

The background of the slide is a stylized, glowing Earth as seen from space. The planet is tilted, and a bright, golden-yellow light streak, possibly representing a comet or a solar flare, cuts across the upper right portion of the globe. The colors are warm, with oranges, yellows, and browns, set against a dark, starry space background.

# Climate response and sensitivity: time scales and late tipping points

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Climate Emulator Workshop, 2023-04-22

## Climate Response

The change in observable due to climate forcing (e.g. CO<sub>2</sub>)

### Equilibrium Climate Sensitivity (ECS)

change in equilibrium temperature  
due to (instantaneous) doubling of CO<sub>2</sub>

### Transient Climate Response (TCR)

change in temperature after 100 years  
with 1% CO<sub>2</sub> increase per year (until doubling)

## Methodology

- DESIGN experimental protocol for GCM
- FIT resulting time series to simple model
- EXTRAPOLATION using simple model

**TODAY:** a few words of caution

# Linear Response

$$\frac{dO}{dt} = \mathcal{L} O + g(t)$$

Evolution of temperature

$$\Delta T(t) = (G * g)(t) = \int_0^t G(s) g(t-s) ds$$

Green Function

forcing

Approximation of Green Function:

$$G(t) = \sum_{m=1}^M \beta_m e^{-t/\tau_m}$$

time scales

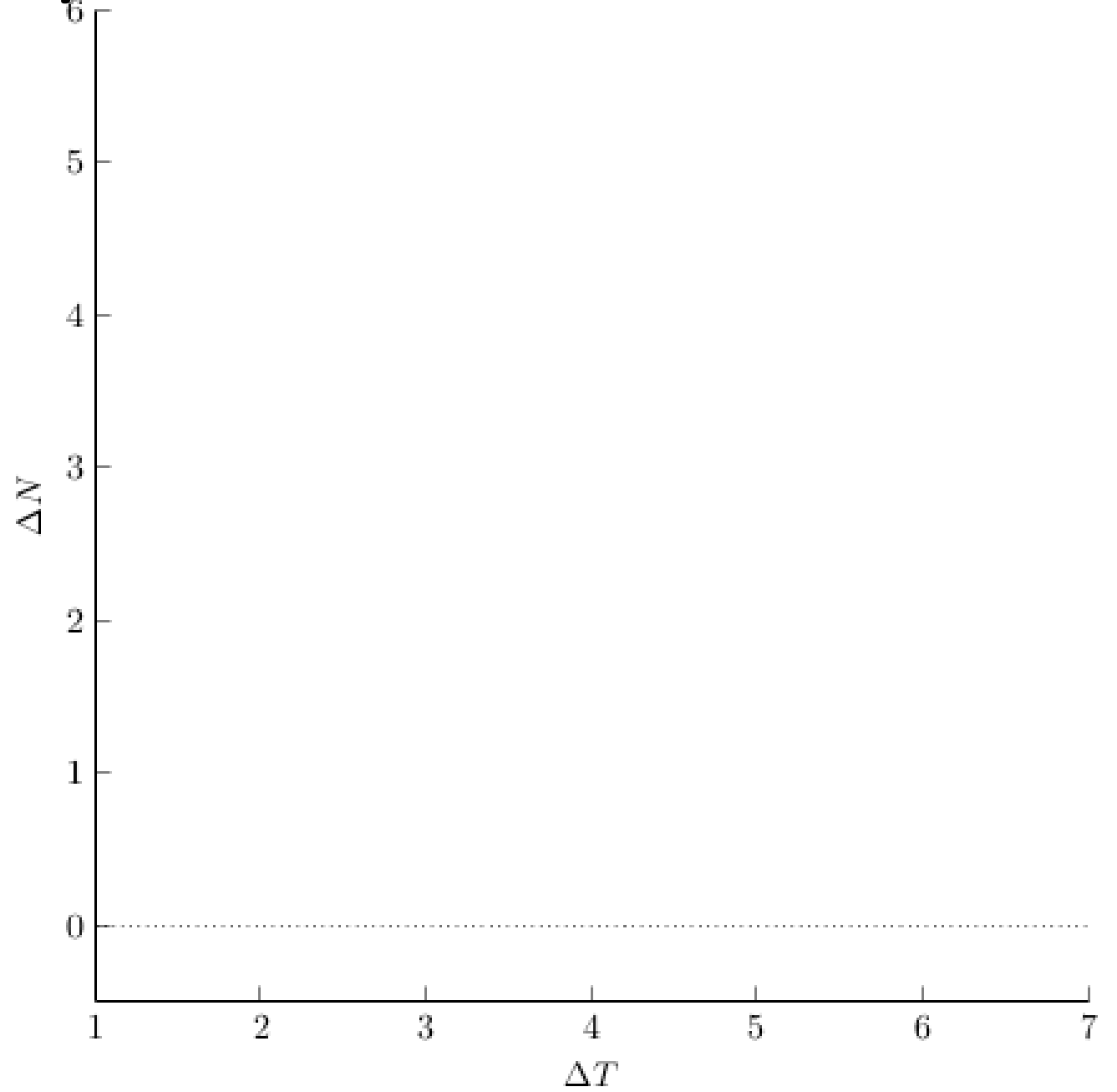
# Gregory Method

Regress data to

$$\Delta N(t) = \mathbf{F} + \lambda \Delta T(t)$$

Since  $\Delta N_* = 0$  in equilibrium,  
ECS estimation is

$$\Delta T_*^{est} = -\lambda^{-1} \mathbf{F}$$



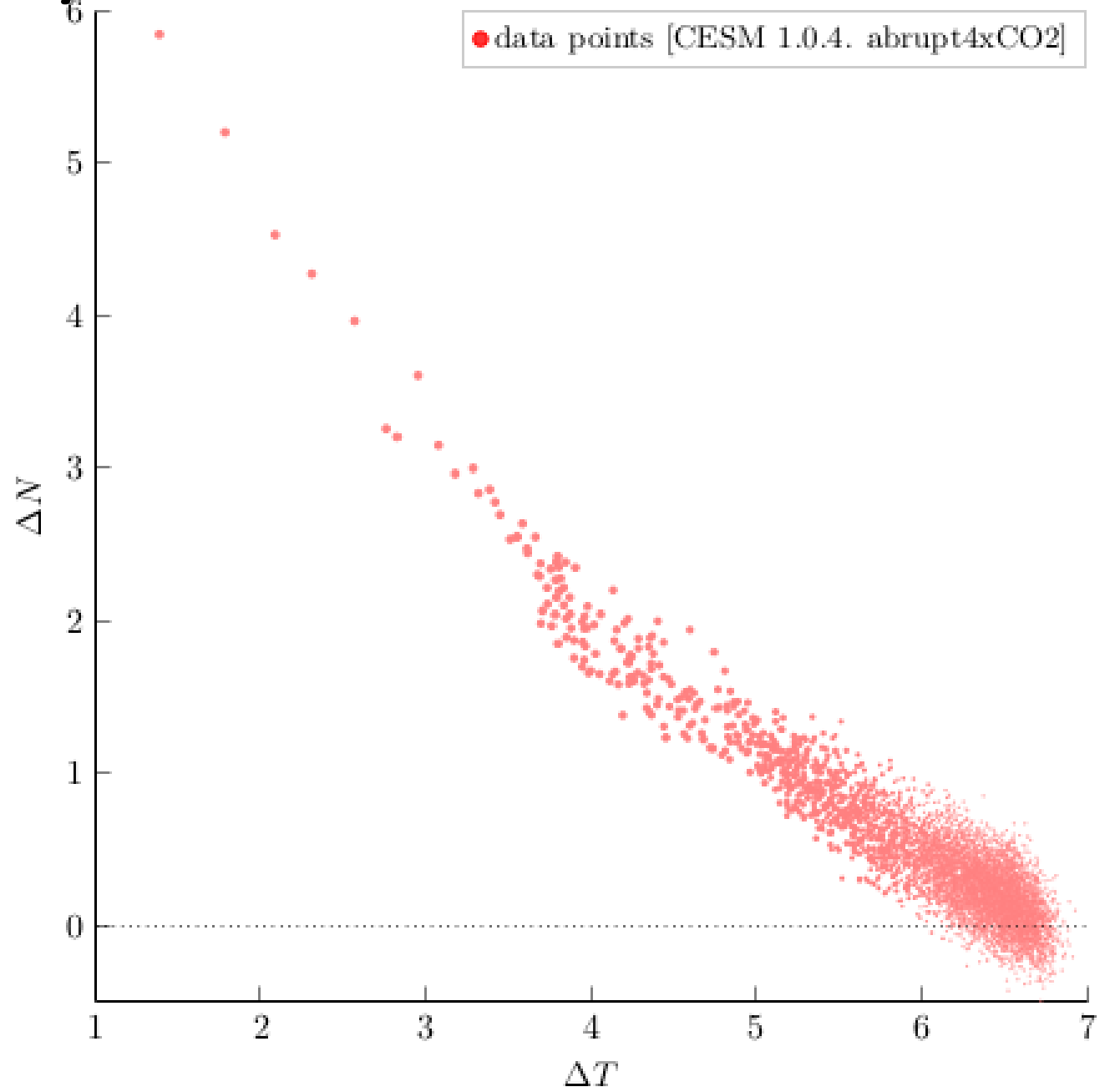
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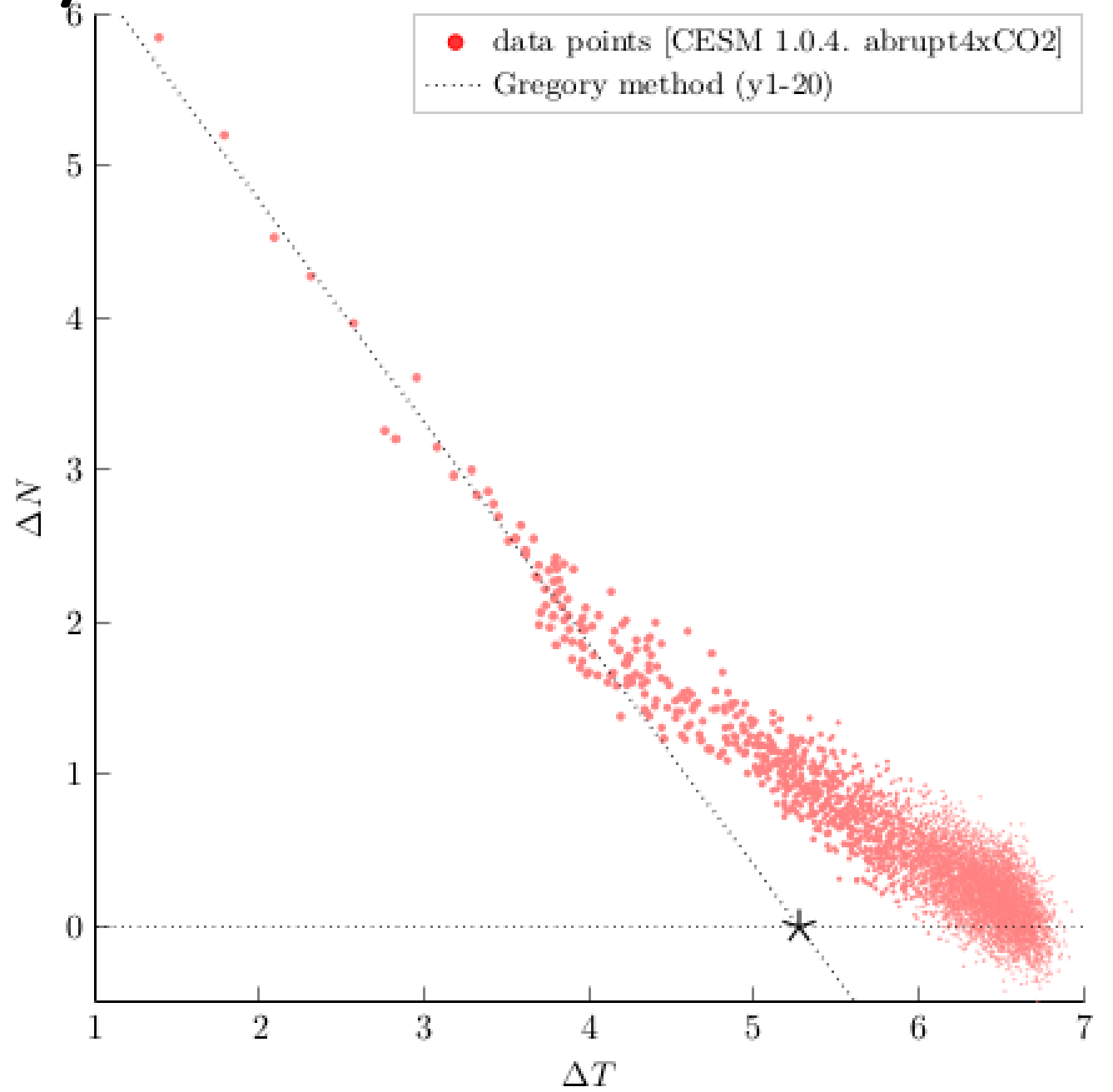
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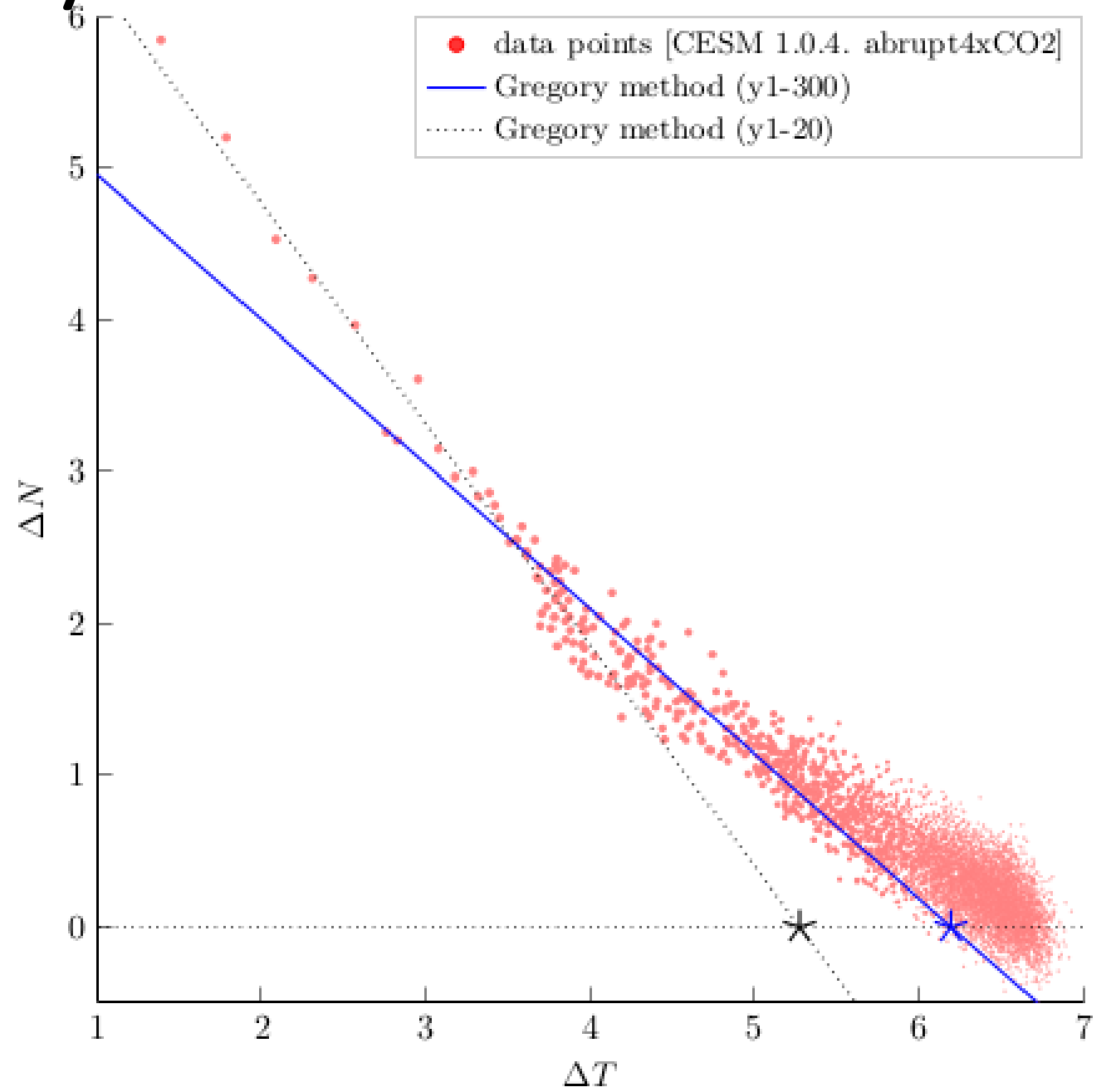
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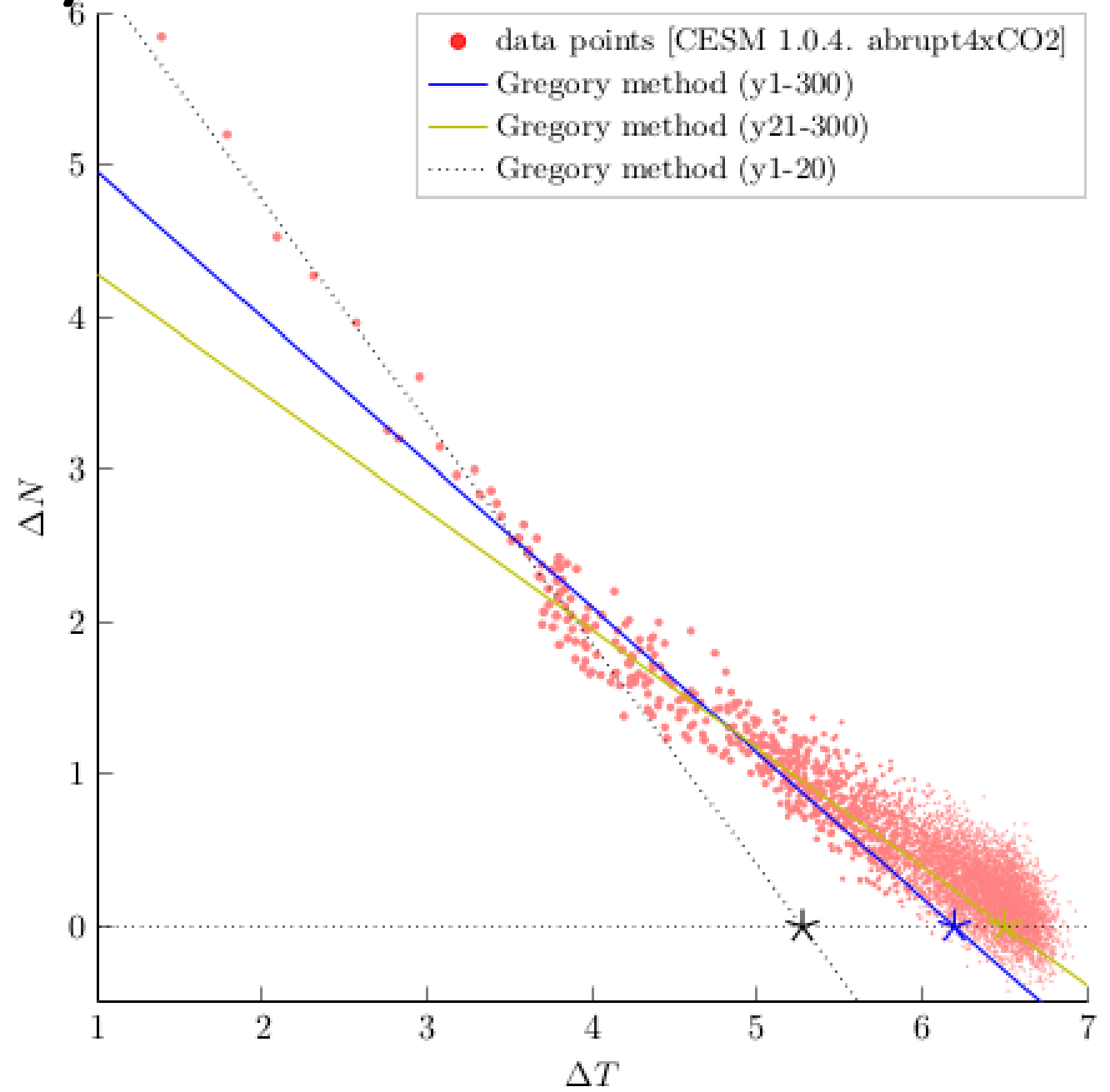
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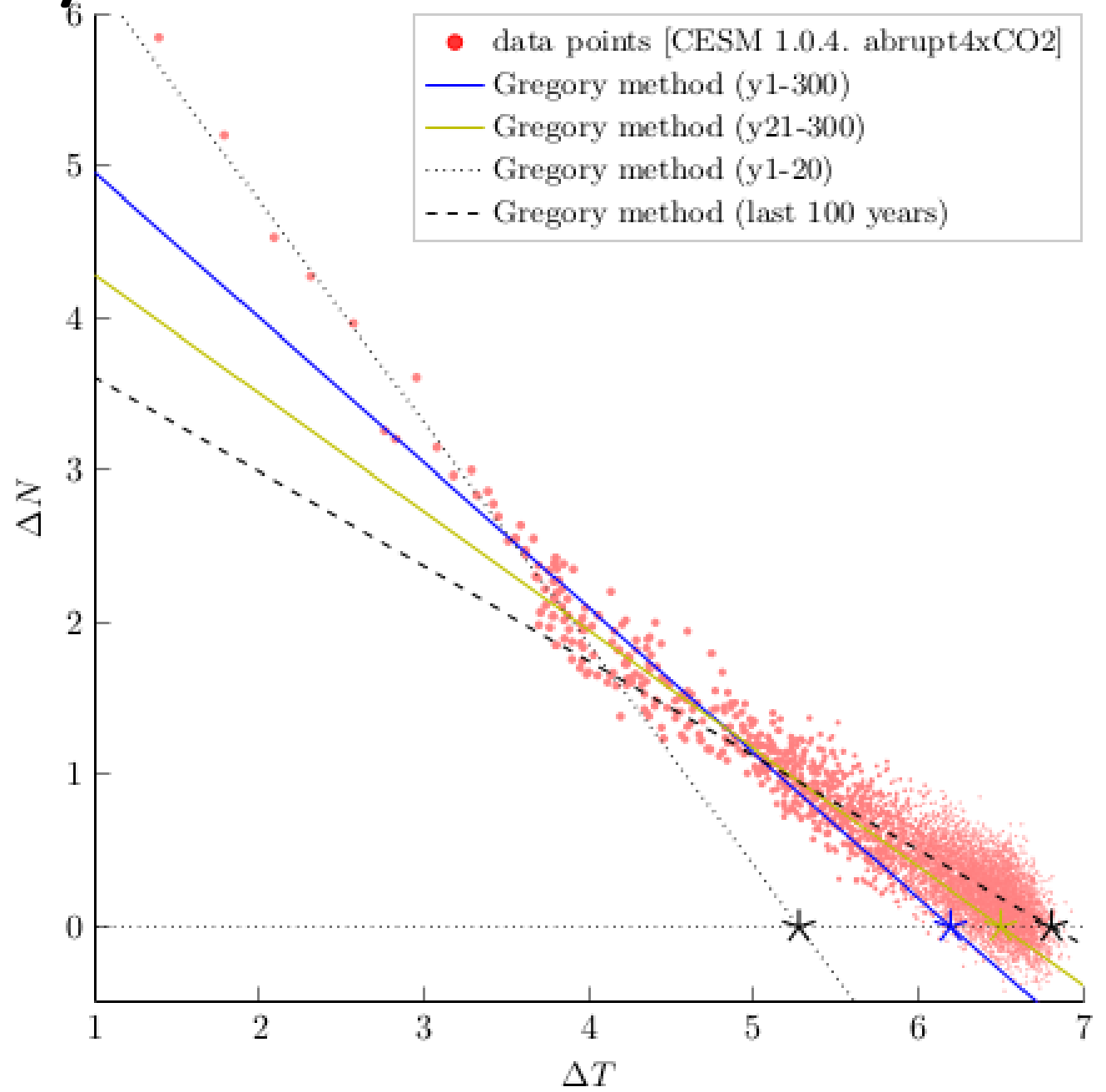
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# New Multicomponent Linear Regression Method

Use additional observables!

Regress to:

$$\overrightarrow{\Delta Y} = \mathbf{A} \overrightarrow{\Delta X} + \overrightarrow{\mathbf{F}}$$

$\overrightarrow{\Delta Y}$ :

observables that  
tend to 0  
in equilibrium

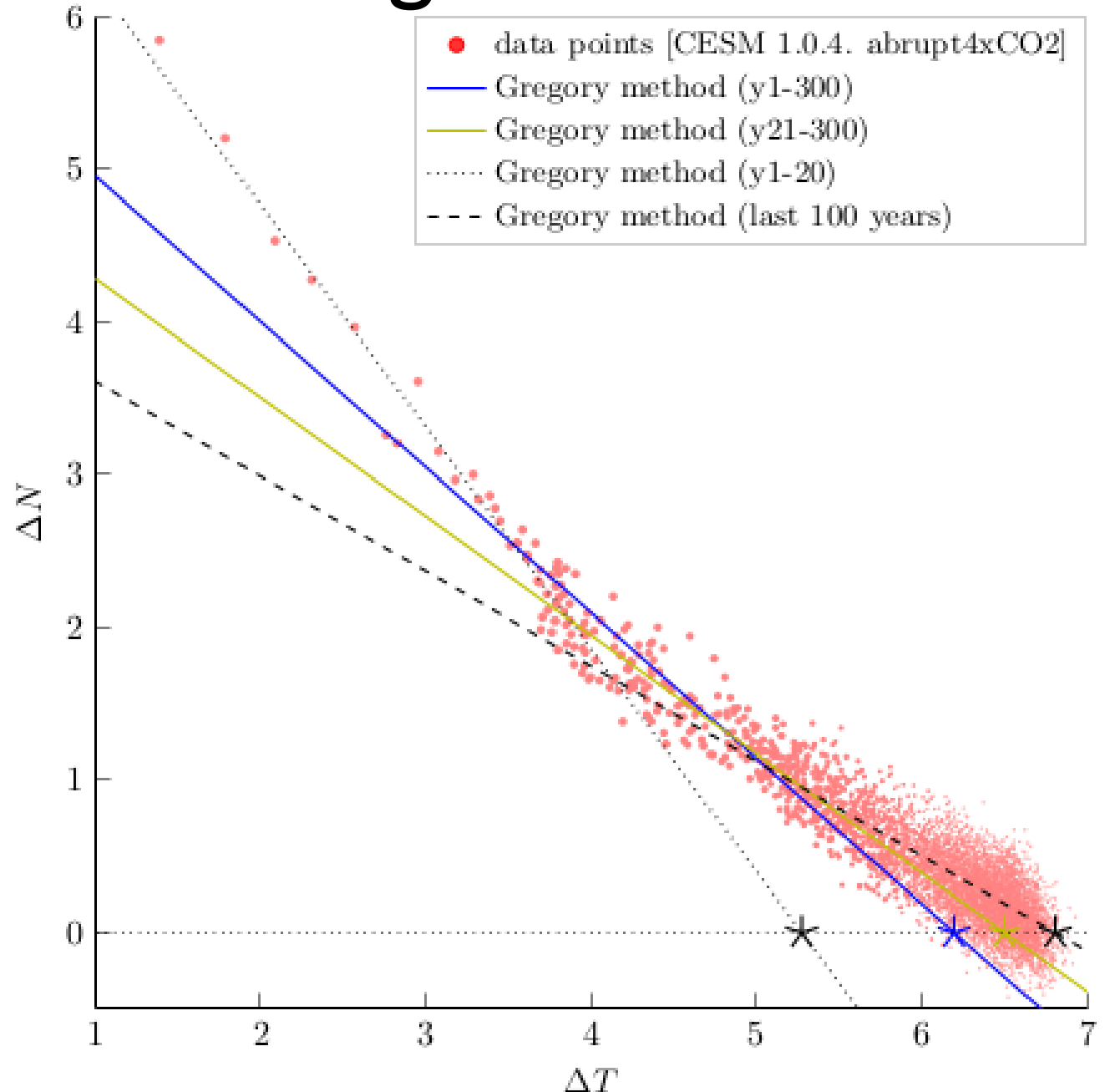
$\overrightarrow{\Delta X}$ :

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Multivariate ECS estimation is

$$\overrightarrow{\Delta X}_*^{est} = -\mathbf{A}^{-1} \overrightarrow{\mathbf{F}}$$

[Bastiaansen et al, 2021]



# New Multicomponent Linear Regression Method

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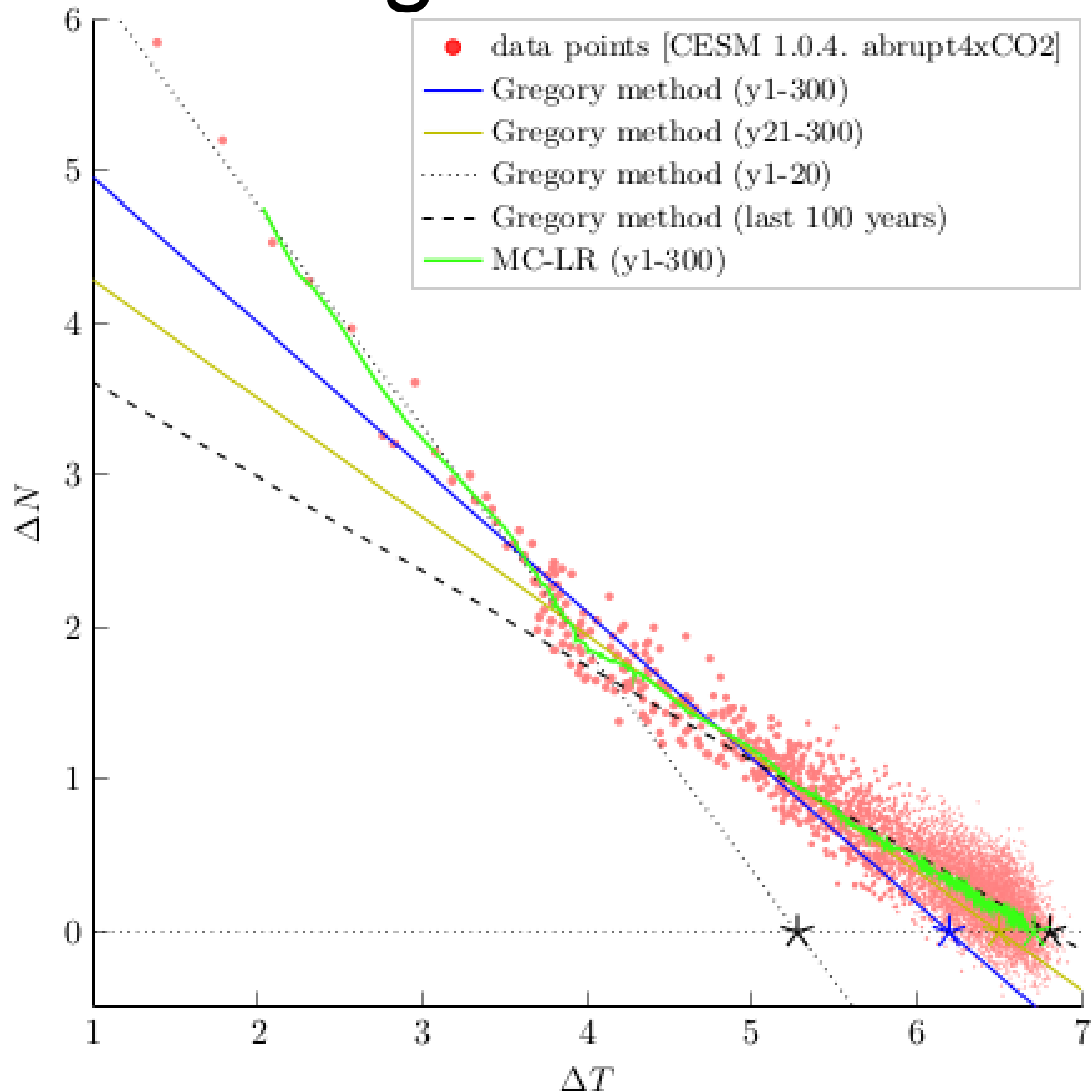
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$\overrightarrow{\Delta X}$ :

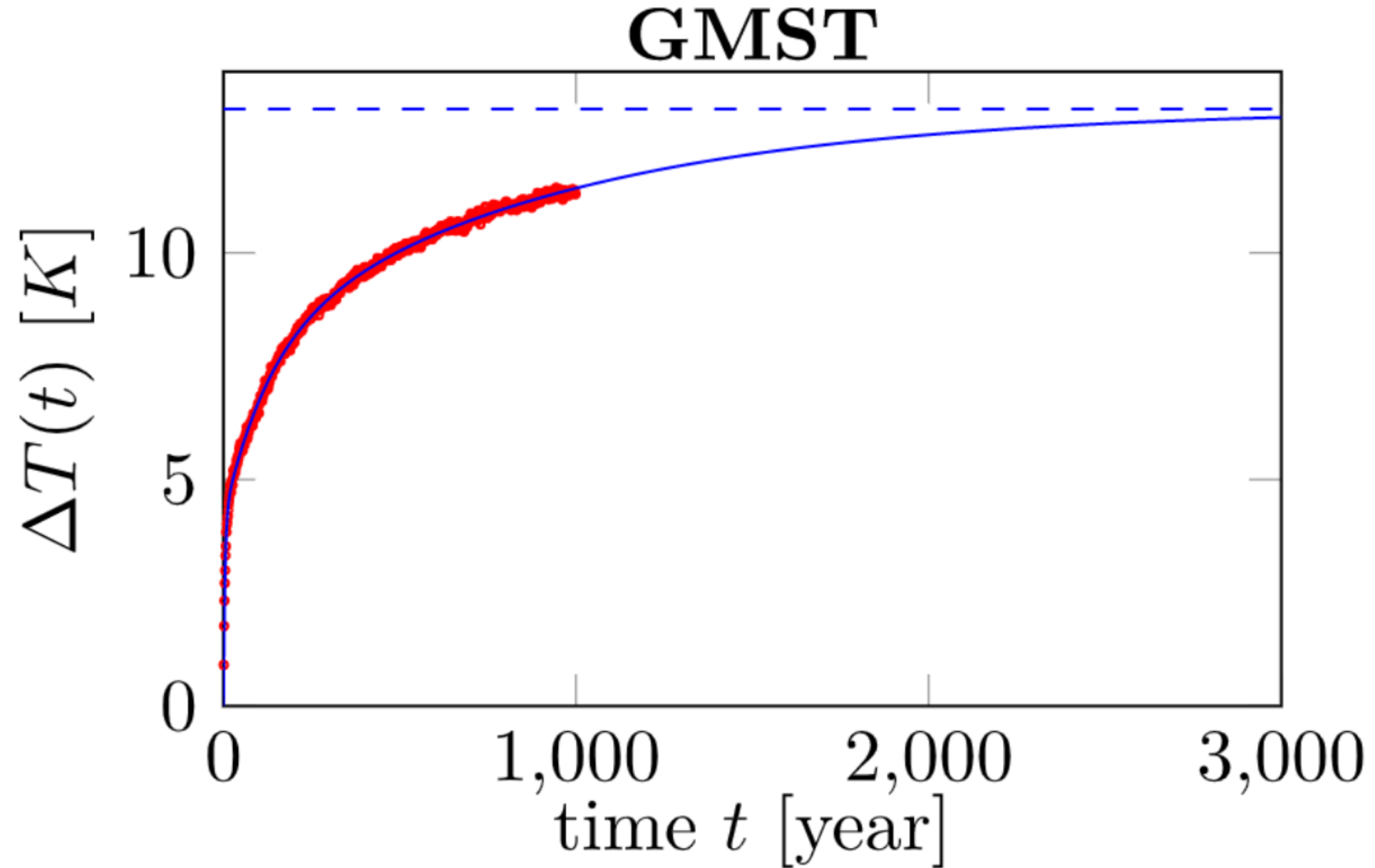
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Multivariate ECS estimation is

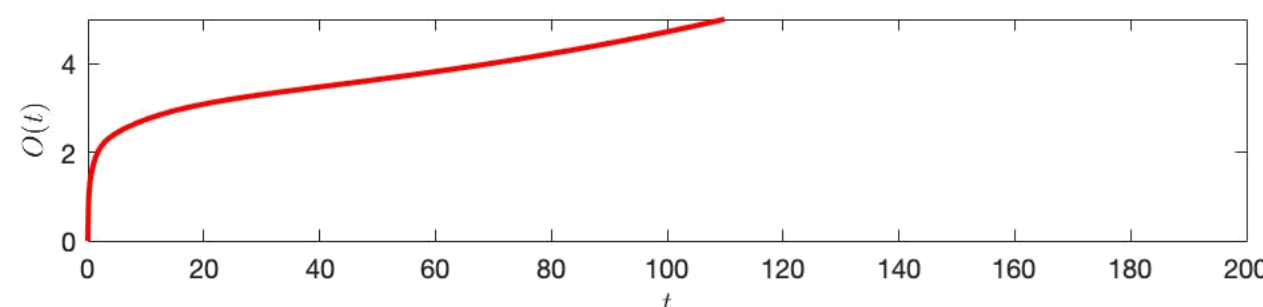
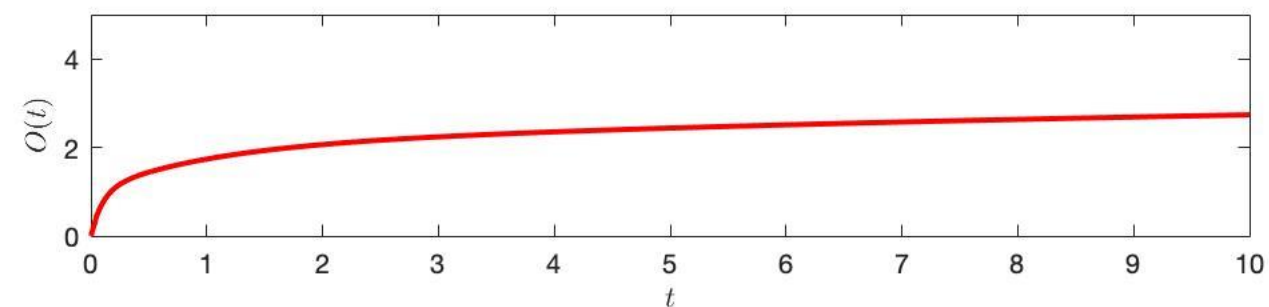
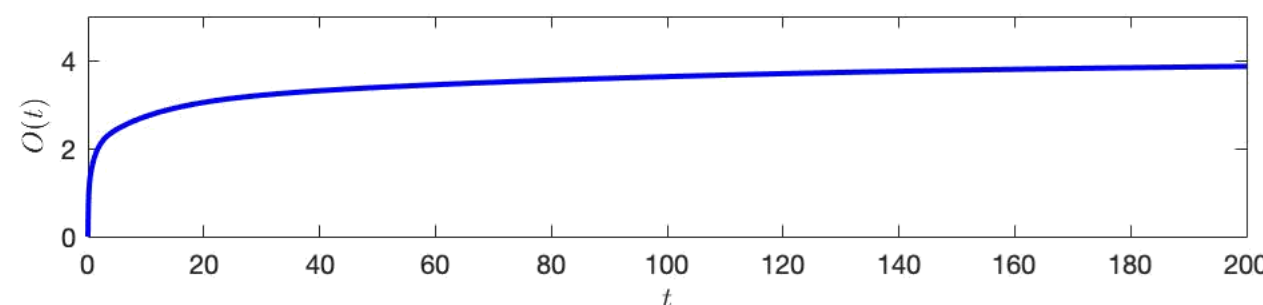
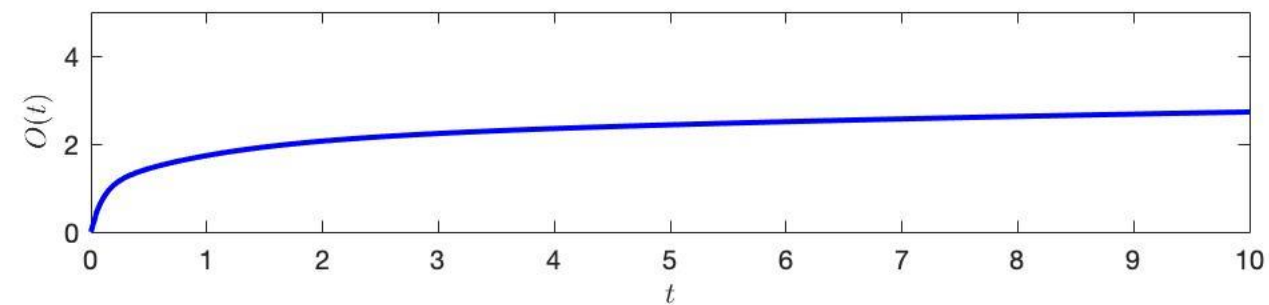
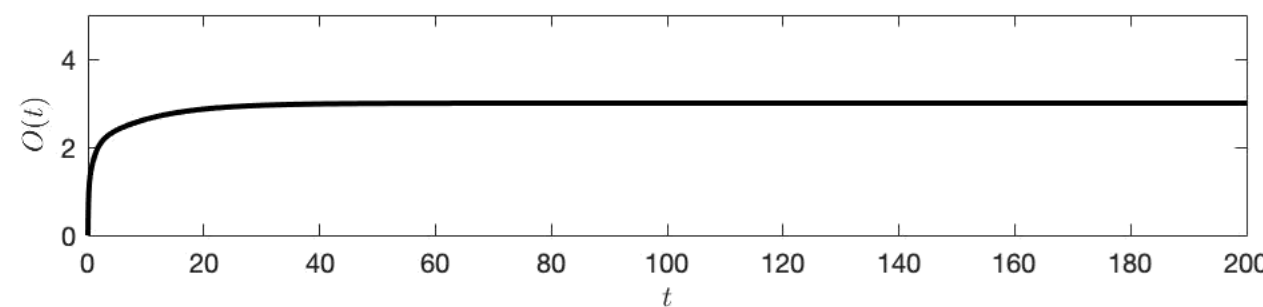
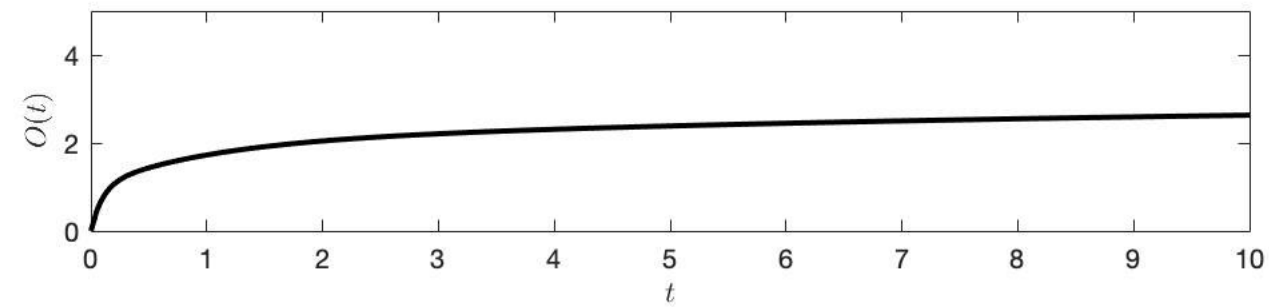
$$\overrightarrow{\Delta X}_*^{est} = -\mathbf{A}^{-1} \overrightarrow{\mathbf{F}}$$



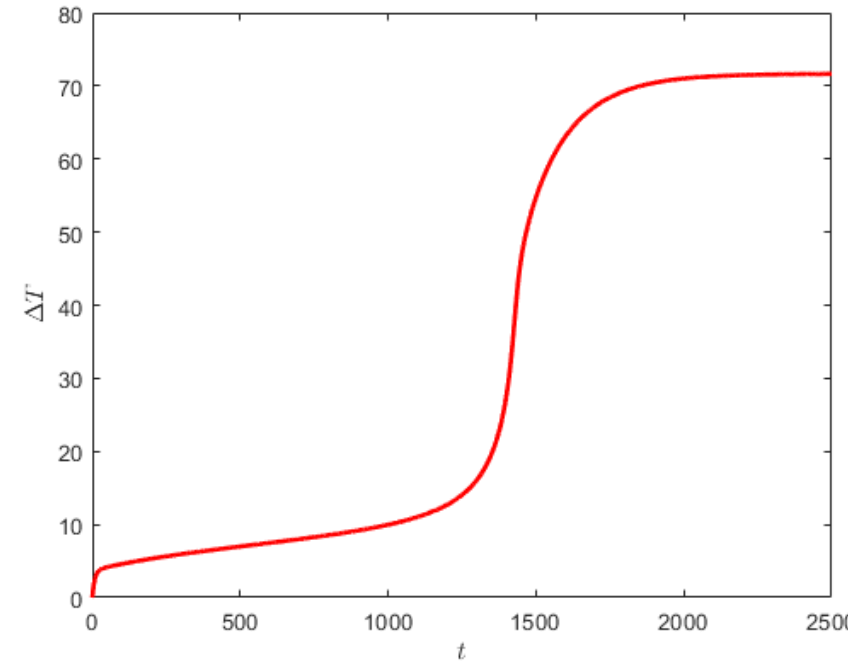
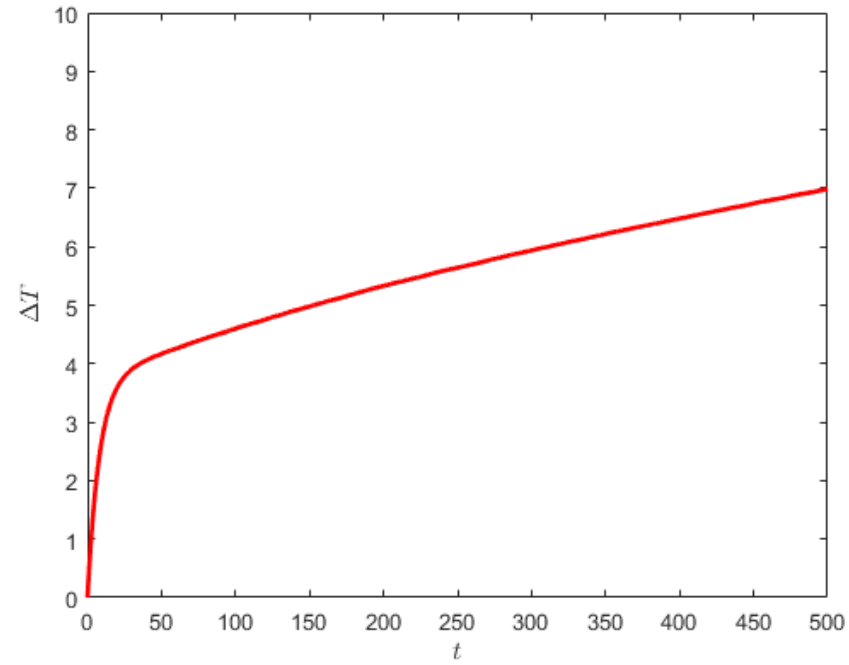
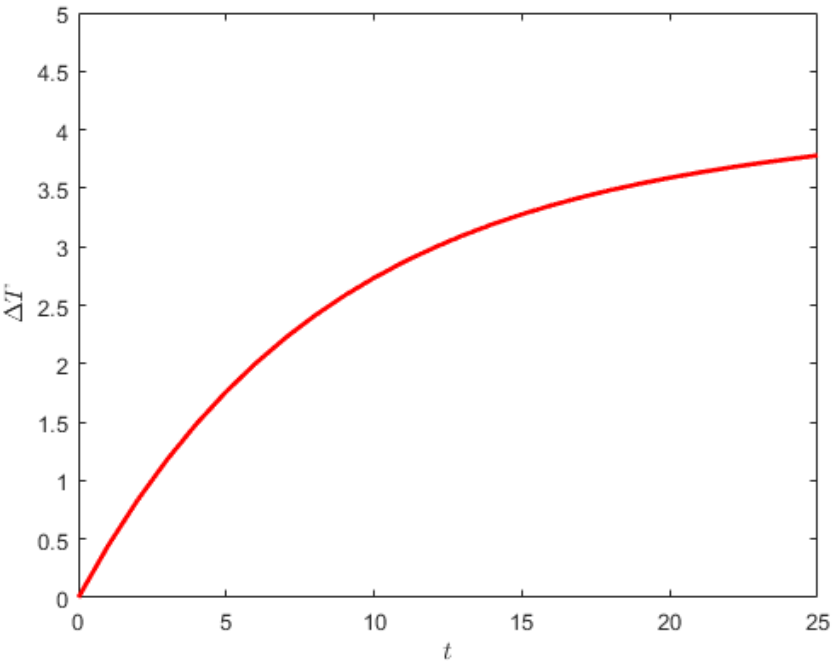
# Projections of the Transient State-Dependency of Climate Feedbacks



# Pitfalls and problems



# Nonlinear Response



# Multiscale Global Energy Balance Model

$$C \frac{dT}{dt} = Q_0(1 - \alpha) - \epsilon\sigma T^4 + \mu$$

Short-Wave

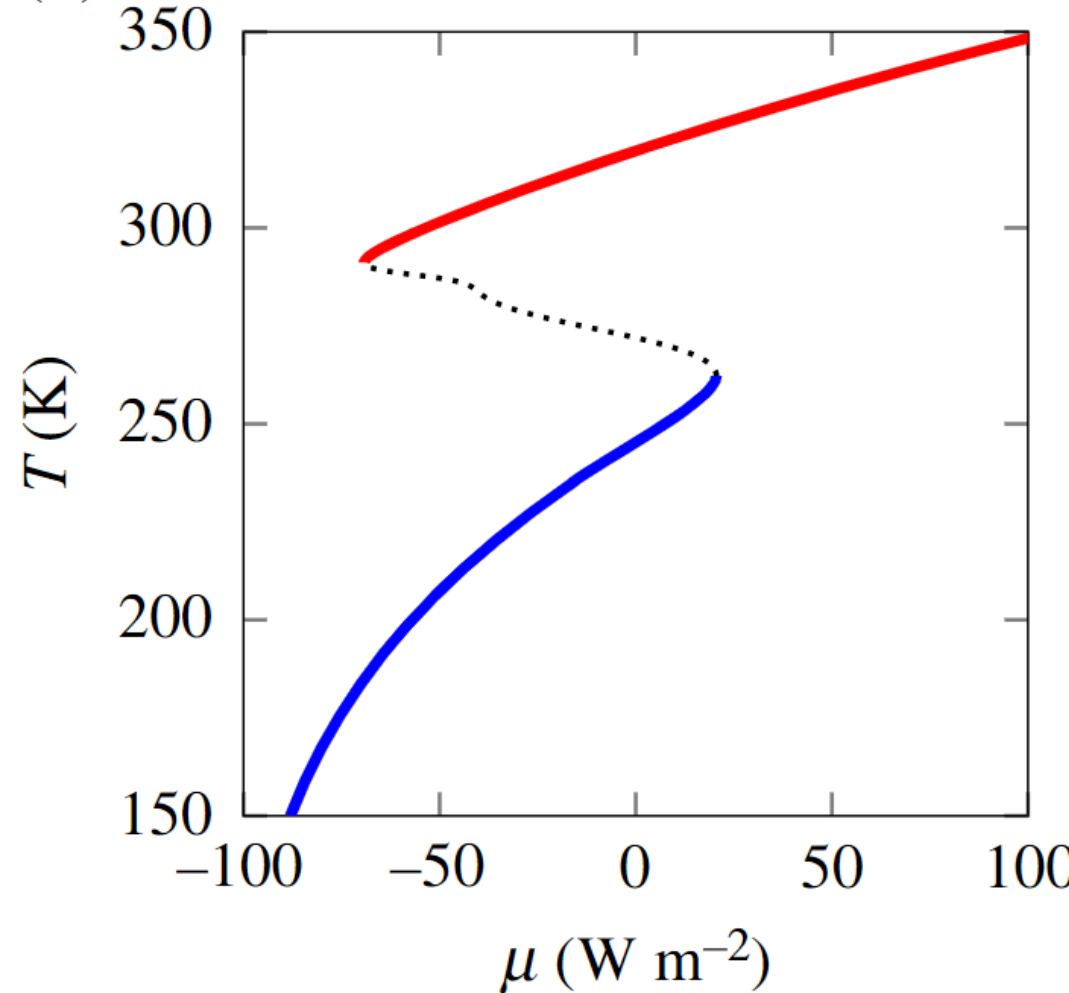
Long-Wave

CO<sub>2</sub>

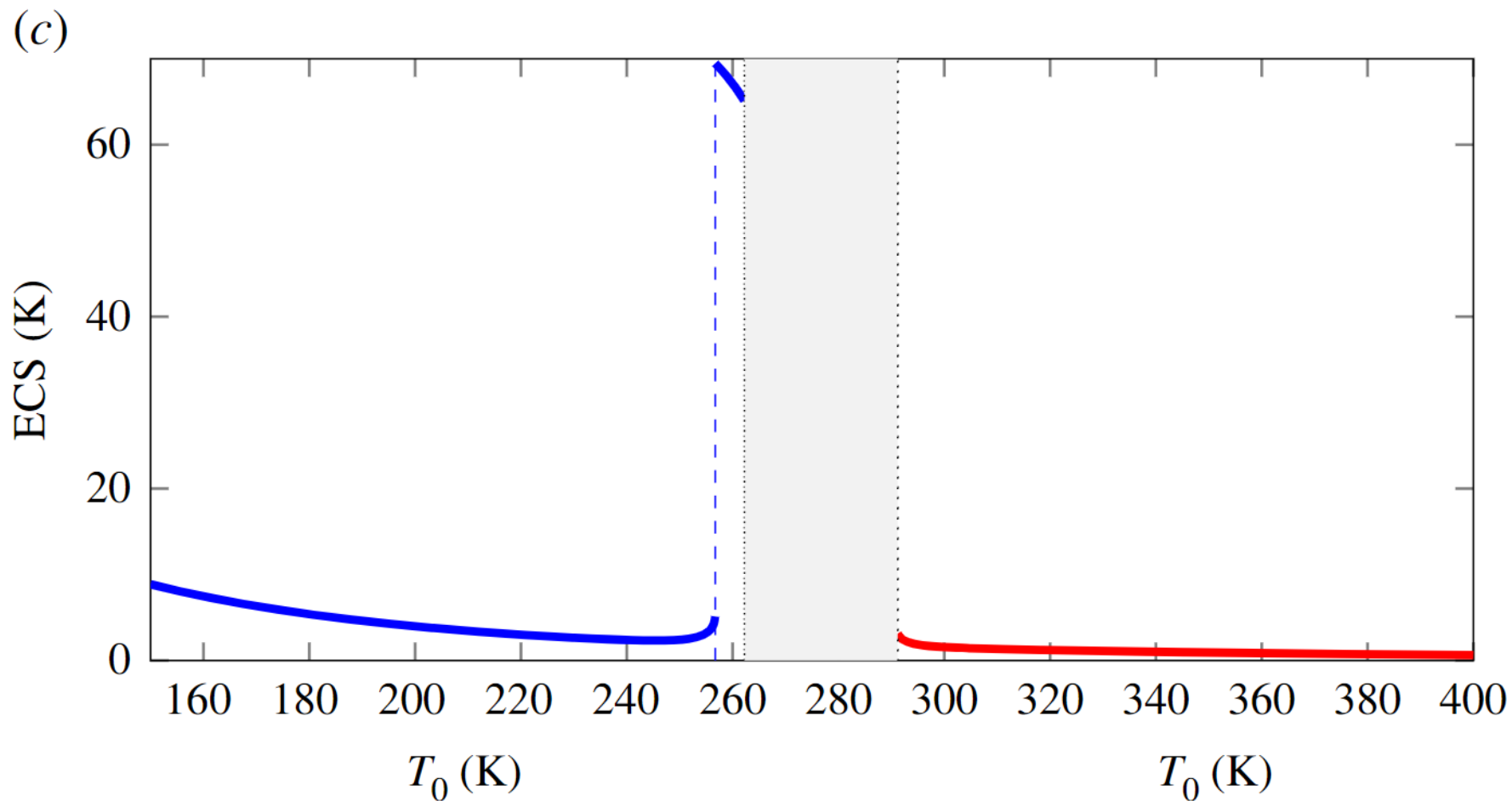
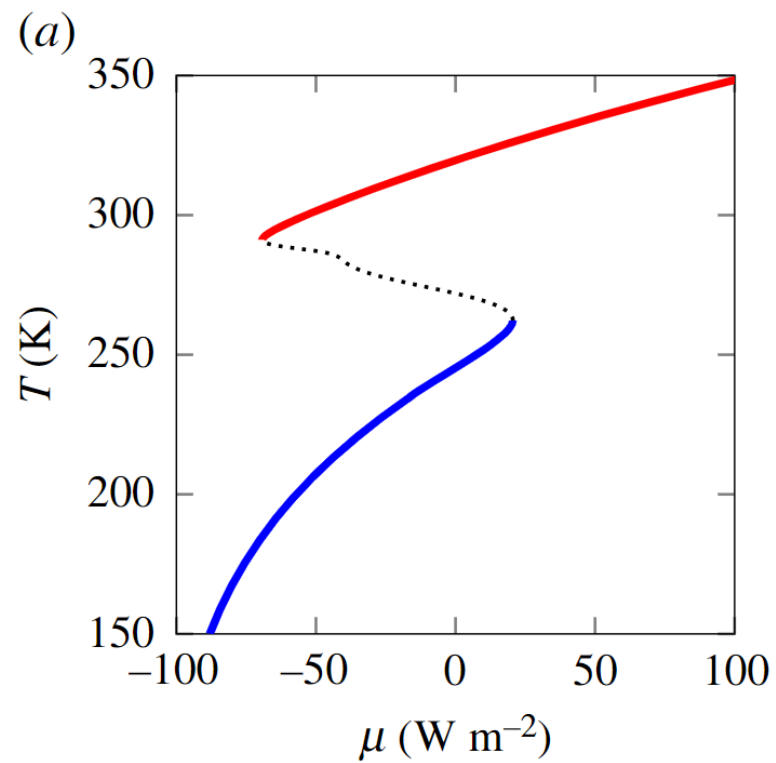
$$\tau_\alpha \frac{d\alpha}{dt} = \alpha_0(T) - \alpha$$

Dynamic albedo

(a)

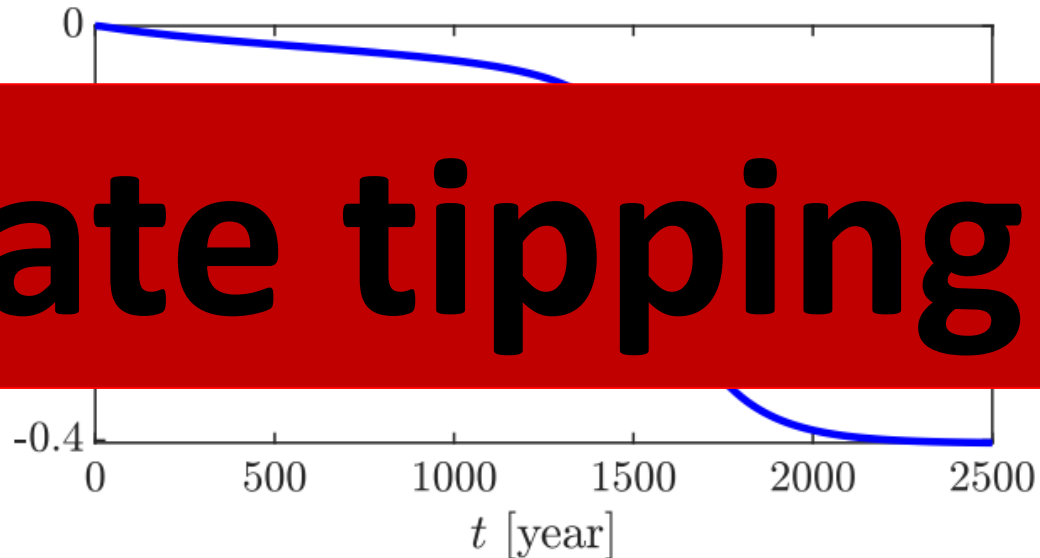
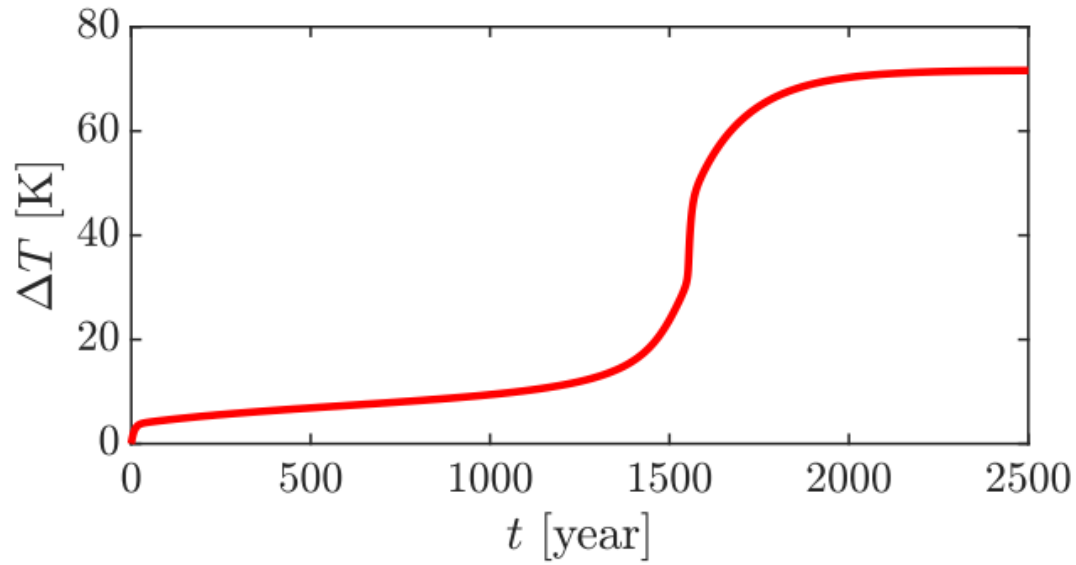


# Background dependency



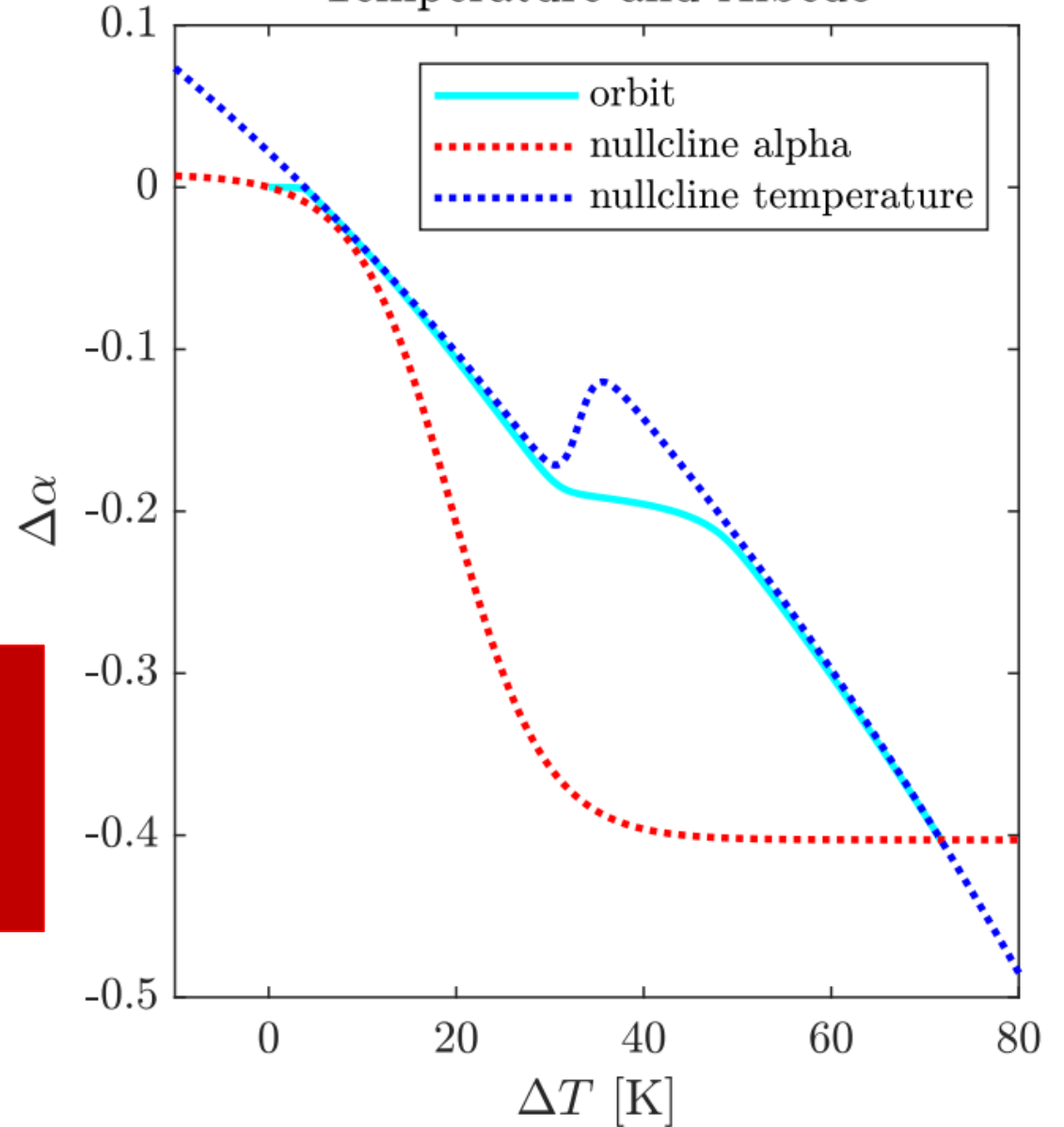


# DYNAMICS: Nonlinear Response

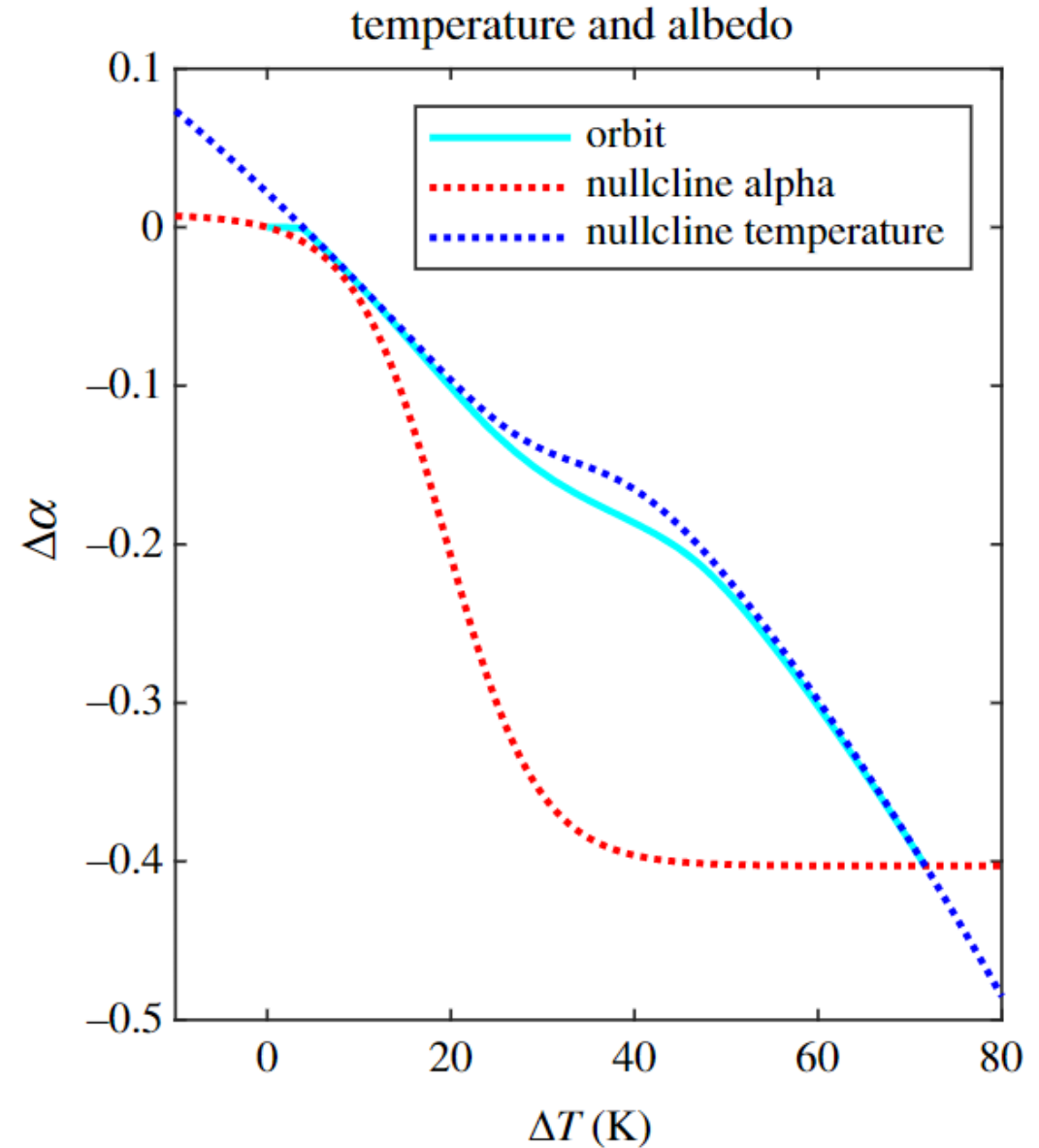
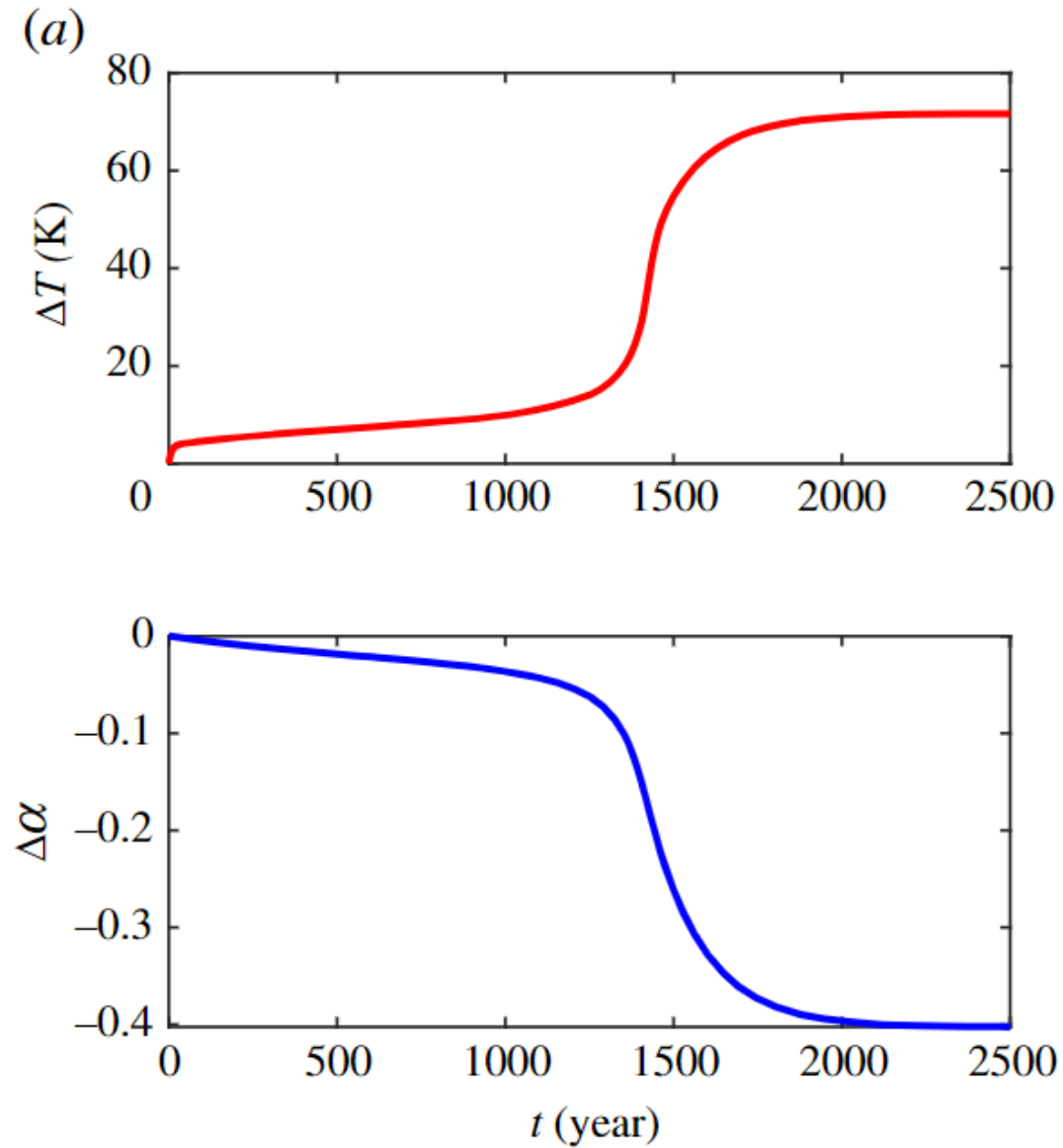


**Late tipping!**

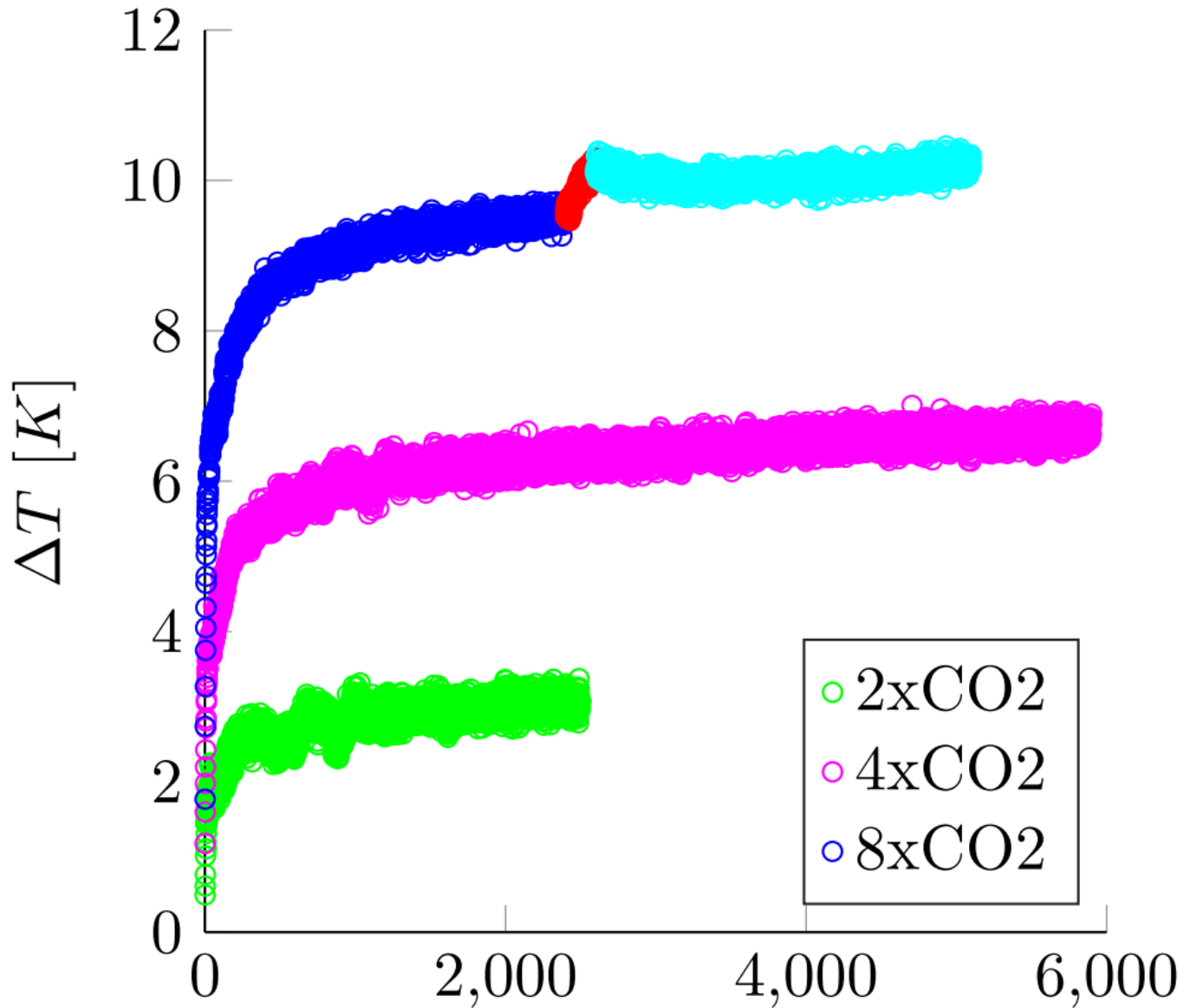
Temperature and Albedo



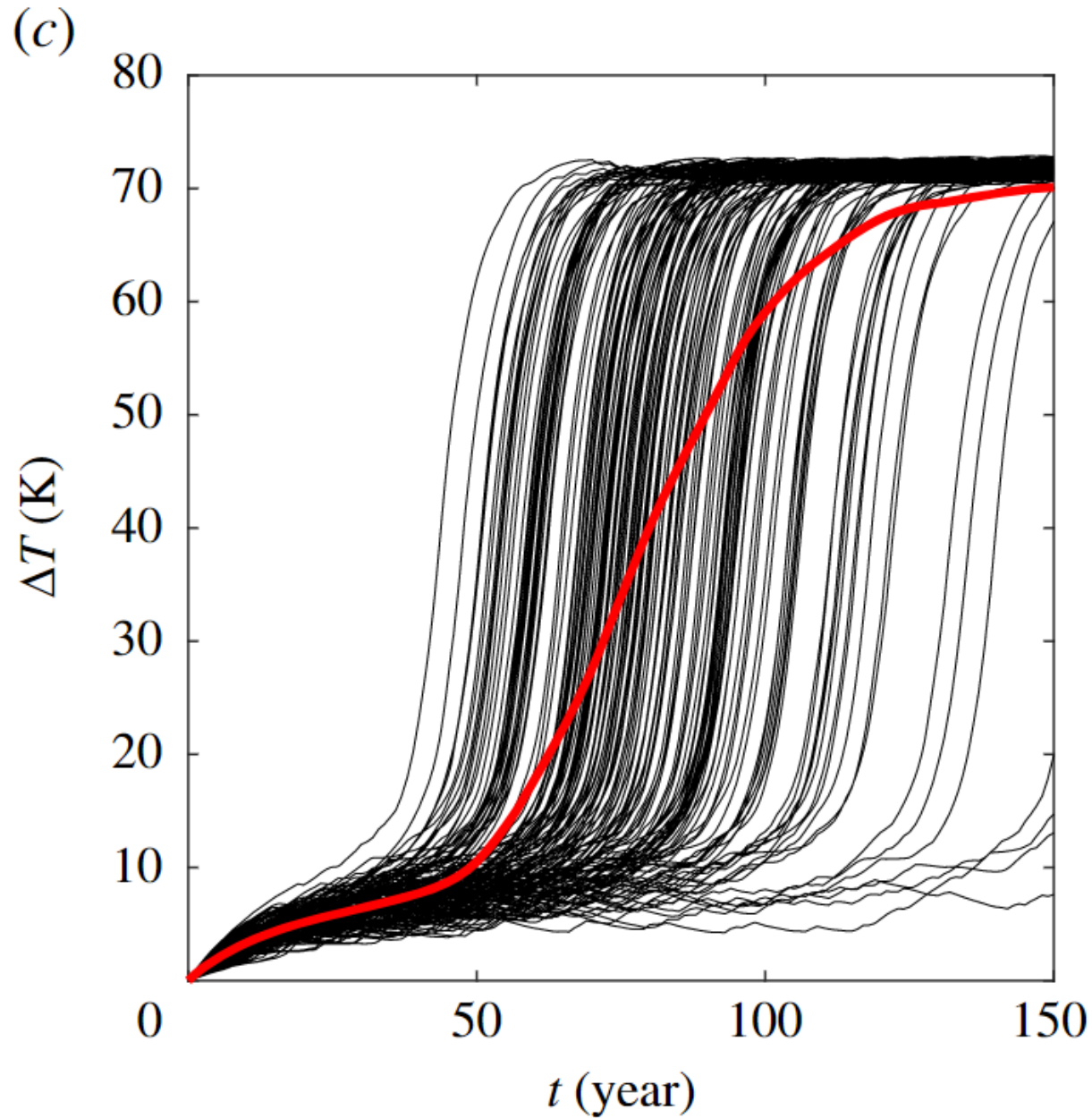
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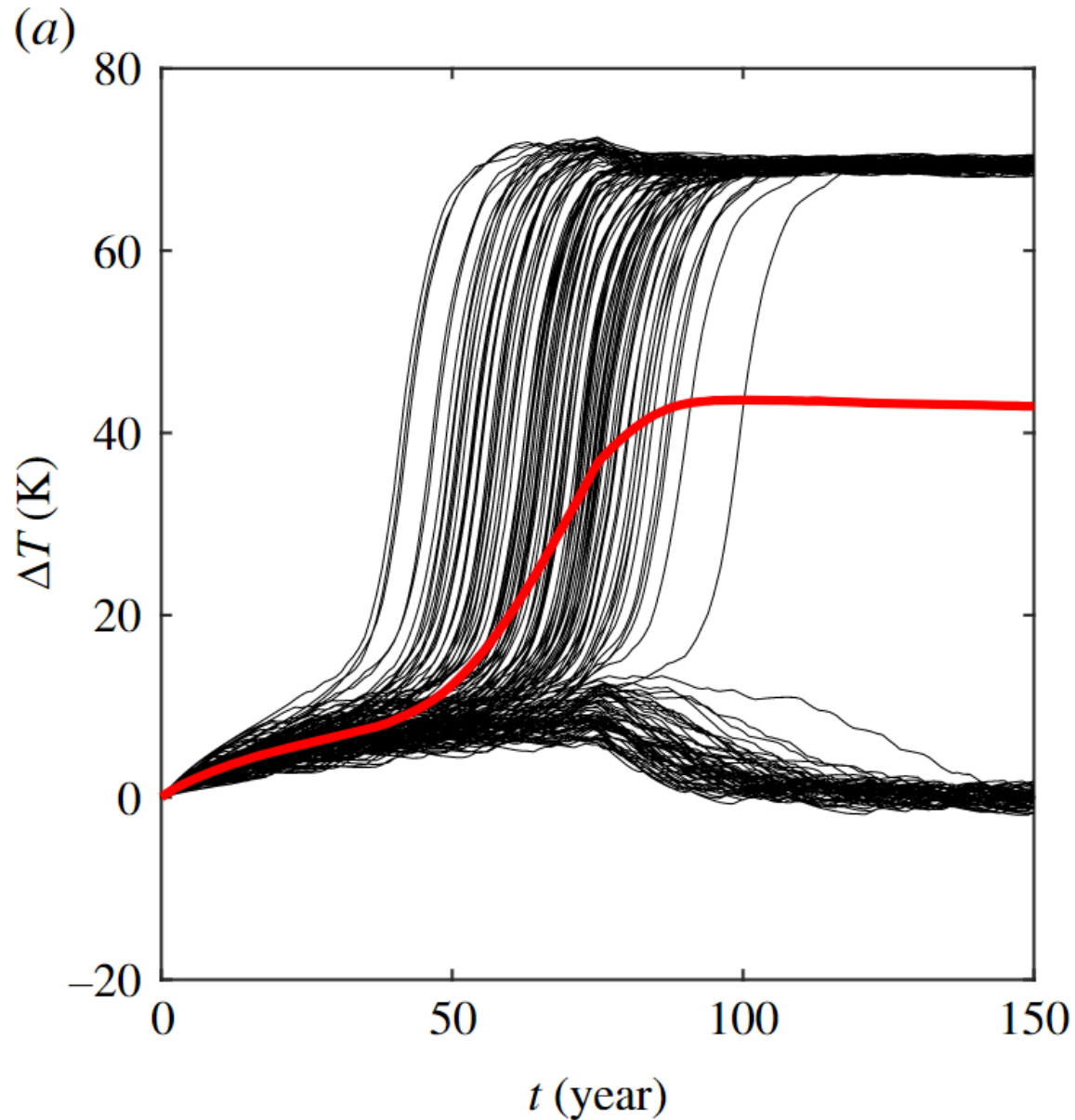
# Nonlinear Response in GCM



# Chaotic Systems & Ensembles



# Chaotic Systems & Ensembles



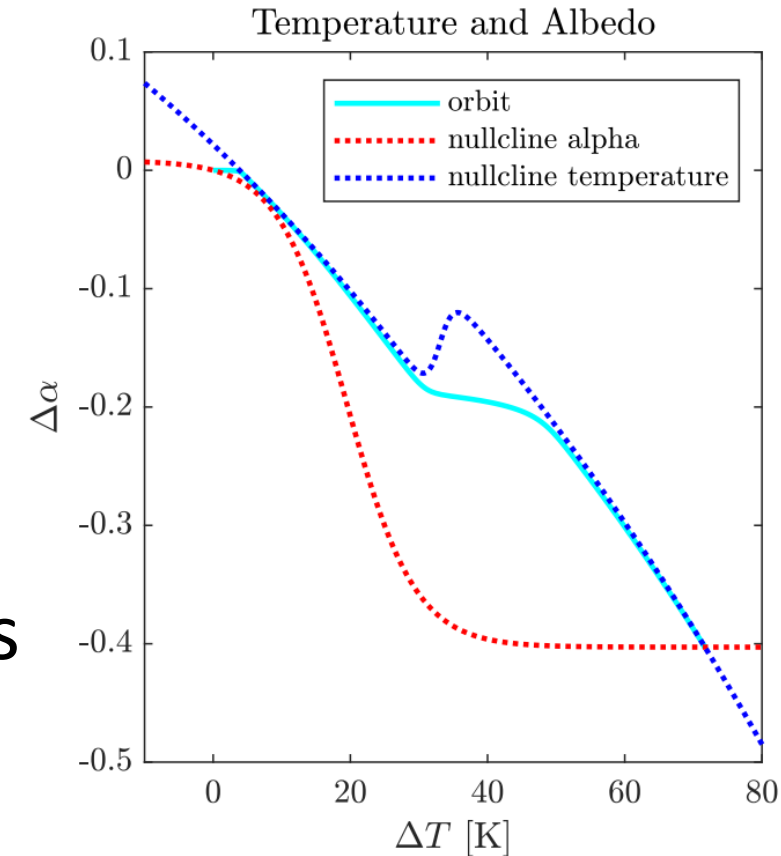
**partial tipping**

Alkhayuon, H. M., & Ashwin, P. (2018)

# Conclusion

## TIME SCALES !

- Models have time scale of validity
  - Extreme example: Late tipping points
- Climate response is sensitive to initial conditions
  - Extreme example: Partial tipping



After long period of SLOW change suddenly FAST change can happen!

### Paper:

Bastiaansen, R., Ashwin, P., & von der Heydt, A. S. (2023). Climate response and sensitivity: time scales and late tipping points. *Proceedings of the Royal Society A*, 479(2269), 20220483.

