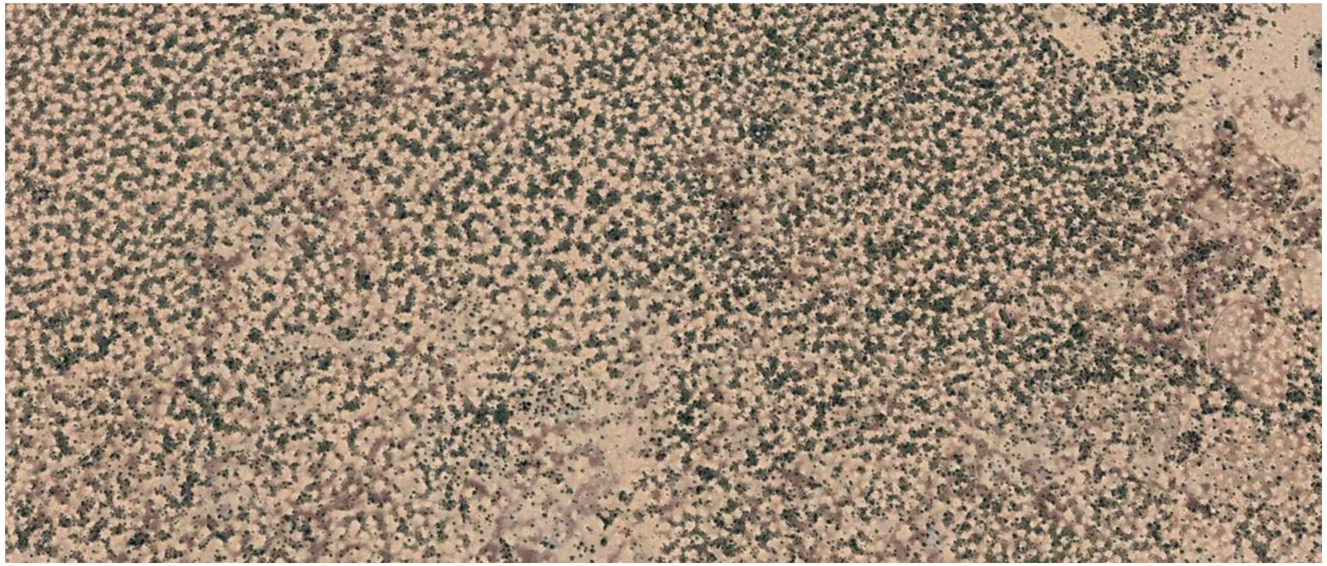


clouds



savannas

melt ponds

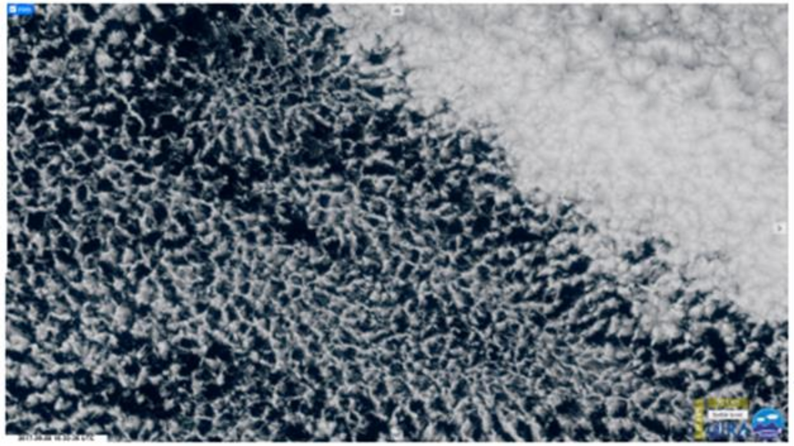


drylands

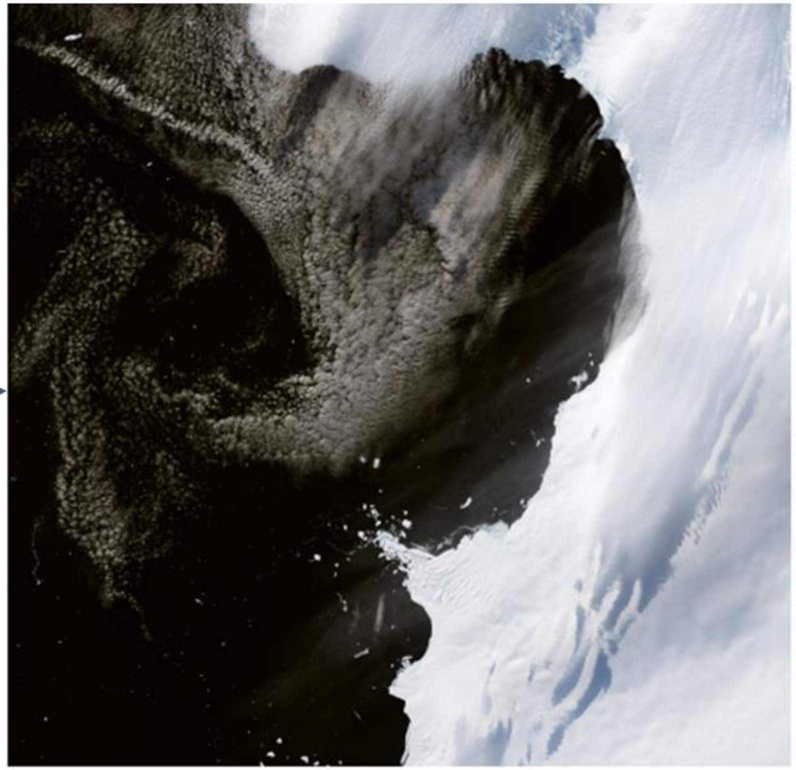
Examples of spatial patterning – spatial interfaces

tropical forest & savanna ecosystems
[Google Earth]

types of stratocumulus clouds
[RAMMB/CIRA SLIDER]



sea-ice & water at Eltanin Bay
[NASA's Earth observatory]



algae bloom in Lake St. Clair
[NASA's Earth observatory]

Part 3-A: Turing Patterns

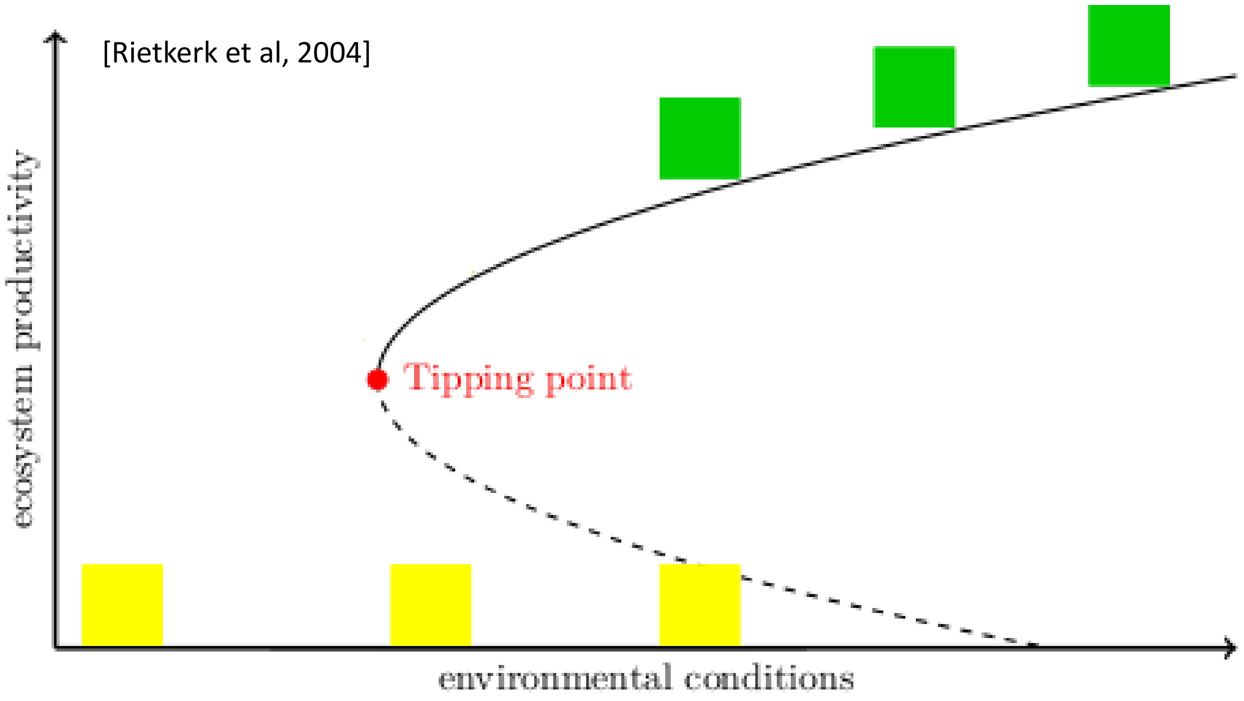
image courtesy: National Science Foundation

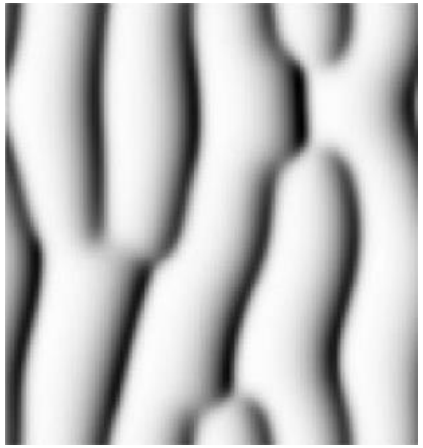
Patterns in models

Add spatial transport:

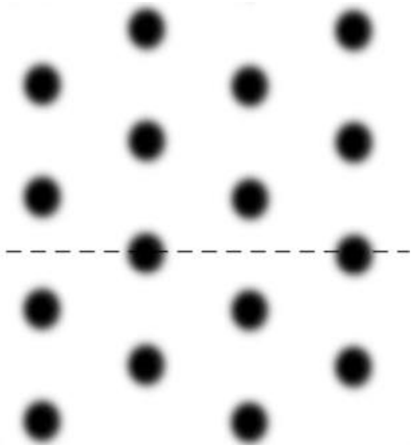
Reaction-Diffusion equations:

$$\begin{cases} \dfrac{du}{dt} = f(u,v) + D_u\,\Delta u \\ \dfrac{dv}{dt} = g(u,v) + D_v\,\Delta v \end{cases}$$

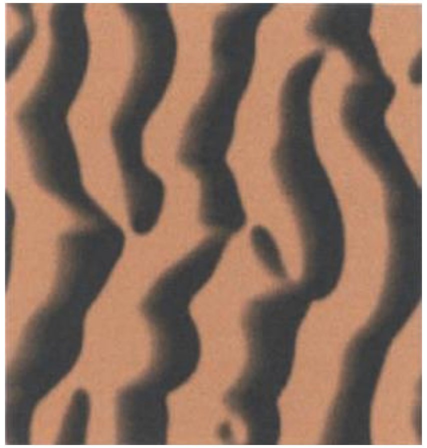




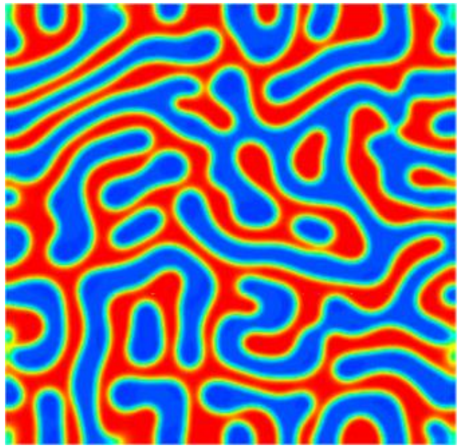
[Klausmeier, 1999]



[Gilad et al, 2004]

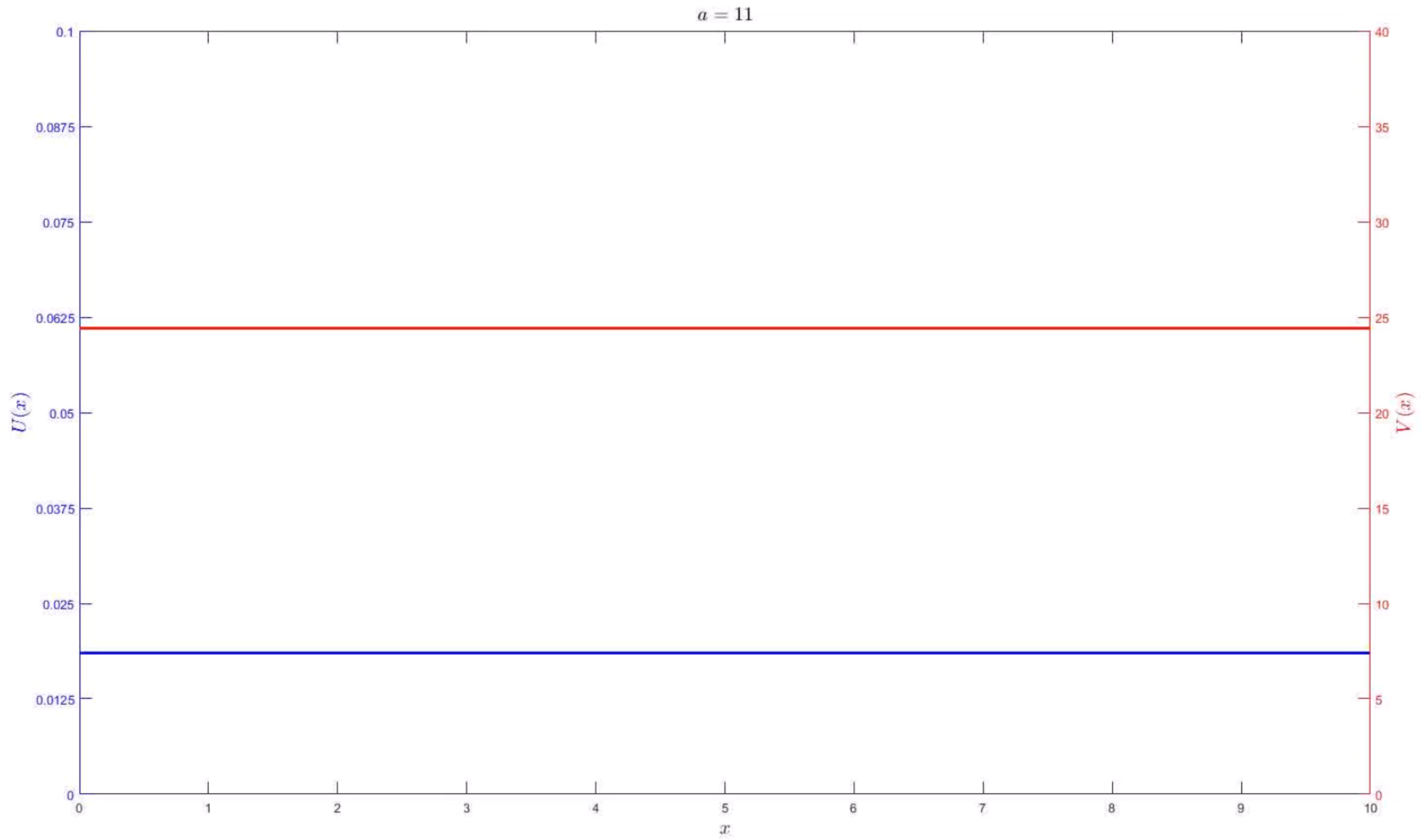


[Rietkerk et al, 2002]



[Liu et al, 2013]

Behaviour of PDEs



Turing patterns

$$\begin{cases} \frac{du}{dt} = f(u,v) + D_u \, \Delta u \\ \frac{dv}{dt} = g(u,v) + D_v \, \Delta v \end{cases}$$

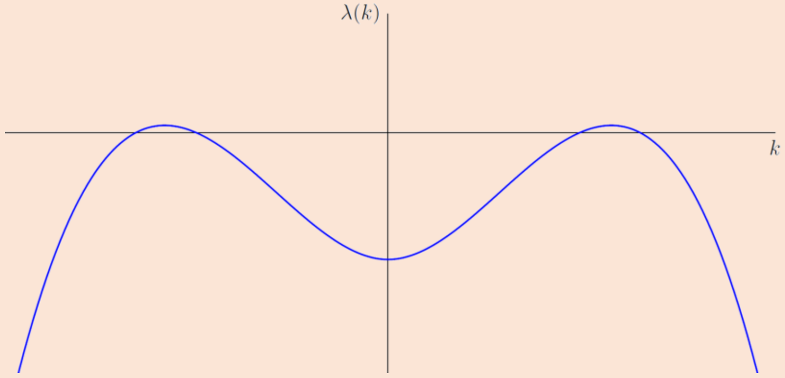
Turing bifurcation

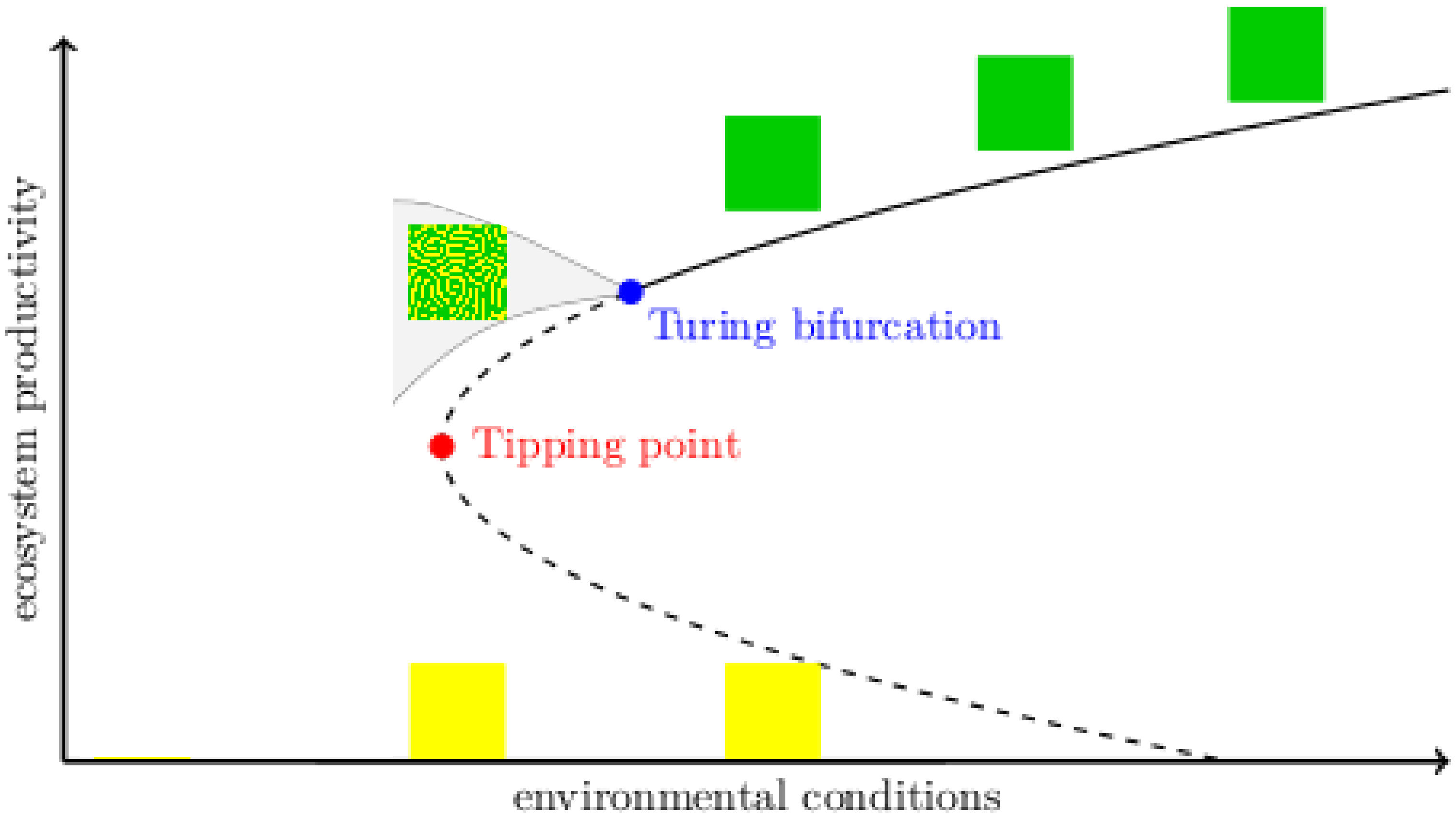
Instability to non-uniform perturbations

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_* \\ v_* \end{pmatrix} + e^{\lambda t} e^{ikx} \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}$$

→ Dispersion relation

$$\lambda(k) = \cdots$$





Weakly non-linear analysis

Ginzburg-Landau equation / Amplitude Equation

& Eckhaus/Benjamin-Feir-Newel criterion

[Eckhaus, 1965; Benjamin & Feir, 1967; Newell, 1974]

Busse balloon

$$\begin{cases} \frac{du}{dt} = f(u,v) + D_u \Delta u \\ \frac{dv}{dt} = g(u,v) + D_v \Delta v \end{cases}$$

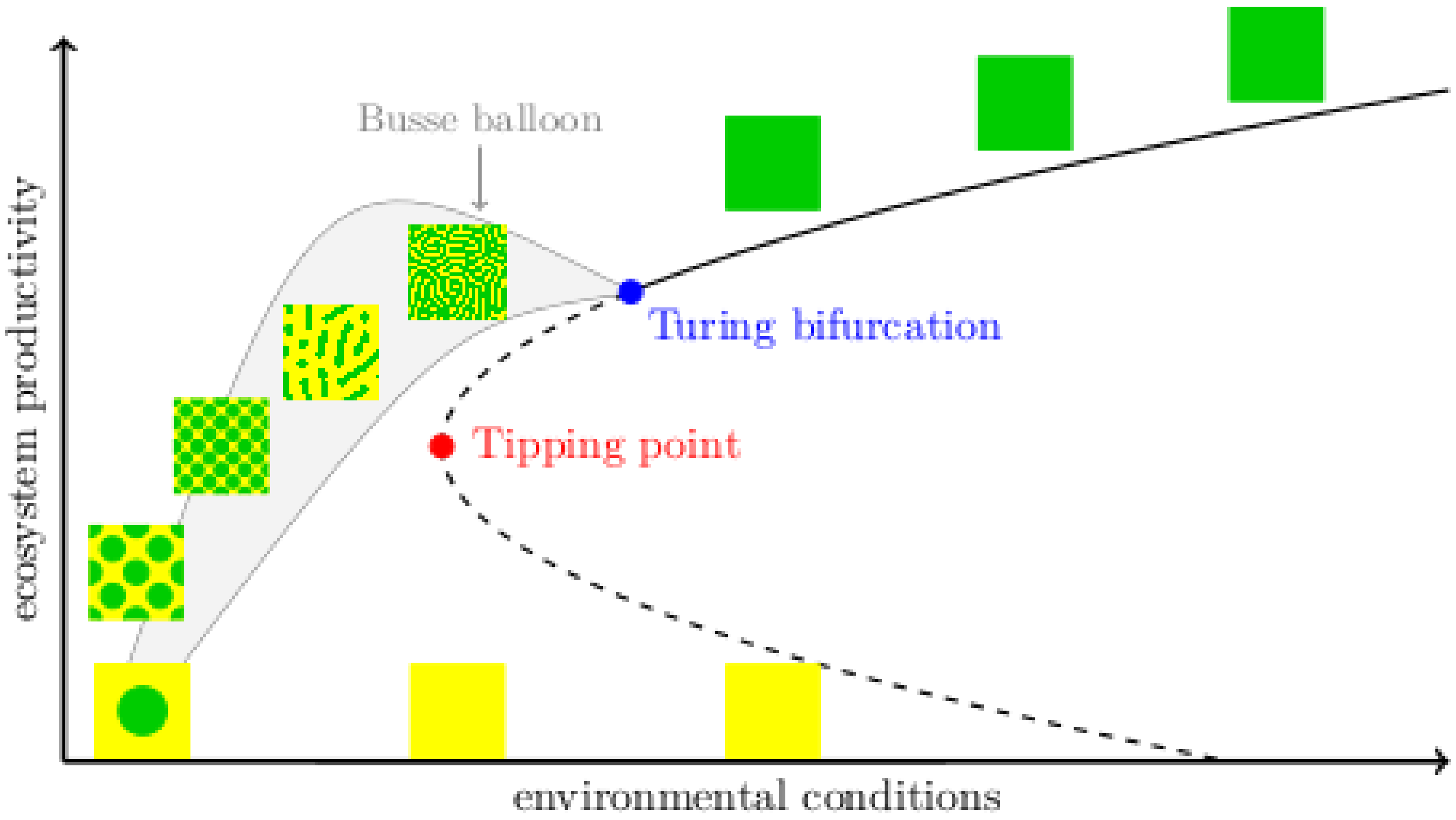
Busse balloon

A model-dependent shape in *(parameter, observable)* space that indicates all stable patterned solutions to the PDE.

Construction Busse balloon

Via numerical continuation

few general results on the shape of Busse balloon

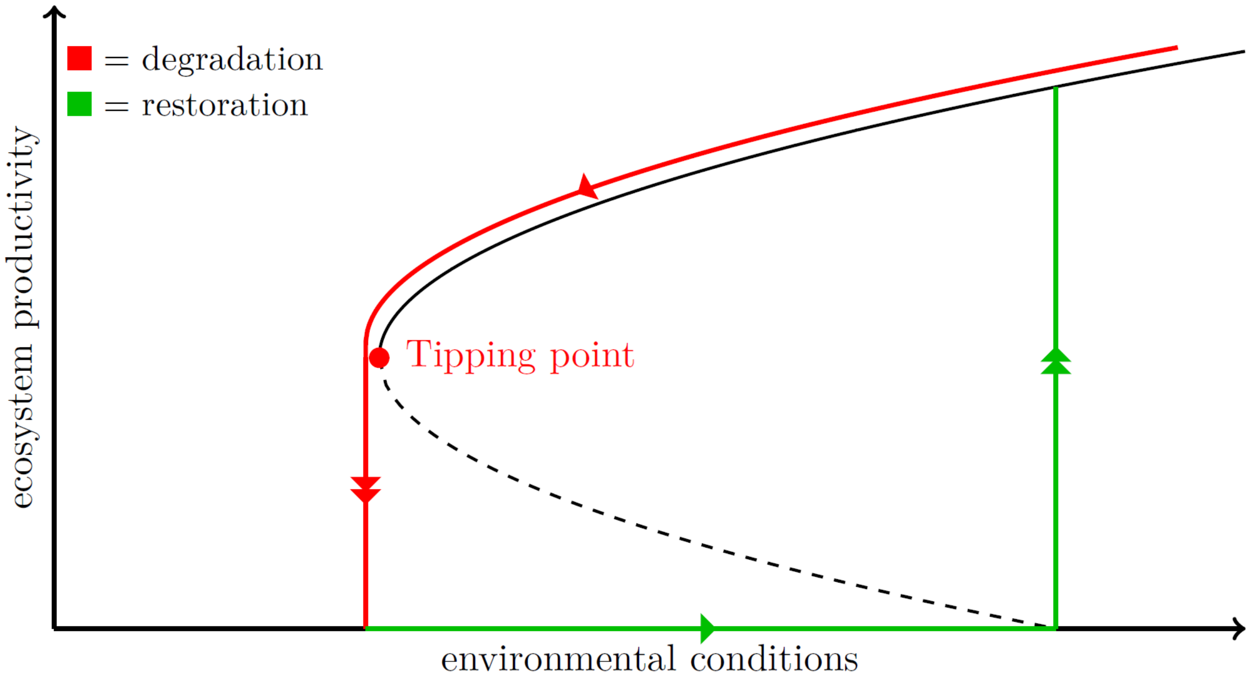


Busse balloon

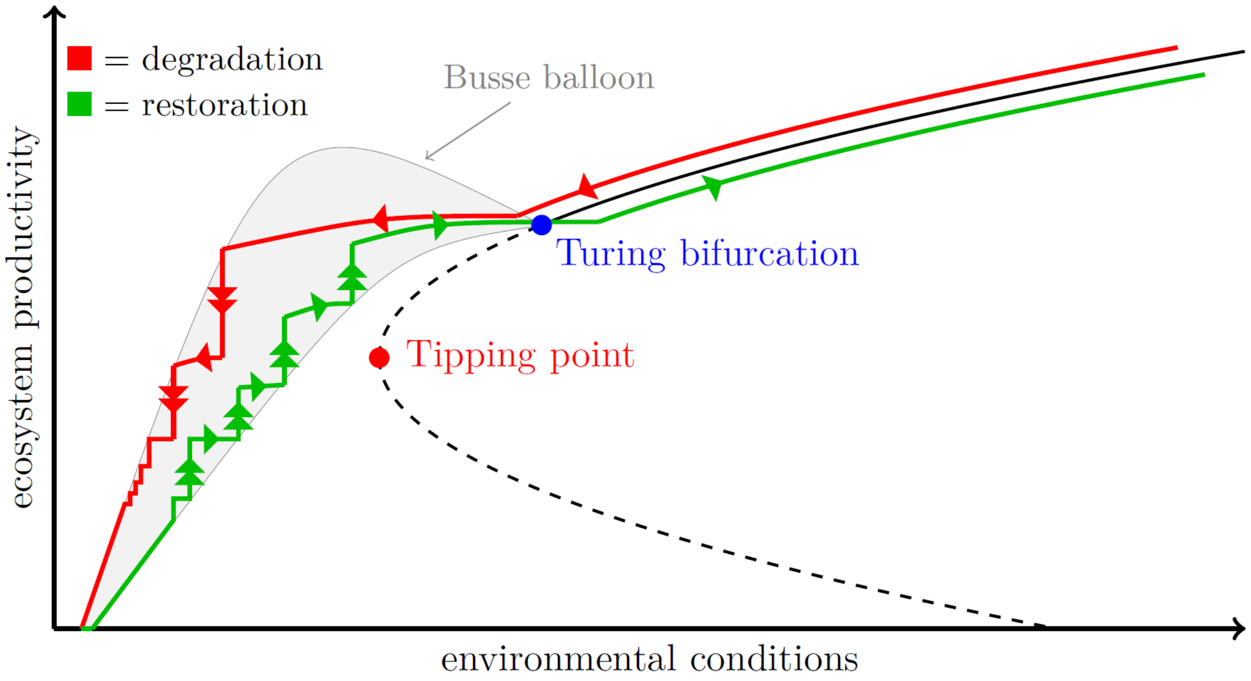
Idea originates from thermal convection

[Busse, 1978]

Tipping of (Turing) patterns

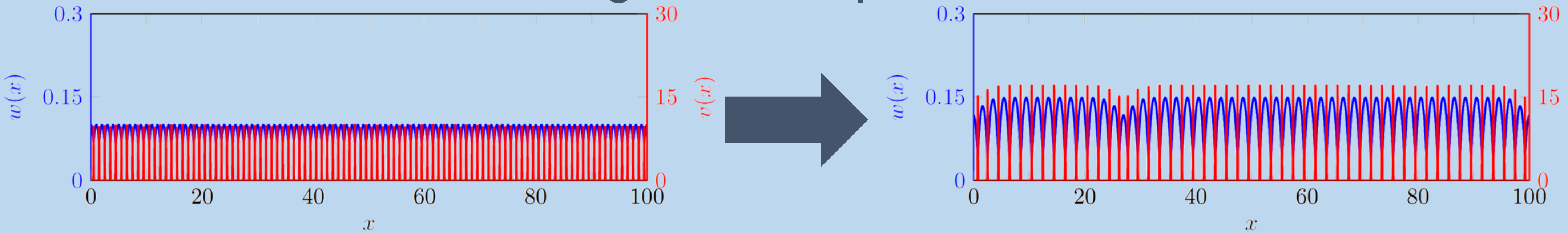


Classic tipping



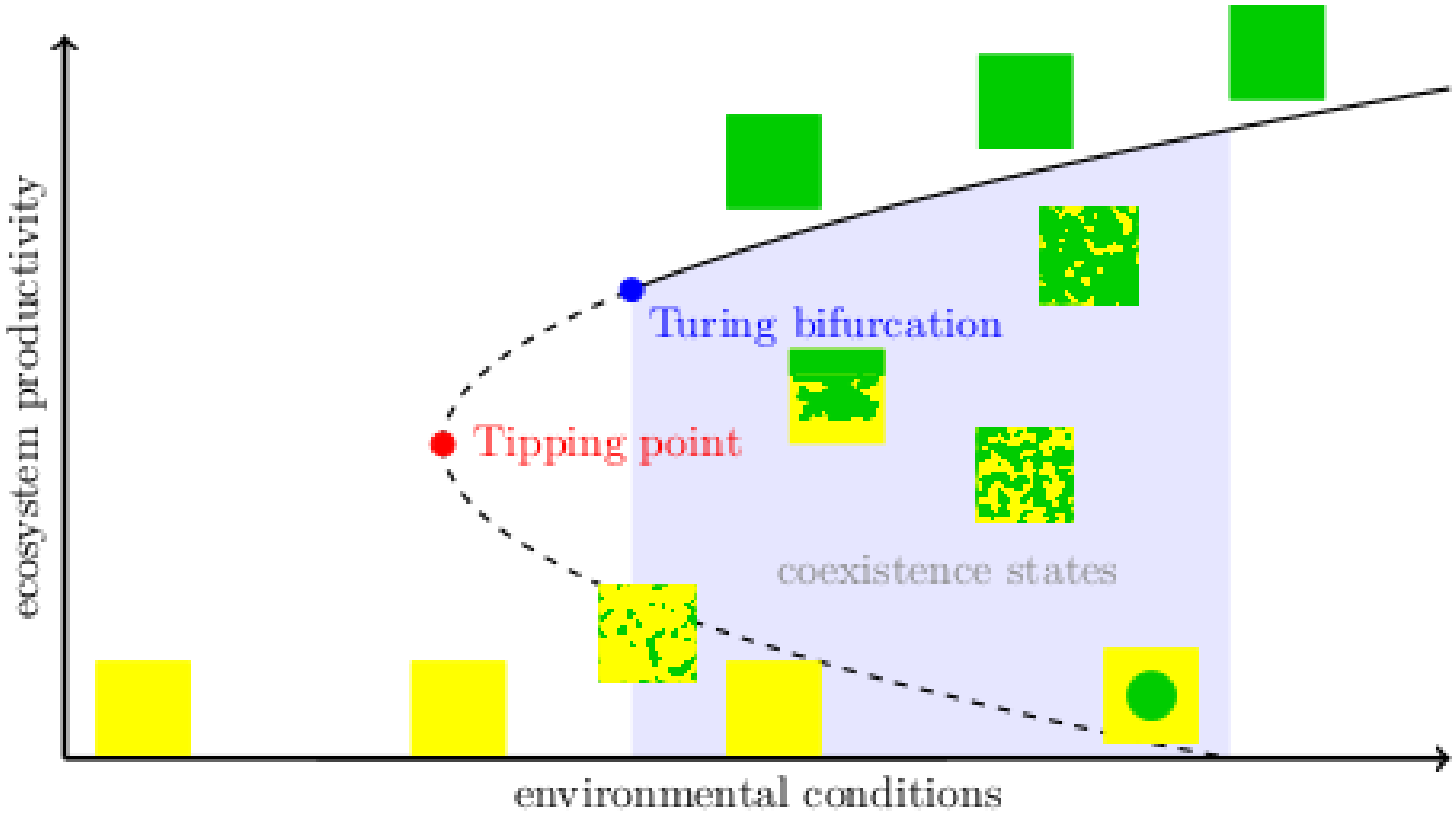
Tipping of patterns

Degradation of patterns



Part 3-B:
Coexistence States

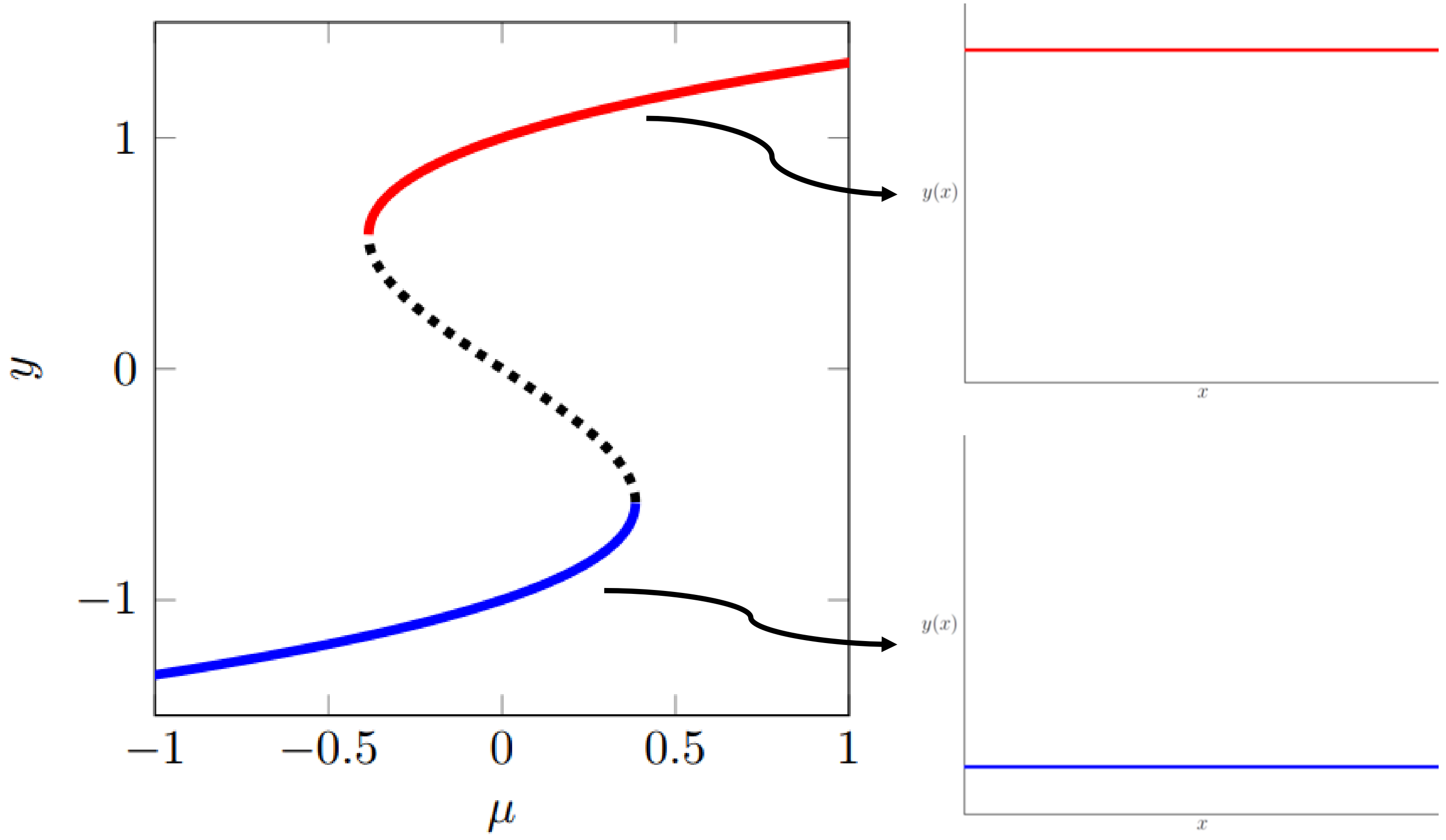
Coexistence states in bifurcation diagram

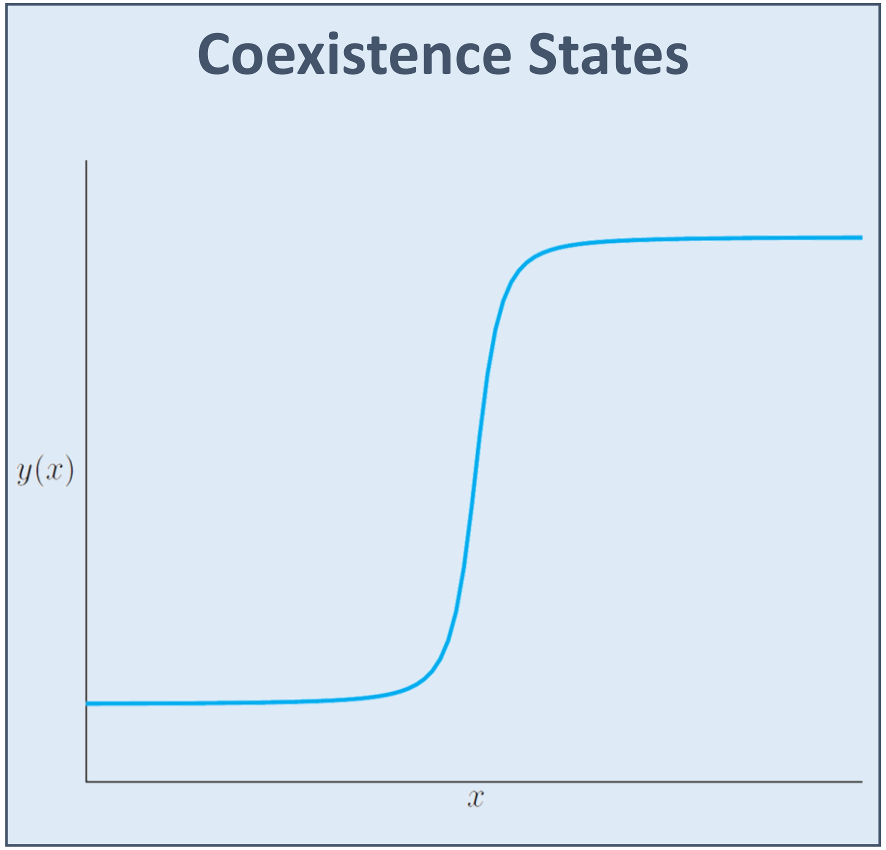


Coexistence states

Bistable (Allen-Cahn/Nagumo) equation:

$$\frac{\partial y}{\partial t} = y(1 - y^2) + \mu + D\,\frac{\partial^2 y}{\partial x^2}$$



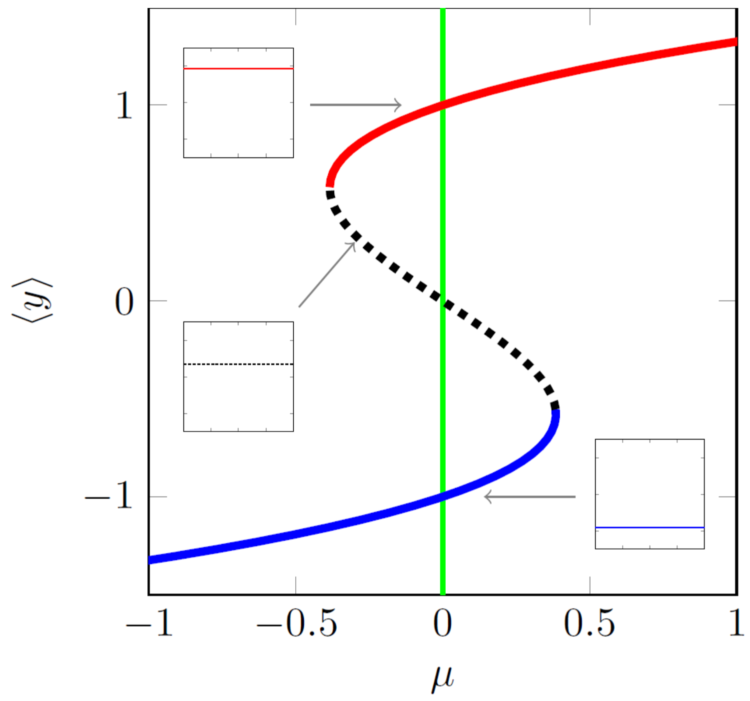


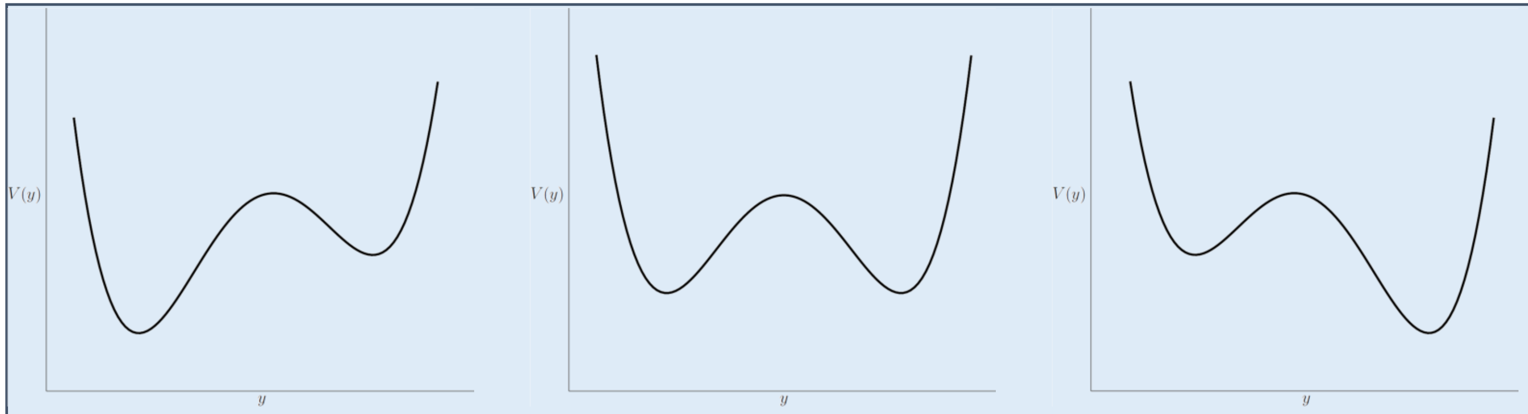
Front Dynamics

$$\frac{\partial y}{\partial t} = D\,\frac{\partial^2 y}{\partial x^2} + f(y;\mu)$$

Potential function $V(y;\mu)$:

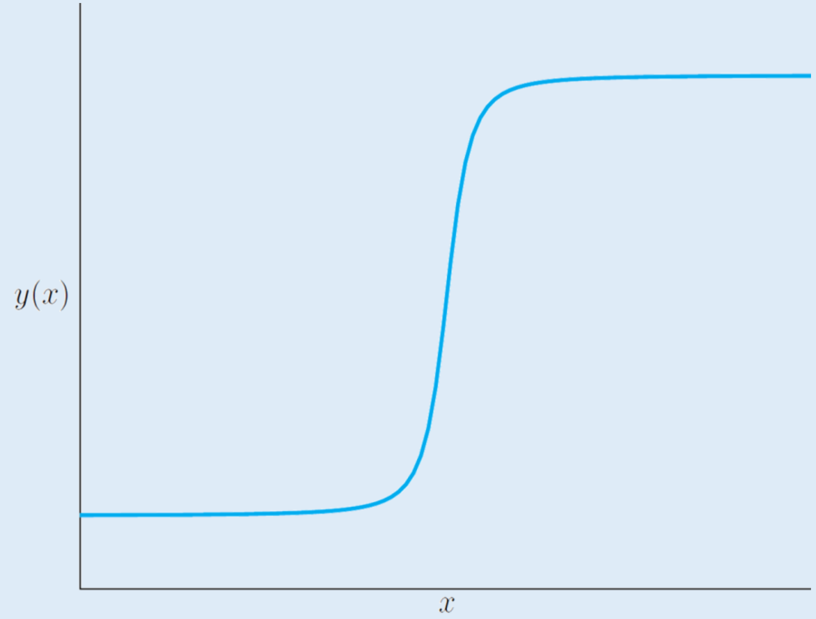
$$\frac{\partial V}{\partial y}(y;\mu) = -f(y;\mu)$$





moves right | stationary | moves left

Maxwell Point $\mu_{maxwell}$



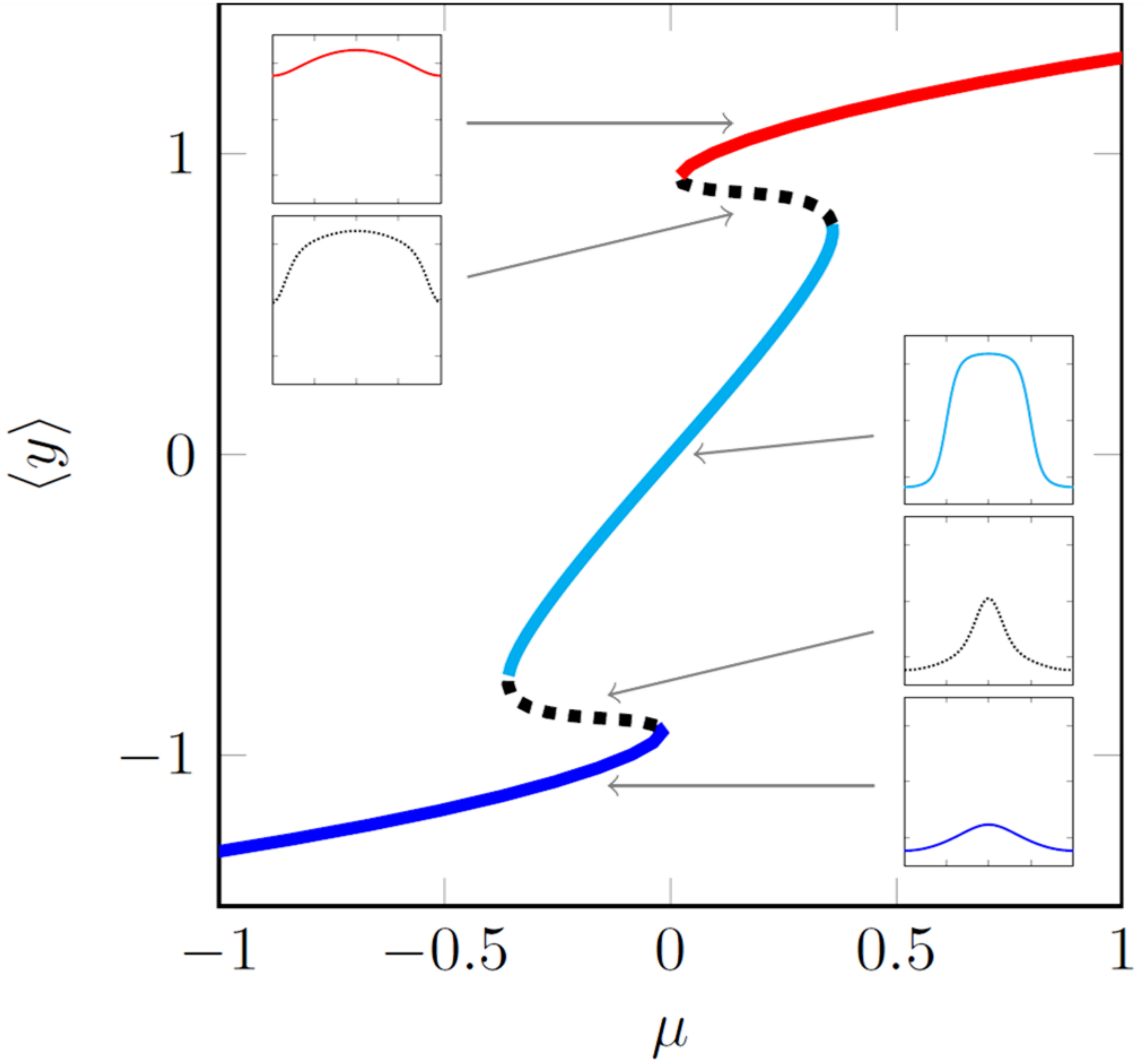
Adding Spatial Heterogeneity

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + f(y, x; \mu)$$

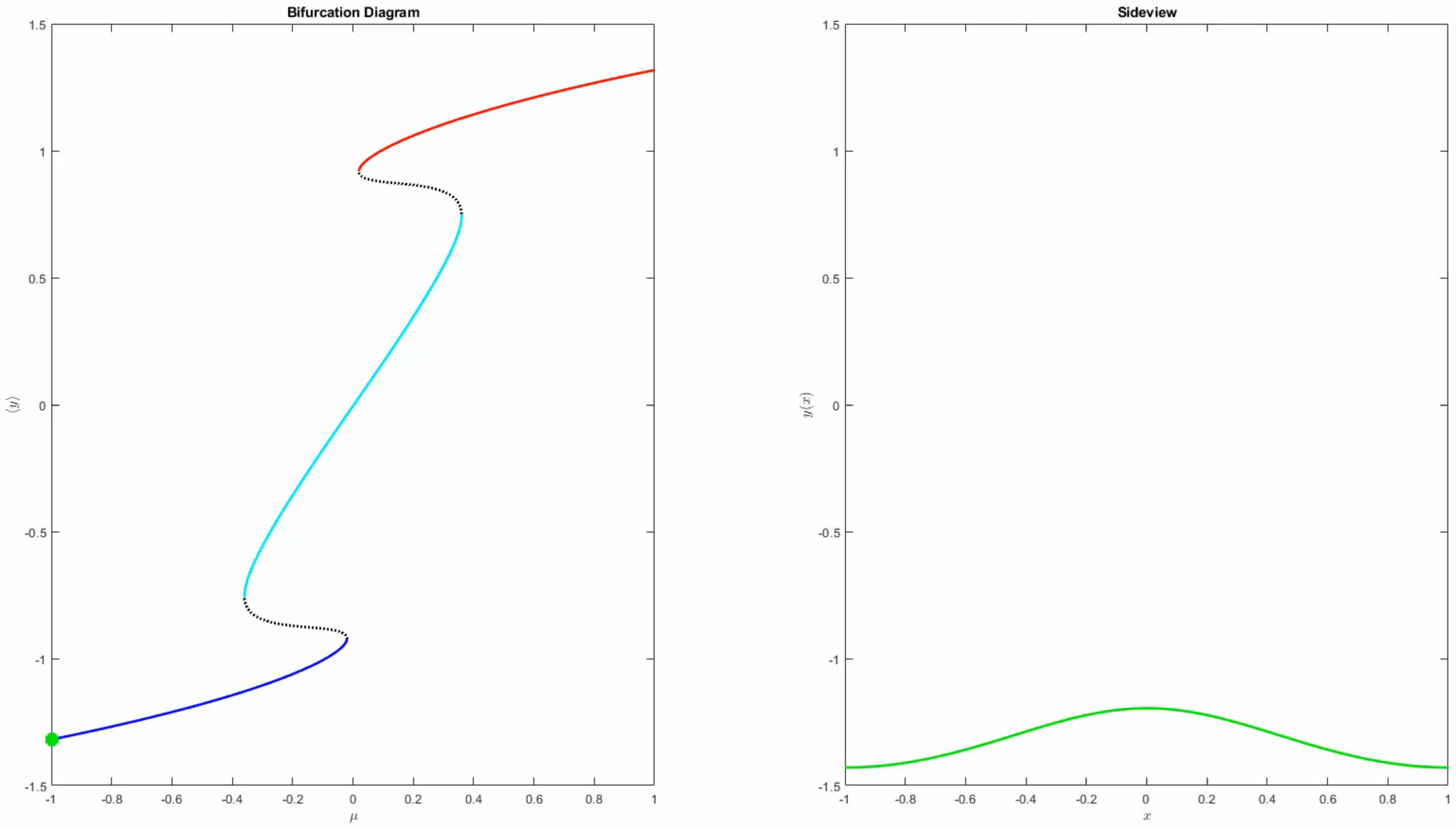
Now, the **local** difference in potentials determines the front movement

New behaviour:

- Multi-fronts can be stationary
- Maxwell point is smeared out

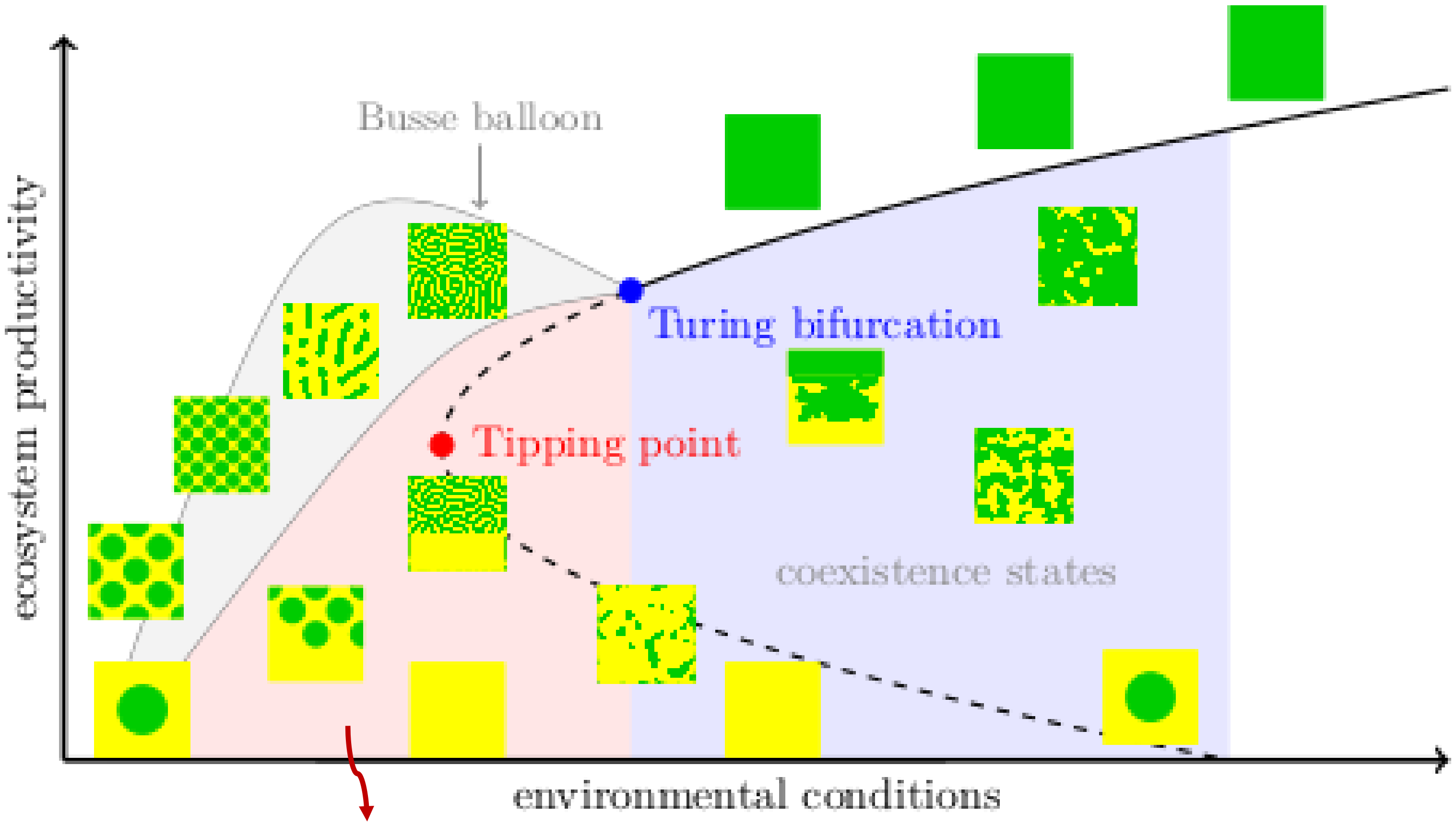


Fragmented Tipping

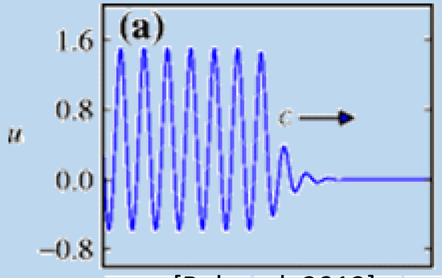



Part 3-C:
Tipping in Spatially
Extended Systems

“Bifurcation Diagram” for spatially extended systems



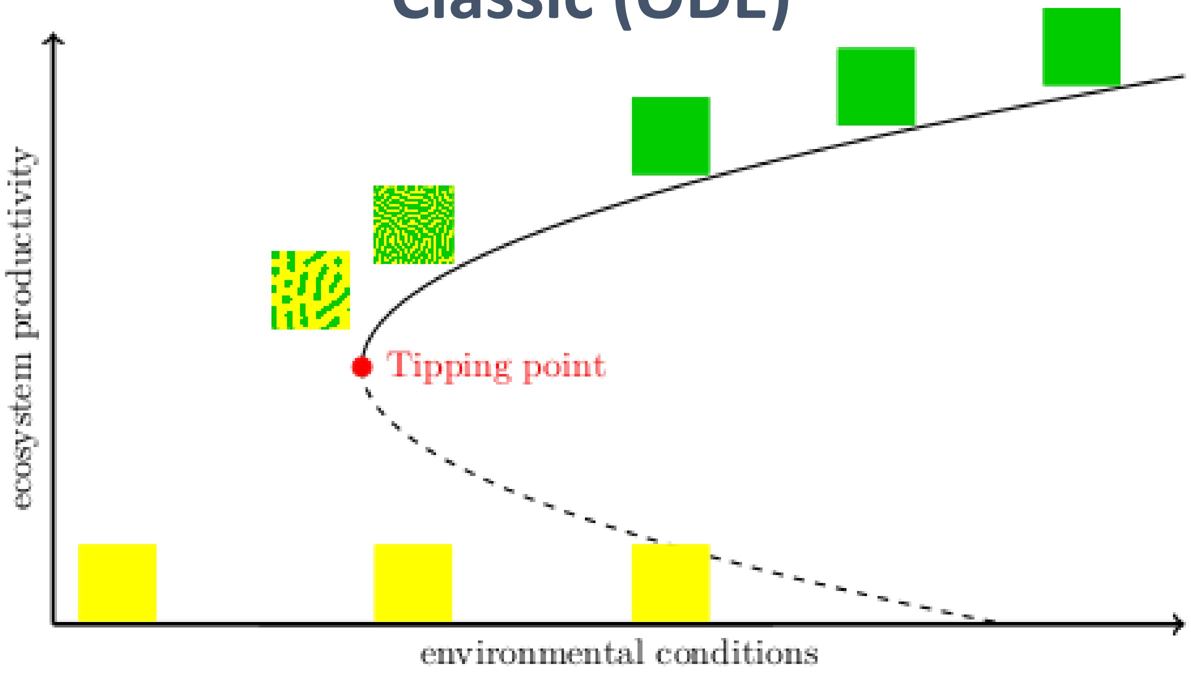
Coexistence states between patterned and uniform states also exist



[Bel et al, 2012]

What if the system tips?

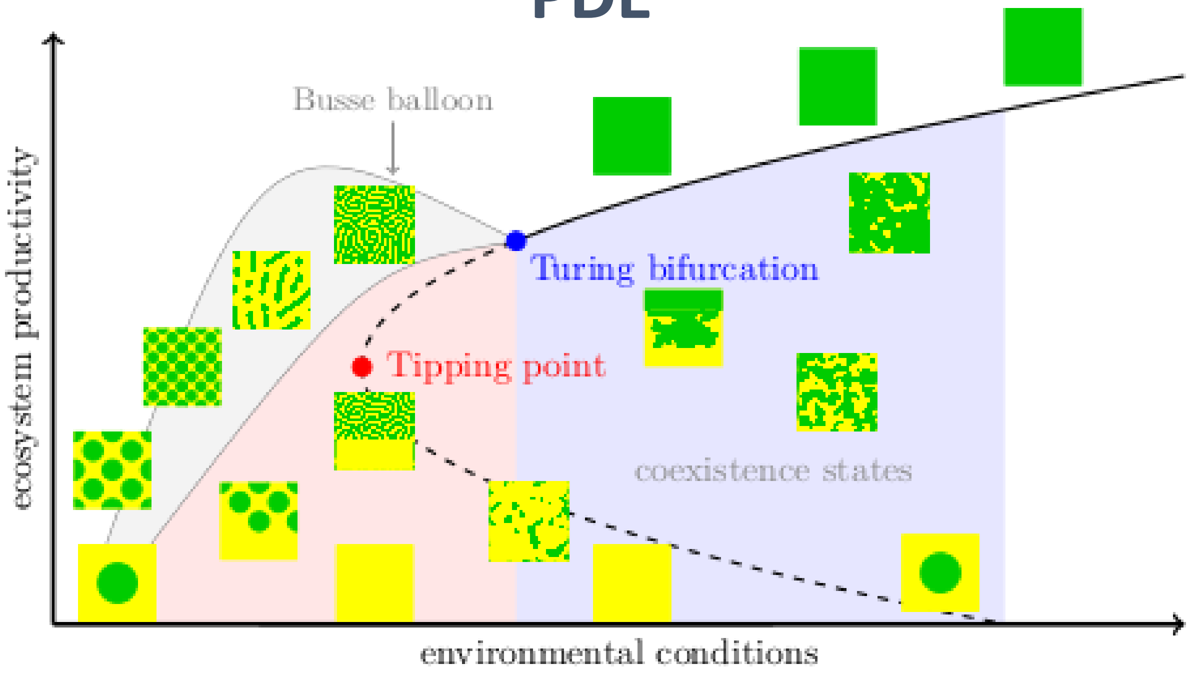
Classic (ODE)



Crossing a Tipping Point:
→ Always full reorganization

Early Warning Signals
signal for WHEN

PDE



Crossing a bifurcation:
Now also possible:
→ Spatial reorganization (Turing patterns)
→ Fragmented tipping (coexistence states)

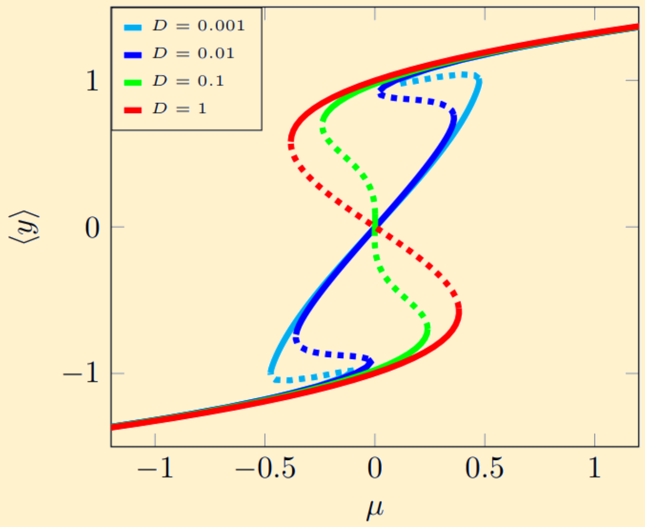
Early Warning Signals
need to signal WHEN & WHAT

Do systems always behave like this? (a.k.a. the small print)

No.

Well-mixed systems

Spatially confined systems



→ Such systems (again) behave like ODEs ←

But even in other systems terms & conditions apply:
System-specific knowledge is required!

Summary

1. Projections for observables are possible using linear response theory.

2. But there are limitations, such as crossings of tipping points.

3. But tipping of spatially extended systems might be local and gradual.