Fragmented tipping in a spatially heterogeneous world

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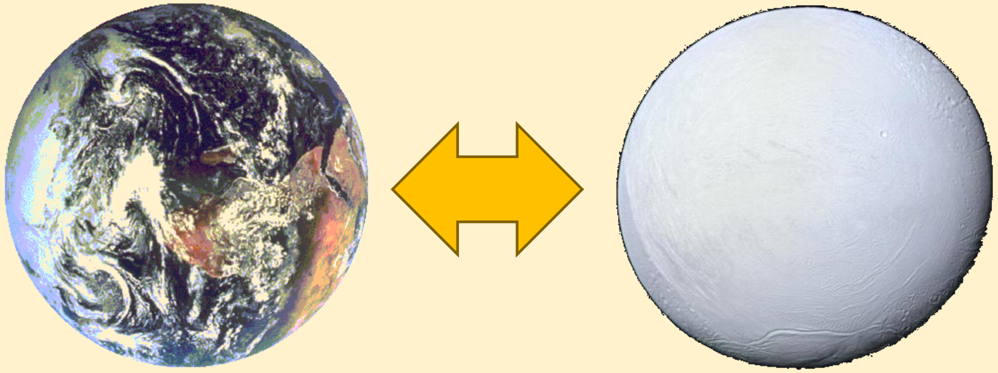
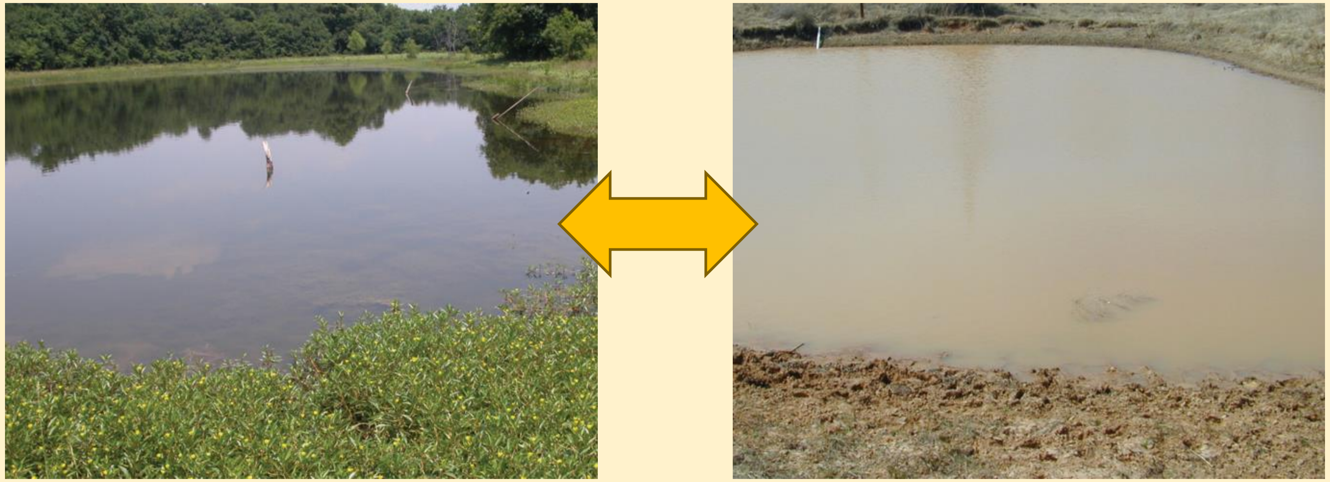
Tipping Points

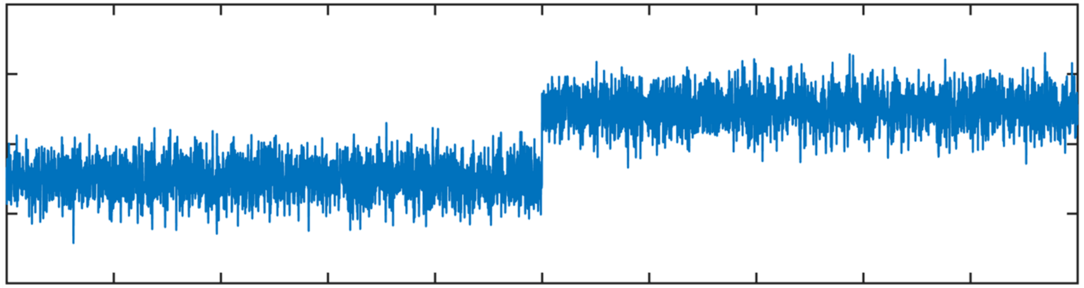
IPCC AR6 (2021) : "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly"



Planetary transitions

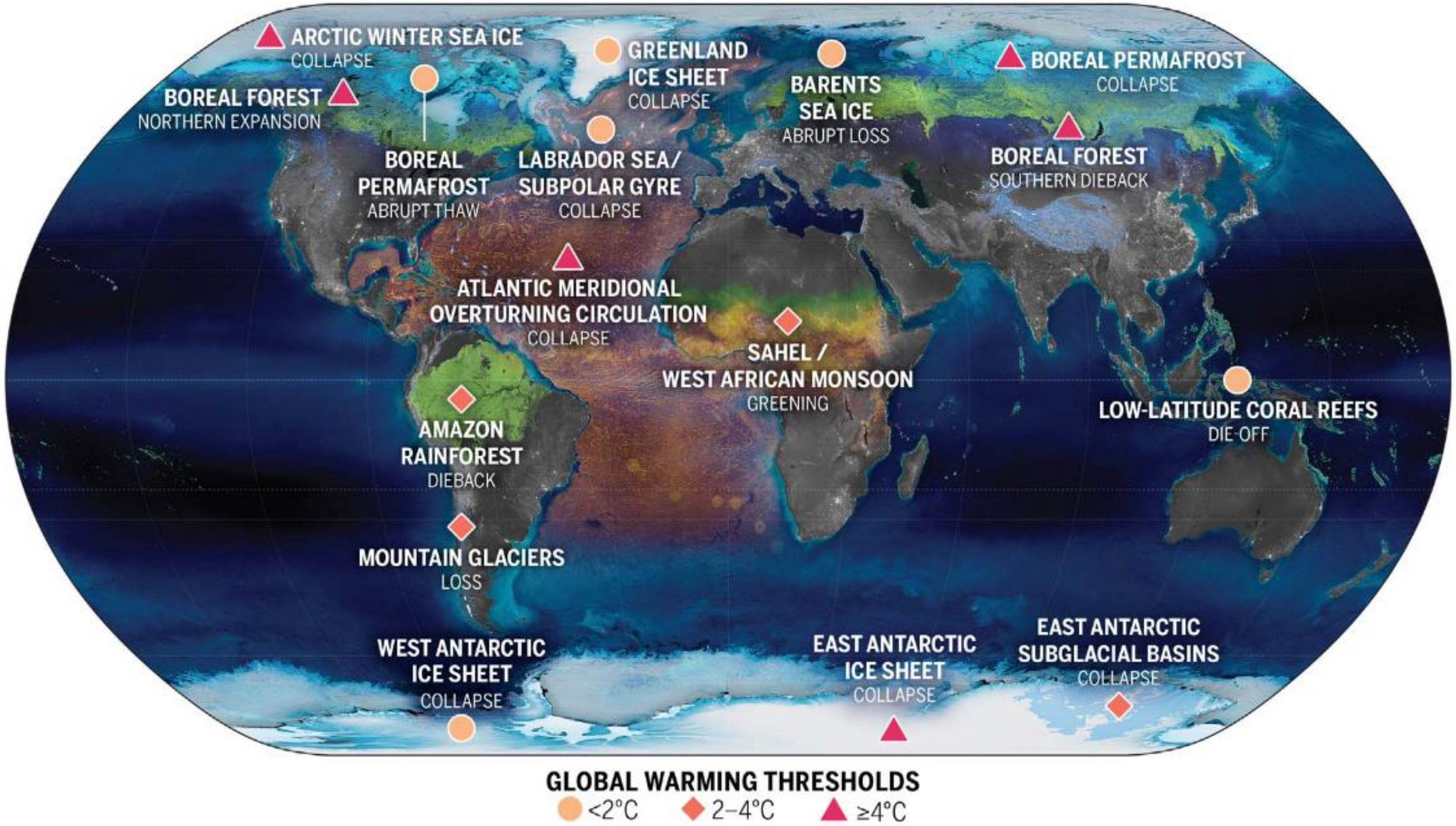
Ecosystem shifts





Tipping Points

IPCC AR6 (2021) : "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly"

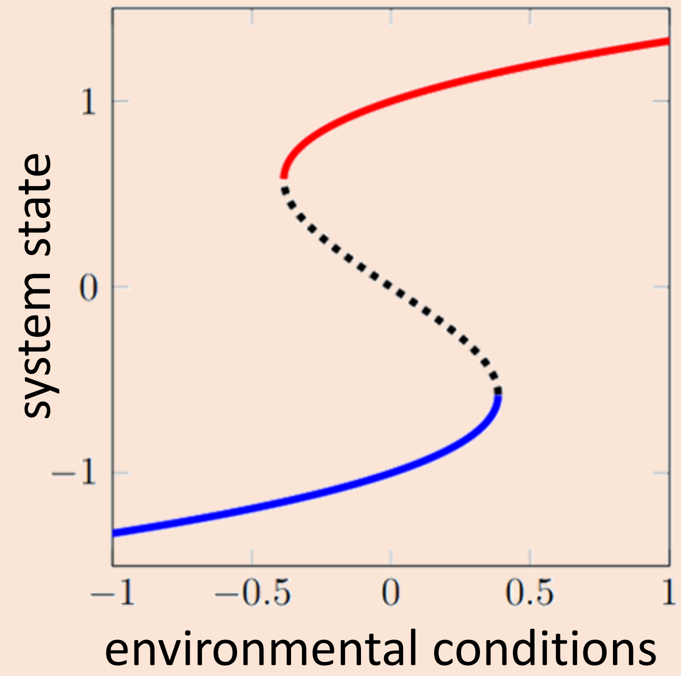


source: McKay et al, 2022

Mathematics

Tipping points ↔ Bifurcations

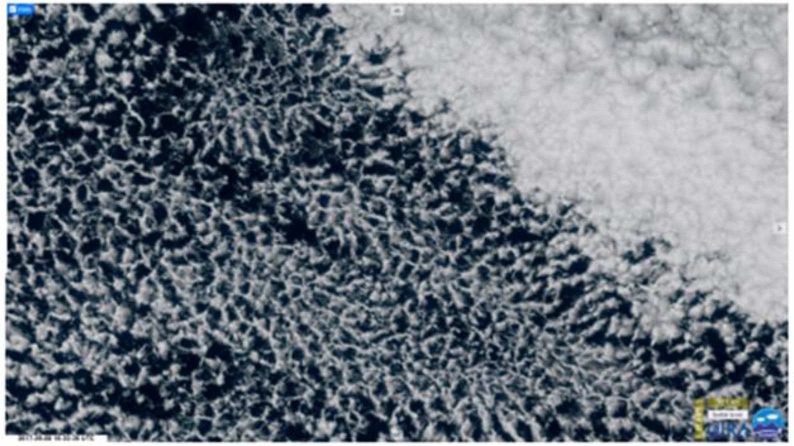
$$\frac{dy}{dt} = f(y, \mu)$$



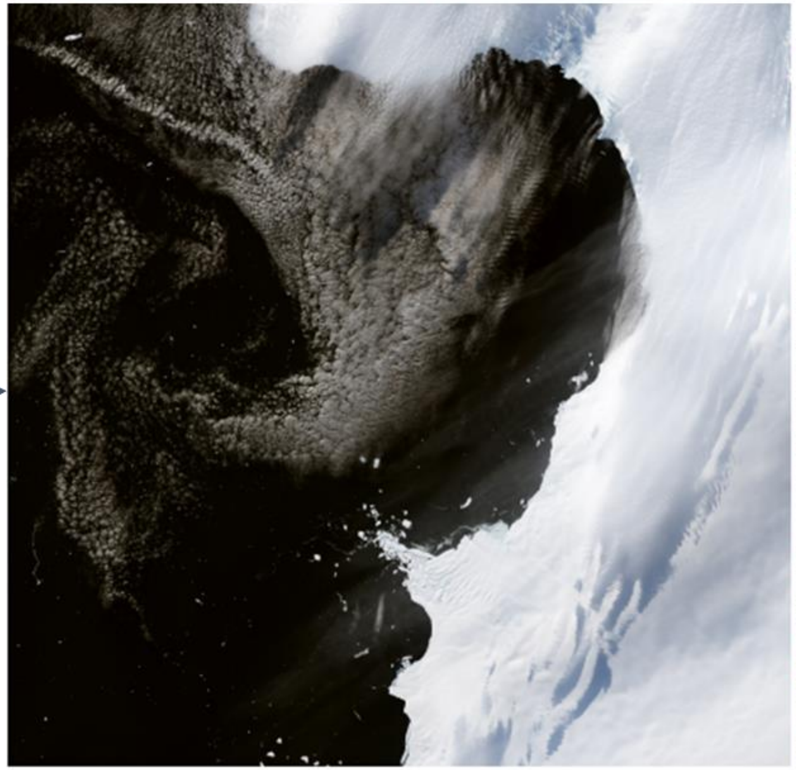
Reality is not always spatially-uniform!

tropical forest & savanna ecosystems
[Google Earth]

types of stratocumulus clouds
[RAMMB/CIRA SLIDER]



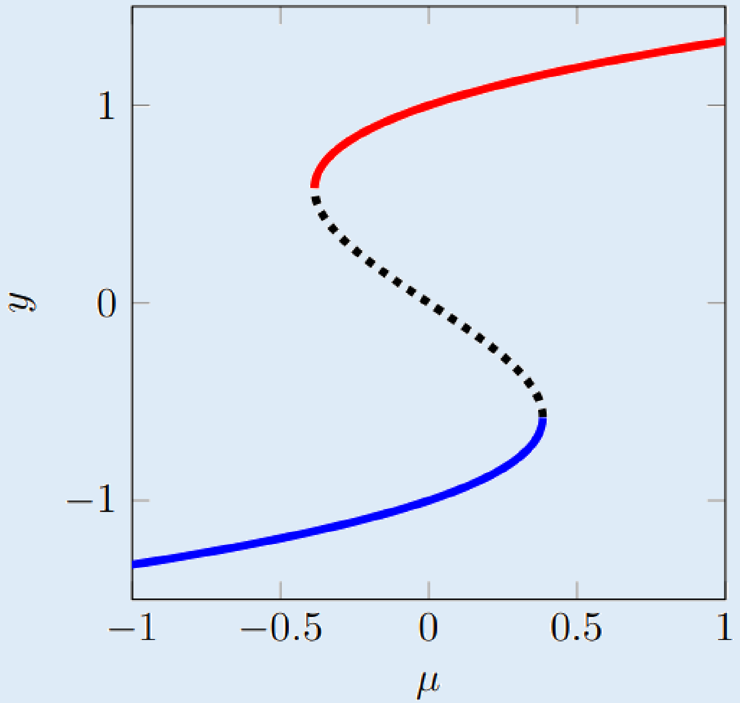
sea-ice & water at Eltanin Bay
[NASA's Earth observatory]



algae bloom in Lake St. Clair
[NASA's Earth observatory]

A spatially heterogeneous world

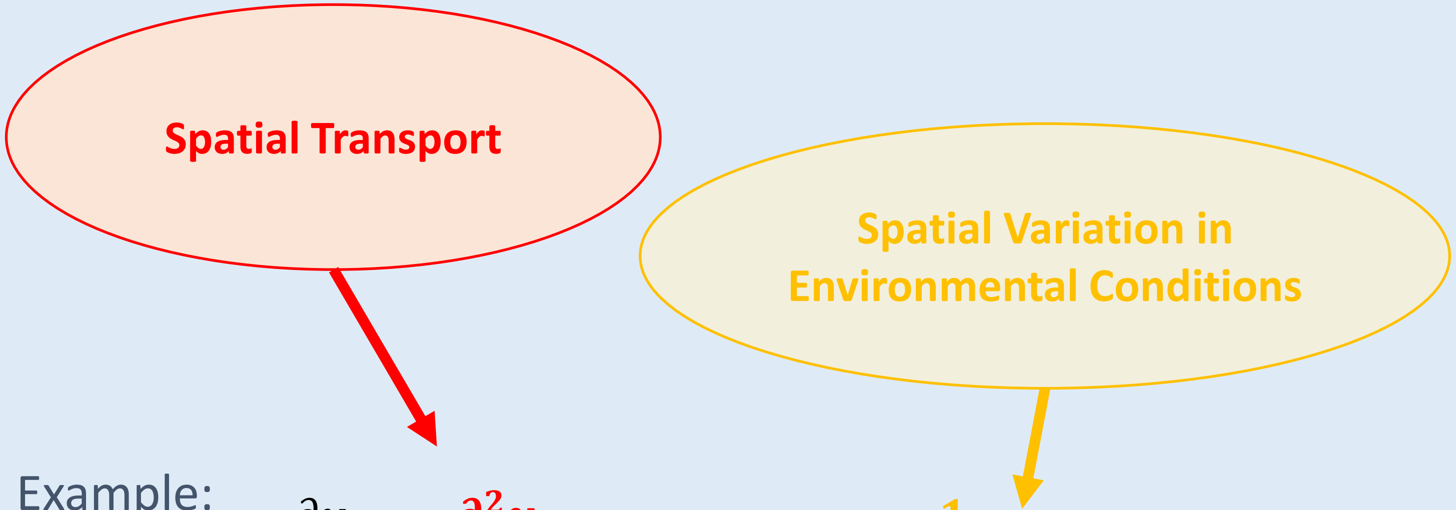
Classic Tipping



Example:

$$\frac{dy}{dt} = y(1 - y^2) + \mu$$

Tipping in Spatially Heterogeneous Systems



Example:

$$\frac{\partial y}{\partial t} = \boldsymbol{D}\frac{\boldsymbol{\partial^2 y}}{\boldsymbol{\partial x^2}} + y(1 - y^2) + \mu + \frac{\mathbf{1}}{\mathbf{2}}\mathbf{cos}(\boldsymbol{\pi x})$$

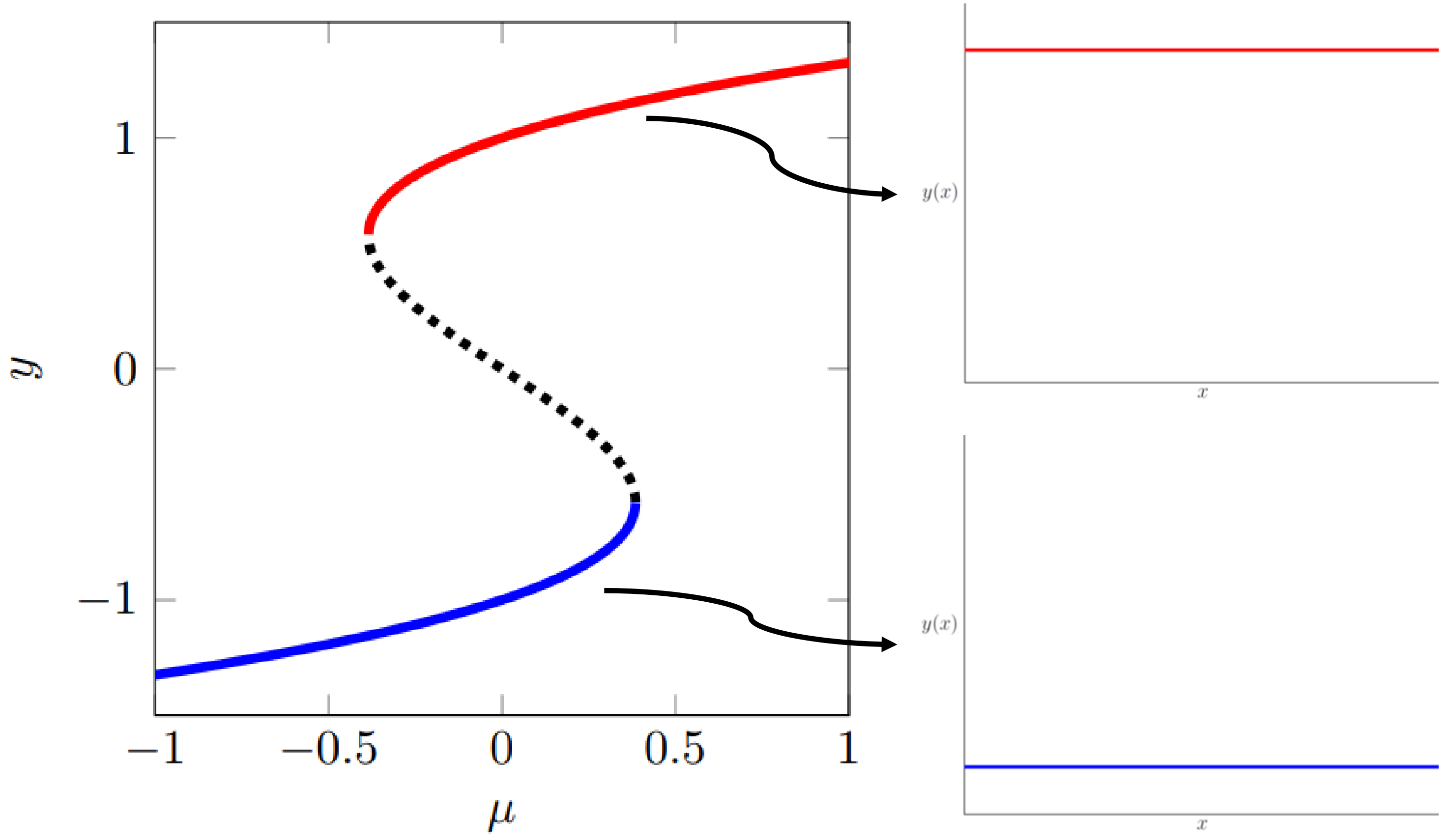
Alternative more math-y title:

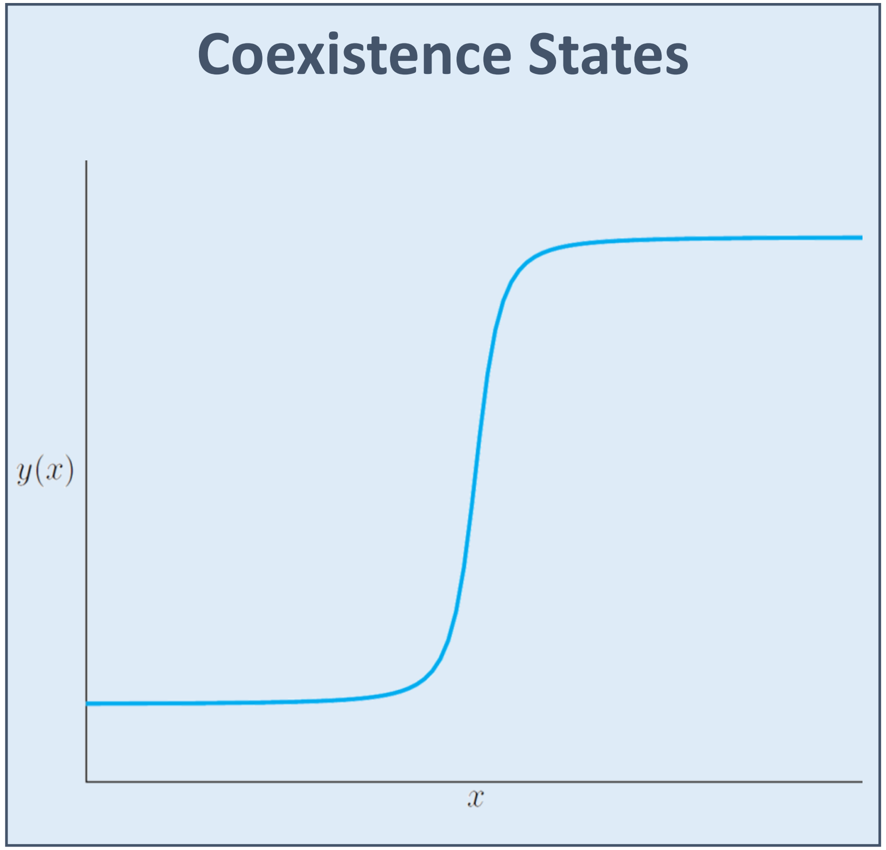
Stationary front solutions in bistable PDEs with coefficients that vary in space

Coexistence states

Bistable (Allen-Cahn/Nagumo) equation:

$$\frac{\partial y}{\partial t} = y(1 - y^2) + \mu + D\,\frac{\partial^2 y}{\partial x^2}$$



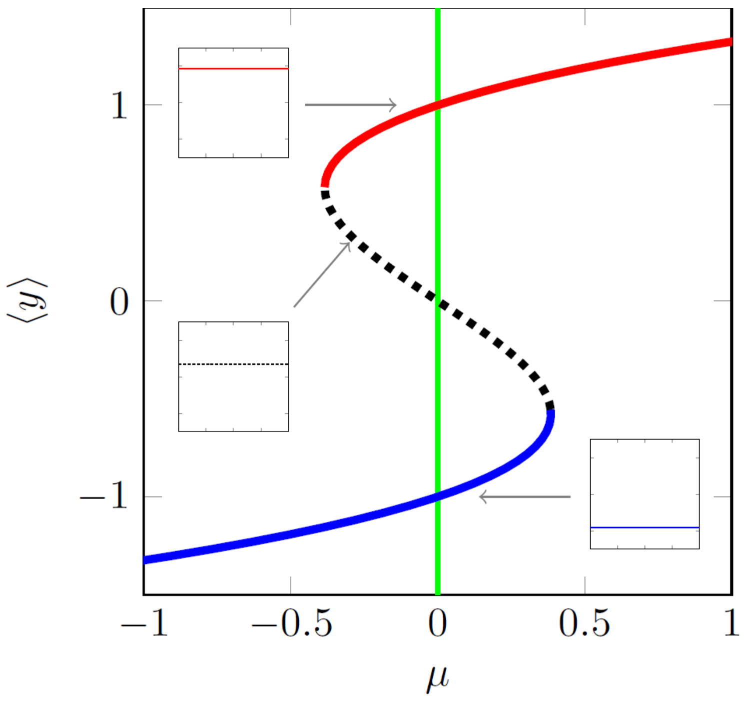


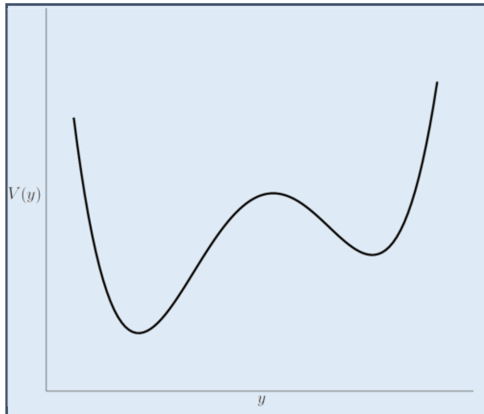
Front Dynamics

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + f(y;\mu)$$

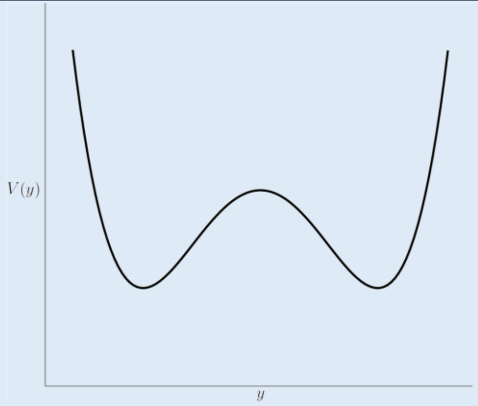
Potential function $V(y;\mu)$:

$$\frac{\partial V}{\partial y}(y;\mu) = -f(y;\mu)$$

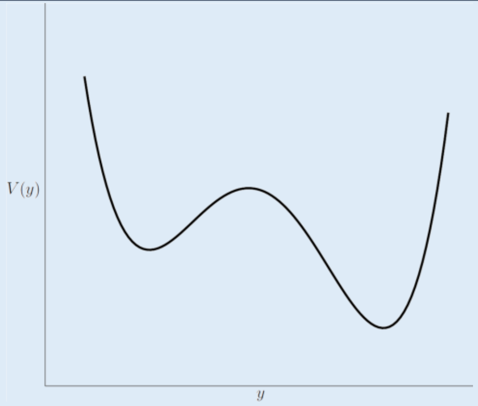




moves right

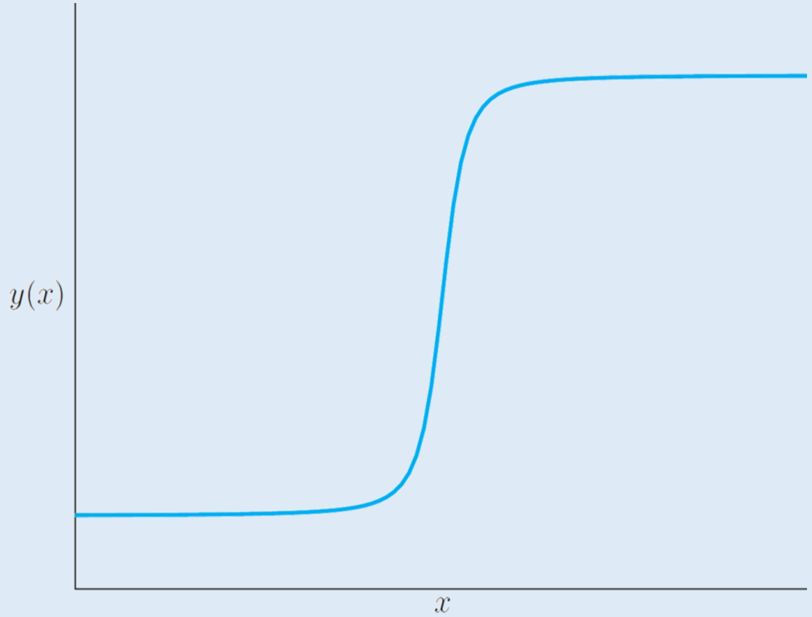


stationary



moves left

Maxwell Point $\mu_{maxwell}$



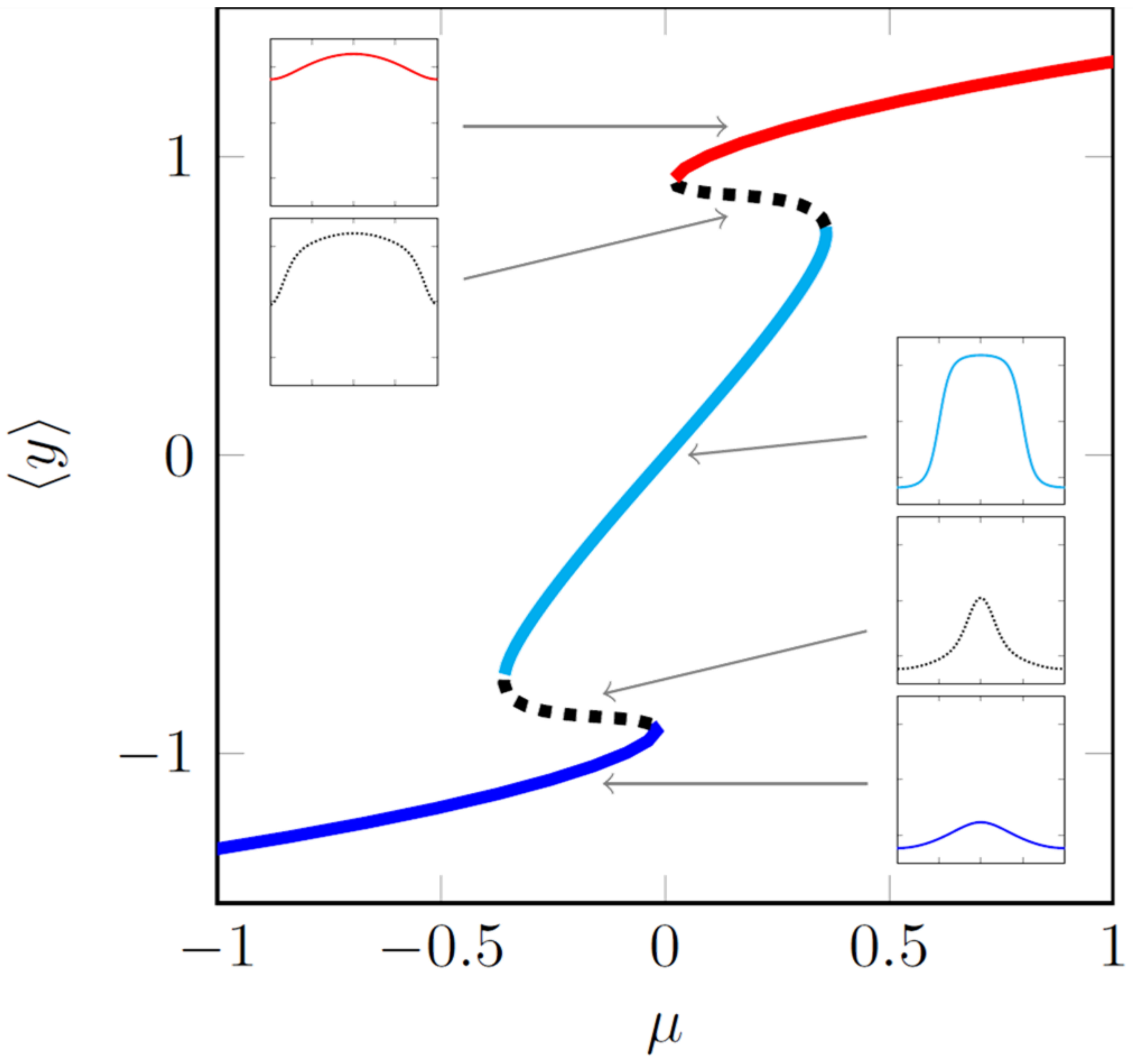
Adding Spatial Heterogeneity

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + f(y, x; \mu)$$

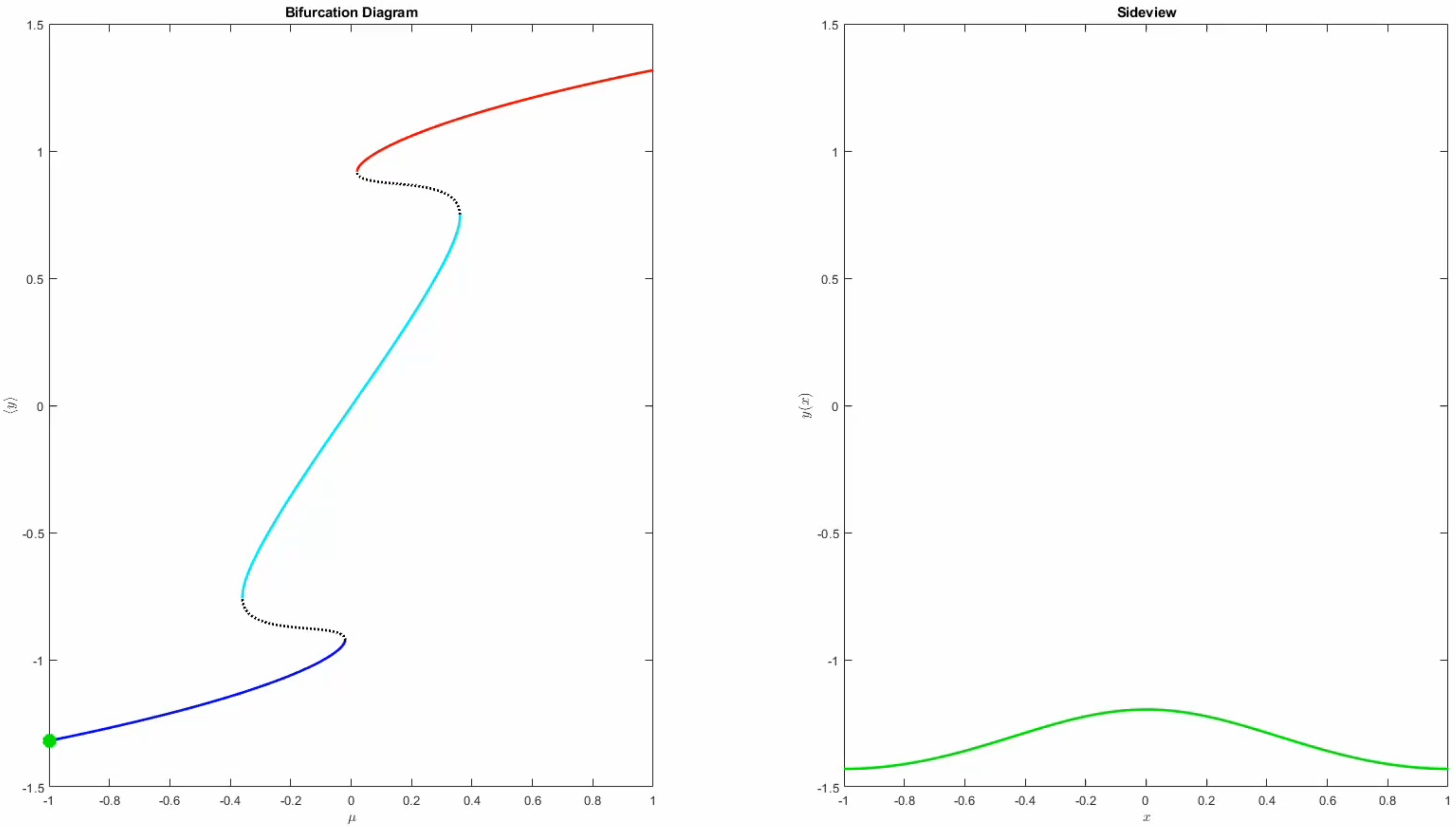
Now, the **local** difference in potentials determines the front movement

New behaviour:

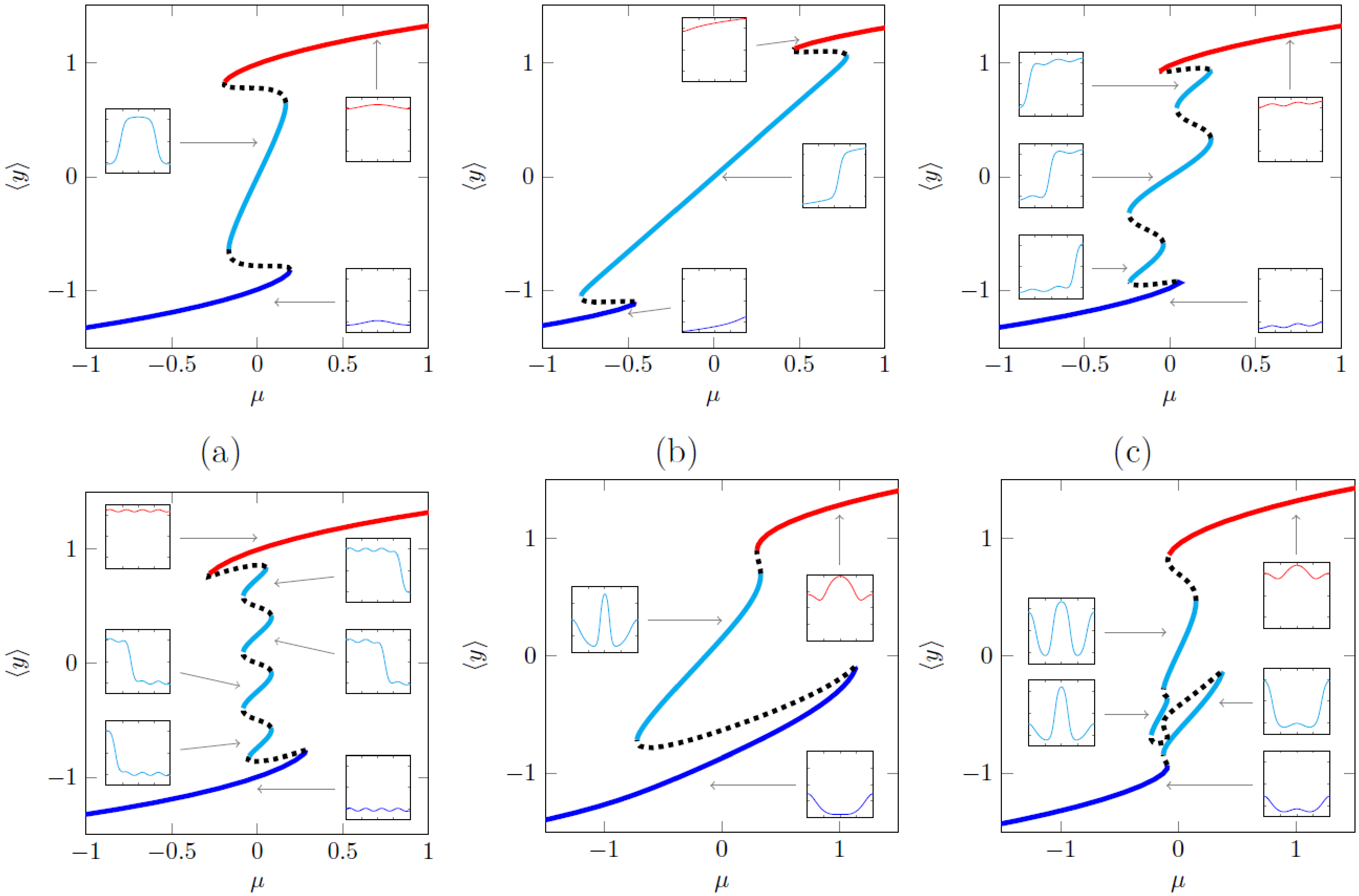
- Multi-fronts can be stationary
- Maxwell point is smeared out



Fragmented Tipping



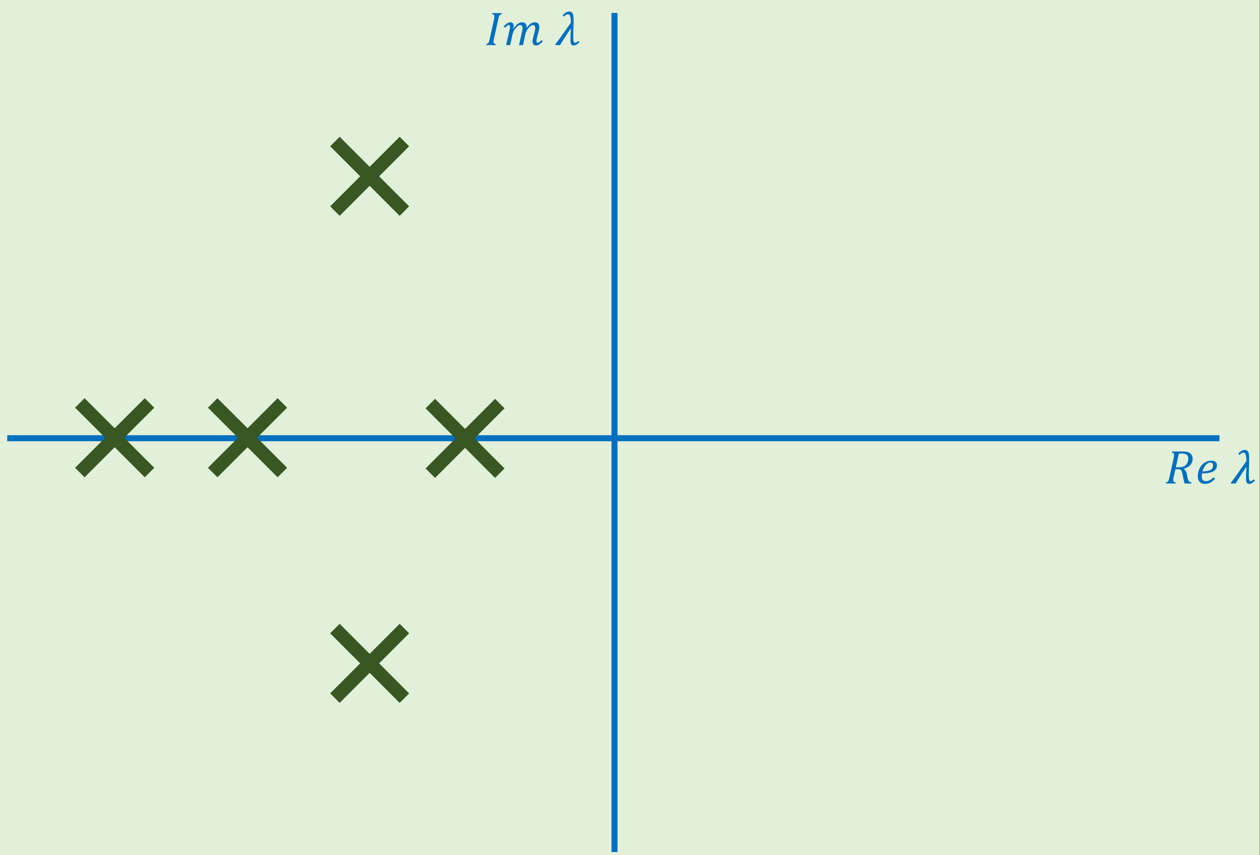
Other Spatial Heterogeneities



Stability of Stationary States

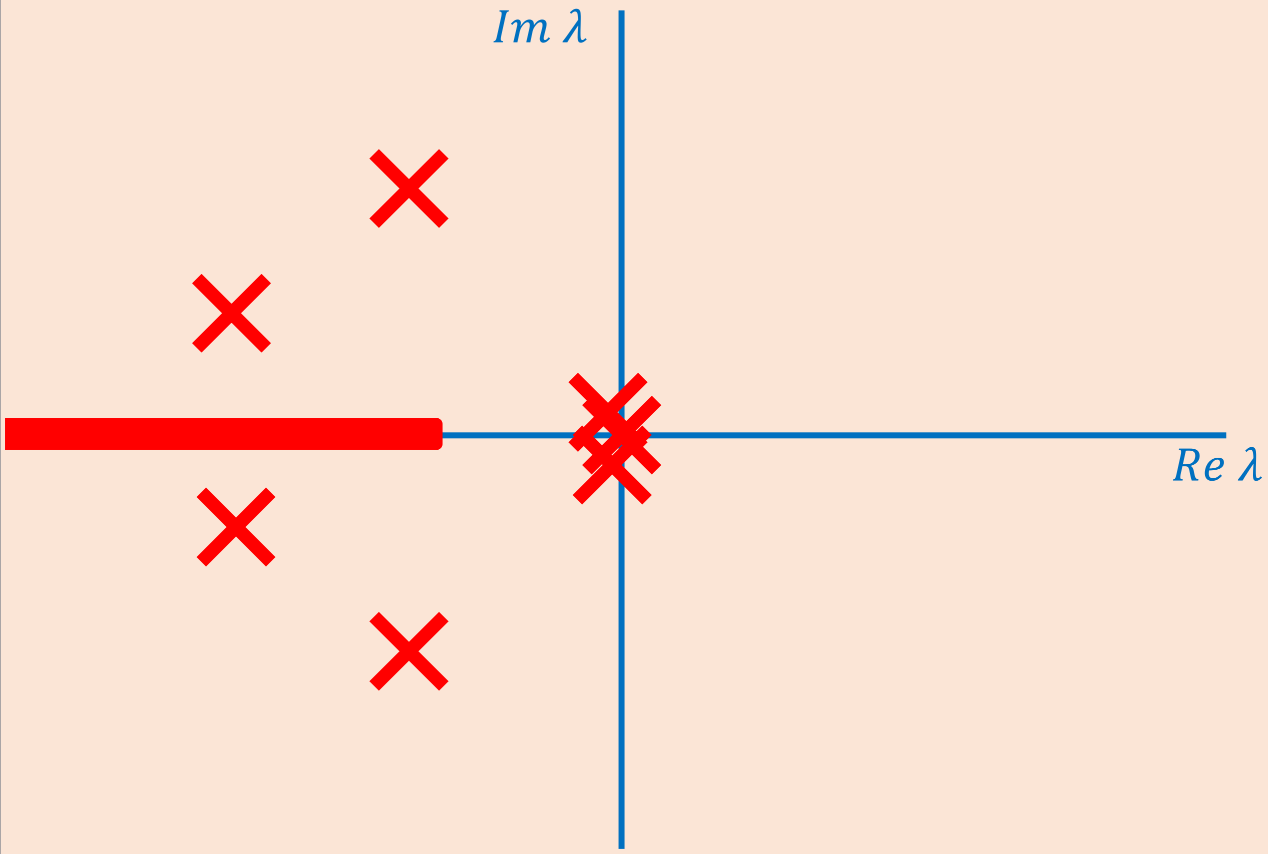
ODE

$$\lambda\,\bar{y} = f_y(y^*;\mu)\,\bar{y}$$

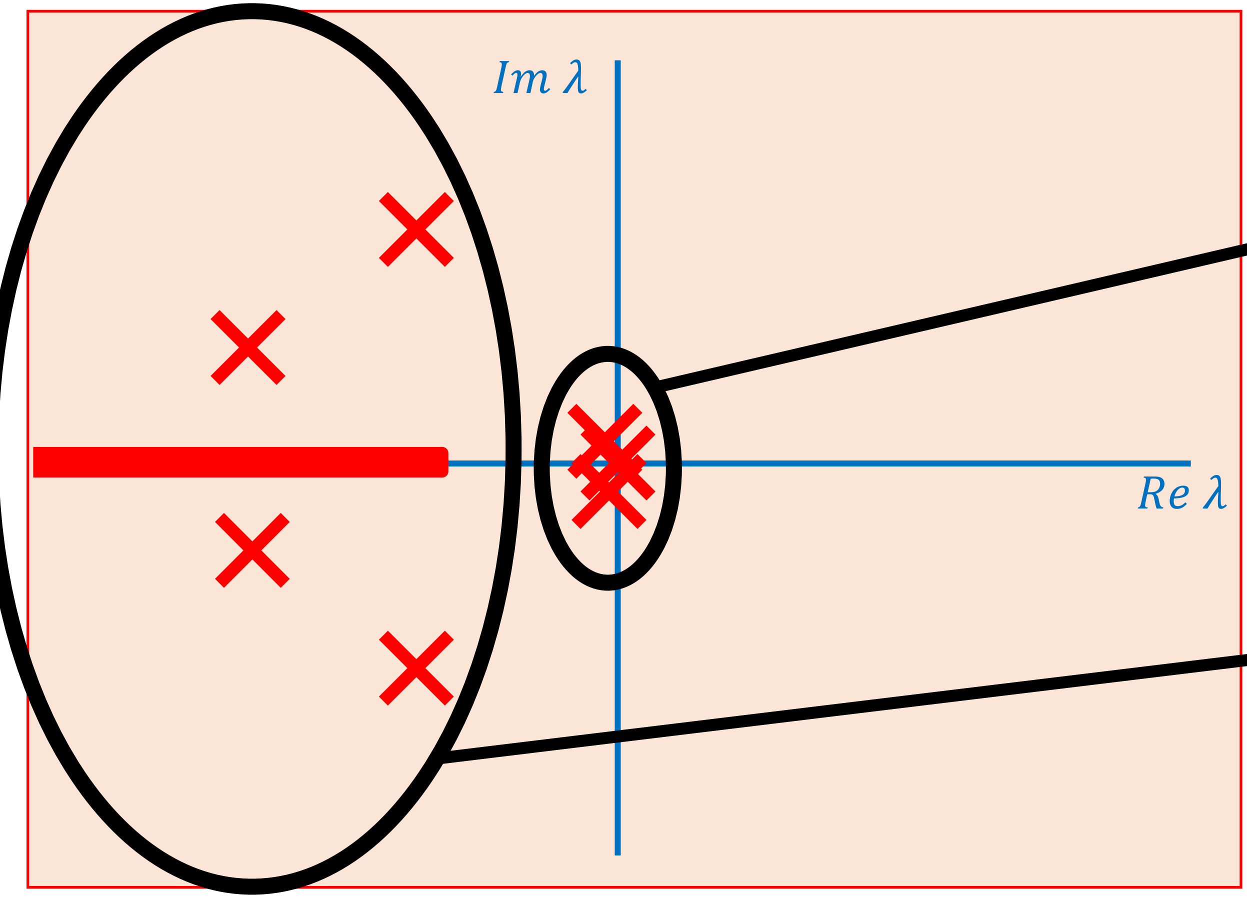


PDE

$$\lambda\,\bar{y} = \bar{y}_{xx} + f_y(y^*(x);\mu)\,\bar{y}$$



Bifurcations



What happens at bifurcation?

1. SLOW Pattern Adaptation

At bifurcation:
→ Location of structure changes

2. FAST Pattern Degradation

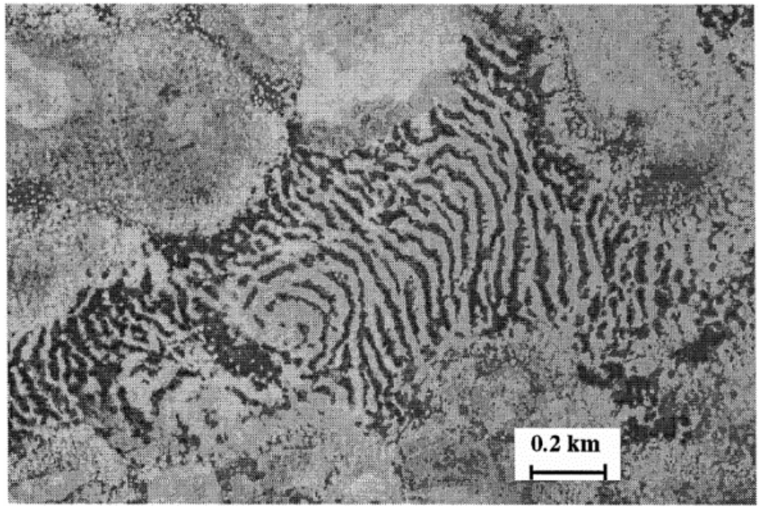
At bifurcation:
→ Structures created or destroyed

1. SLOW pattern adaptation

Somaliland, 1948 [Macfadyen, 1950]

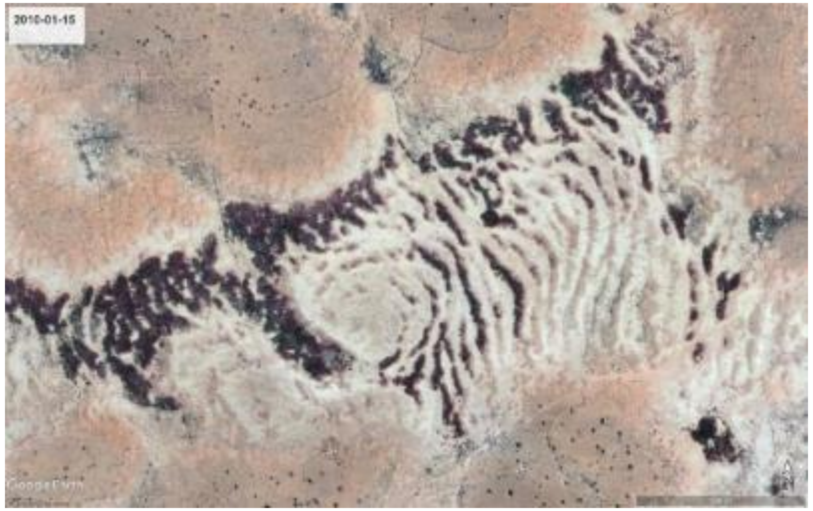
Somaliland, 2008

2. FAST Pattern Degradation

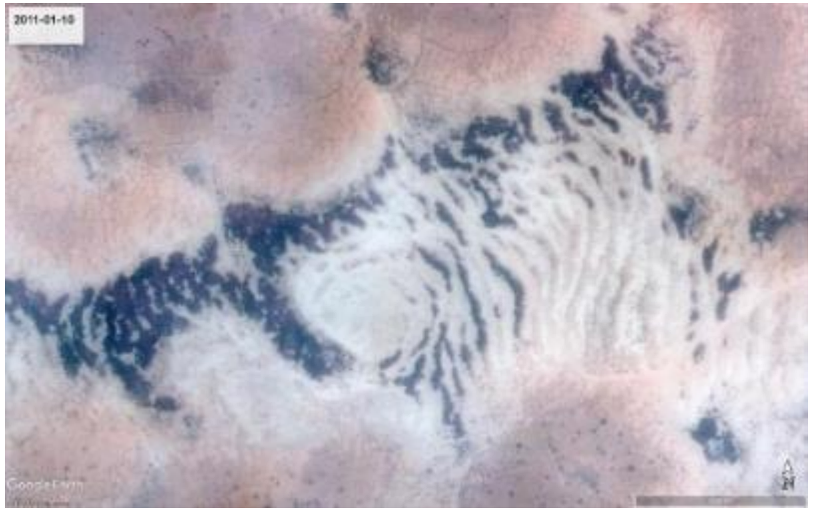


Niger, 1950 [Valentin, 1999]

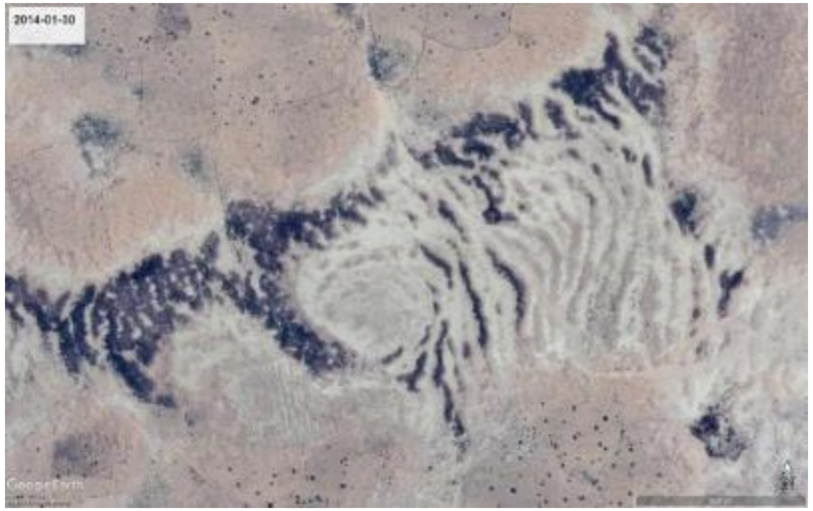
Niger, 2008



Niger, 2010



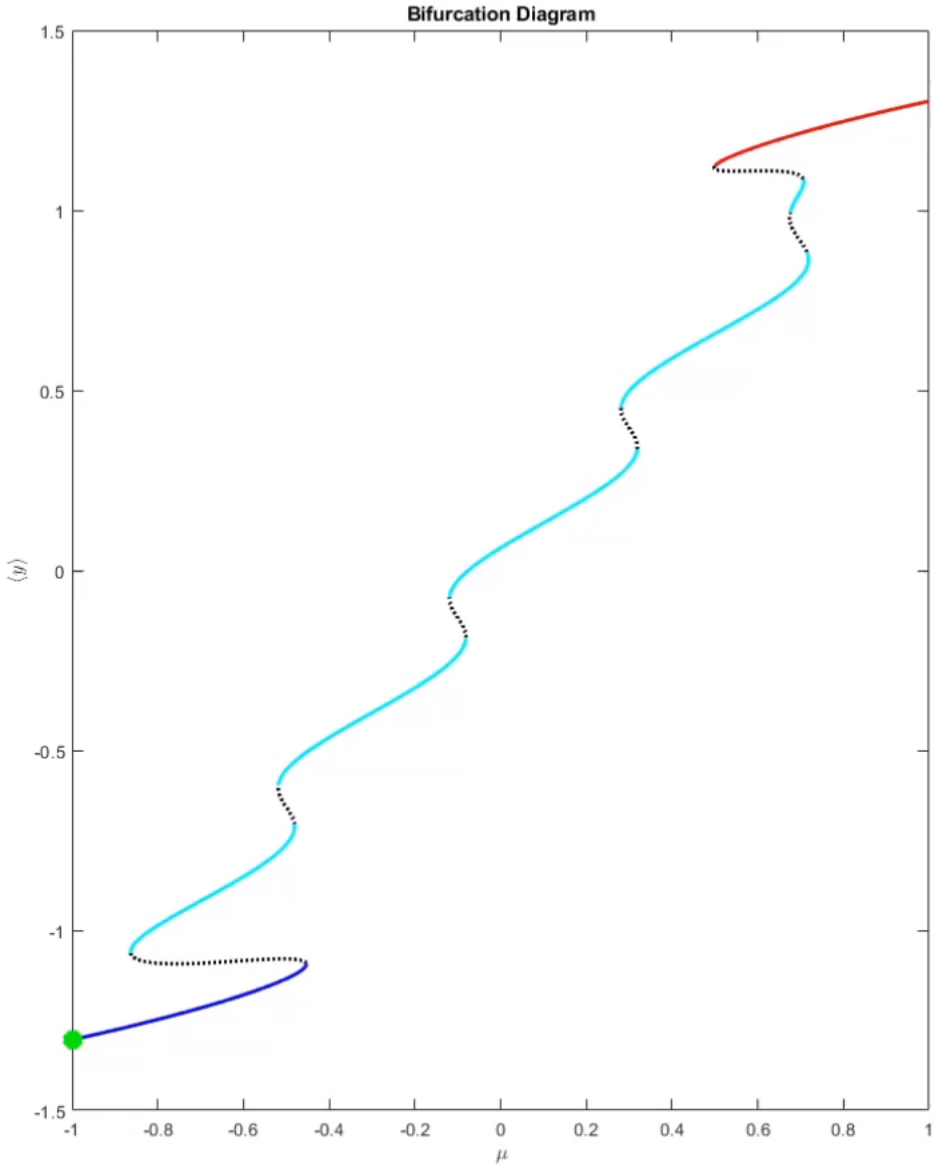
Niger, 2011

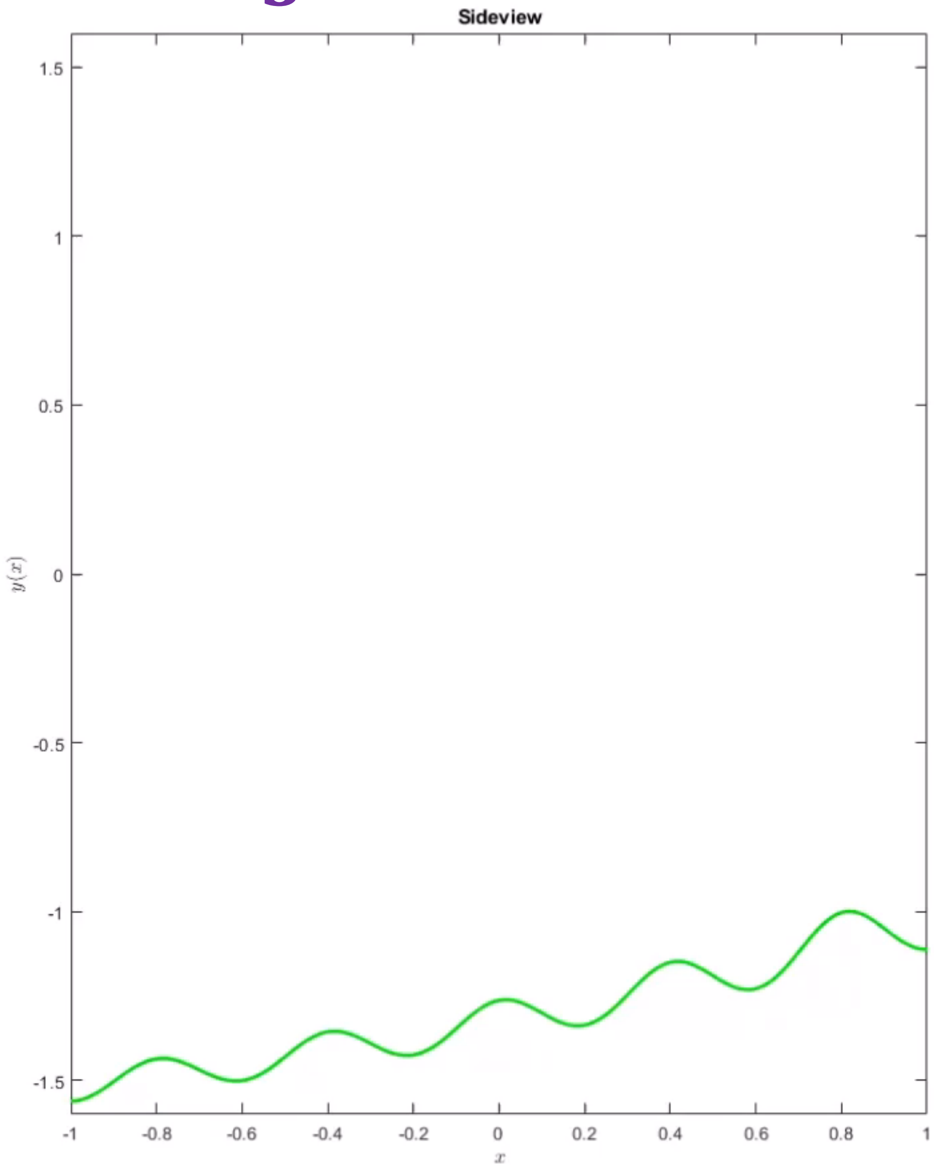


Niger, 2014

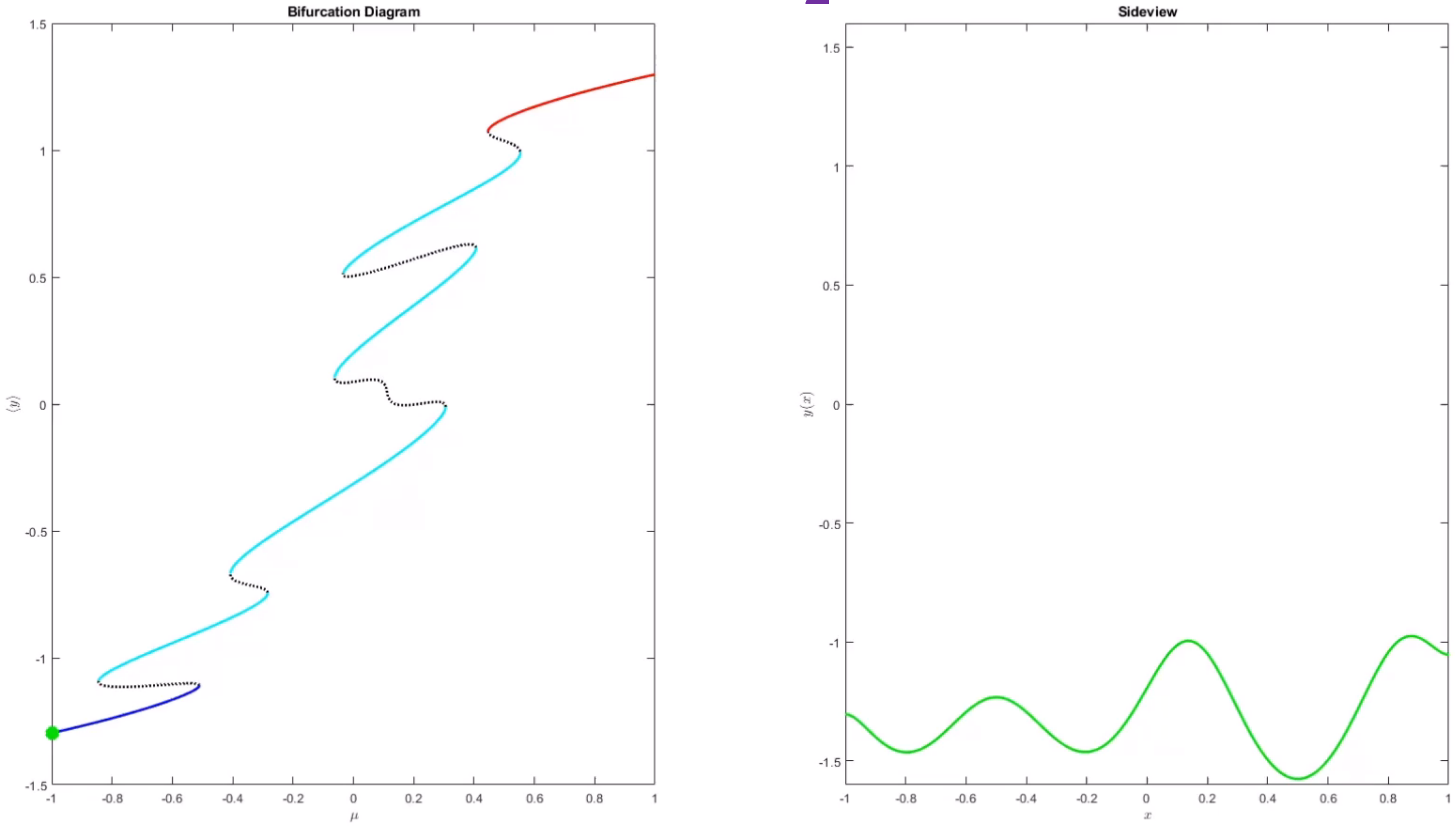
Niger, 2016

$$y_t = D\, y_{xx} + y(1-y^2) + \mu + x + \frac{2}{5}\cos(5\pi x)$$



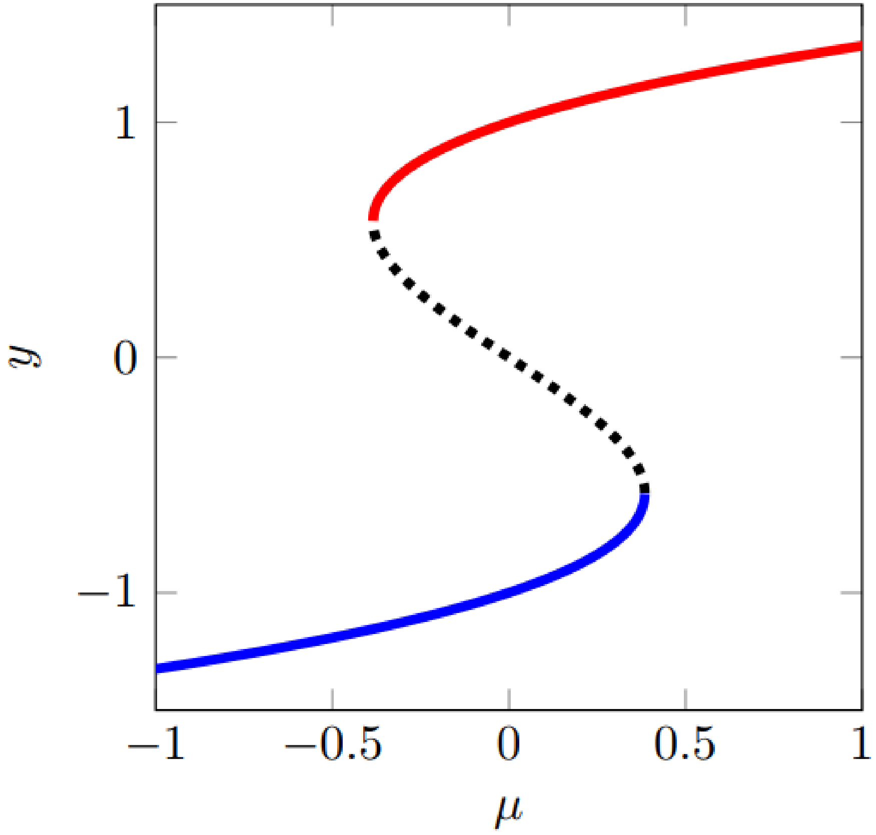


$$y_t = D\, y_{xx} + y\left(1 - y^2\right) + \mu + \frac{1}{2}\cos(2\pi x) + \sin(3\pi x)$$



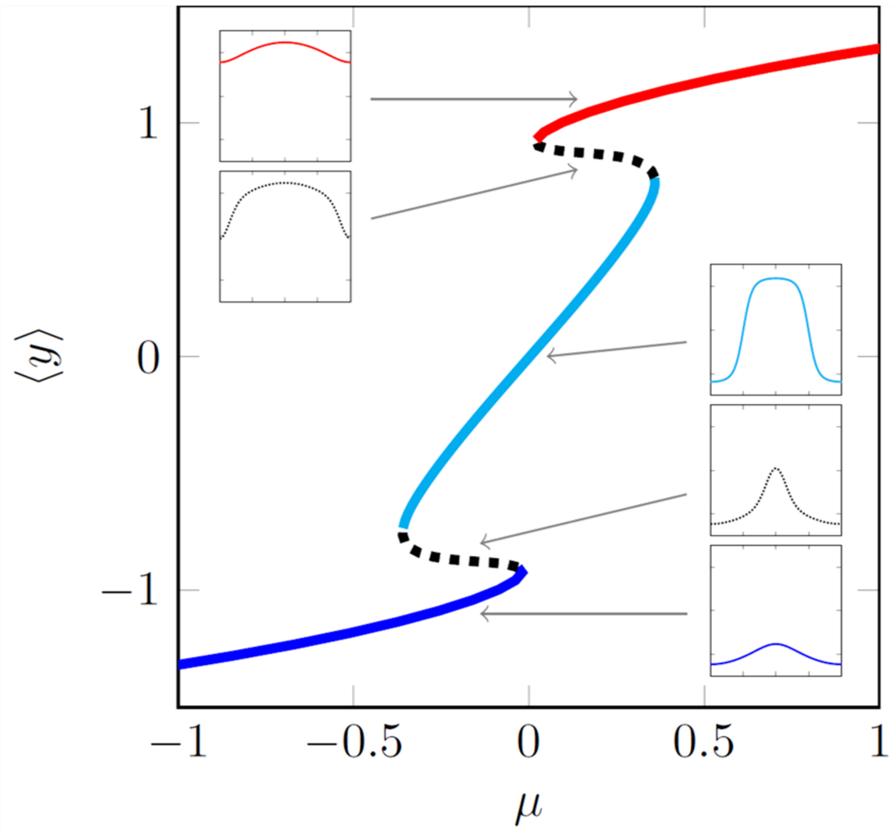
Fragmented Tipping

Classic tipping



Tipping leads to full reorganisation

Tipping in a heterogeneous world



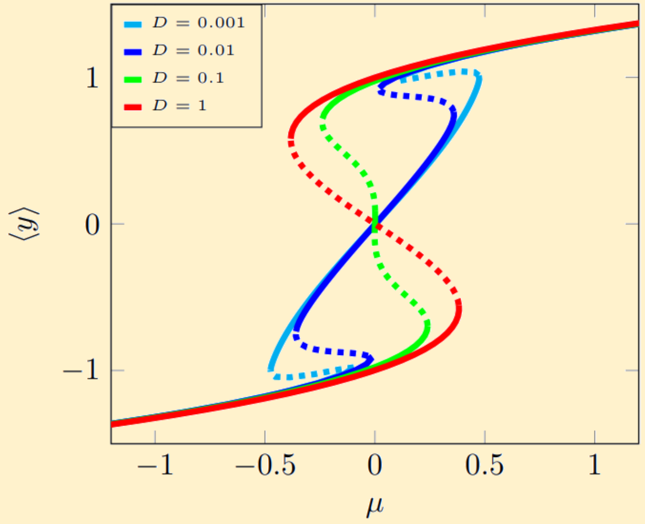
Fragmented tipping possible:
Only part of the domain reorganises

Do systems always behave like this? (a.k.a. the small print)

No.

Well-mixed systems

Spatially confined systems

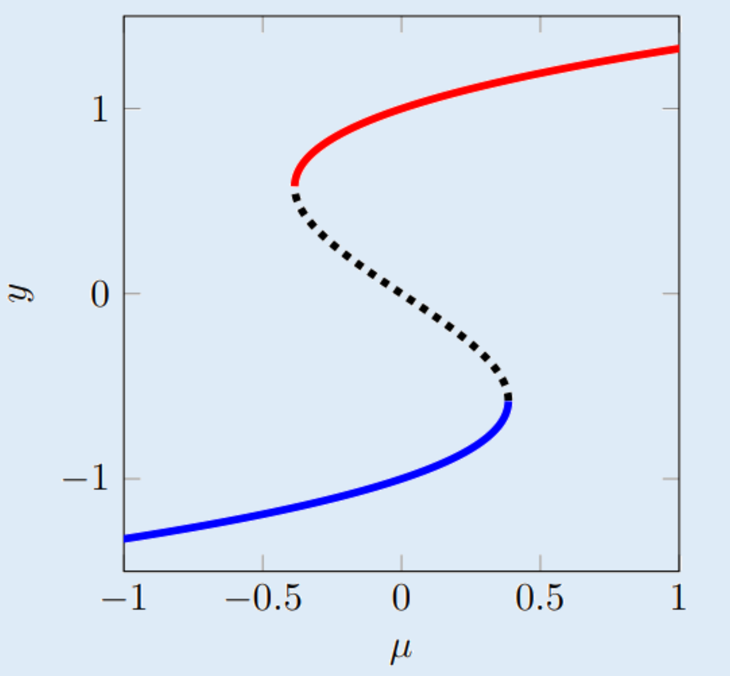


→ Such systems (again) just have one global tipping point ←

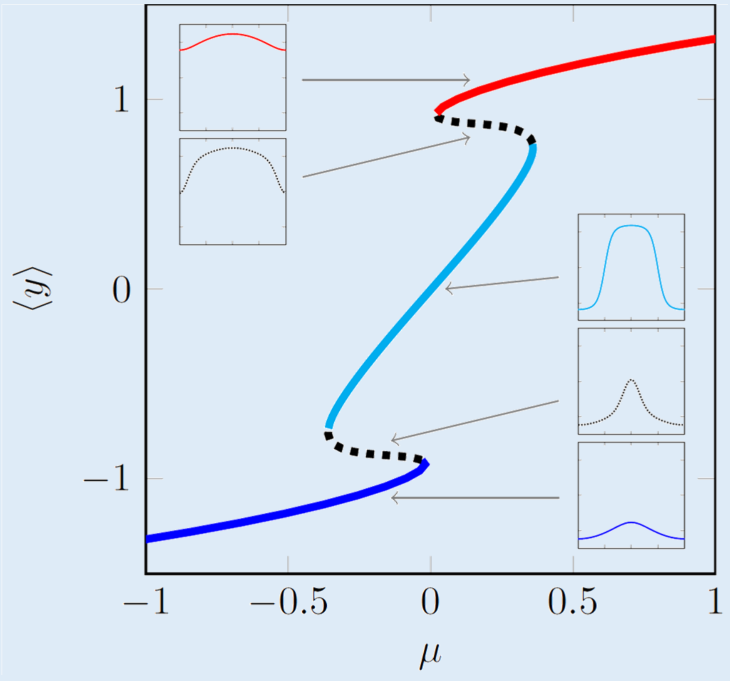
But even in other systems terms & conditions apply:
System-specific knowledge is required!

Fragmented Tipping in a spatially heterogeneous world

Classic Tipping Picture



Fragmented tipping in spatially heterogeneous systems



Bastiaansen, R., Dijkstra, H. A., & von der Heydt, A. S. (2022). Fragmented tipping in a spatially heterogeneous world. *Environmental Research Letters, 17, 045006*