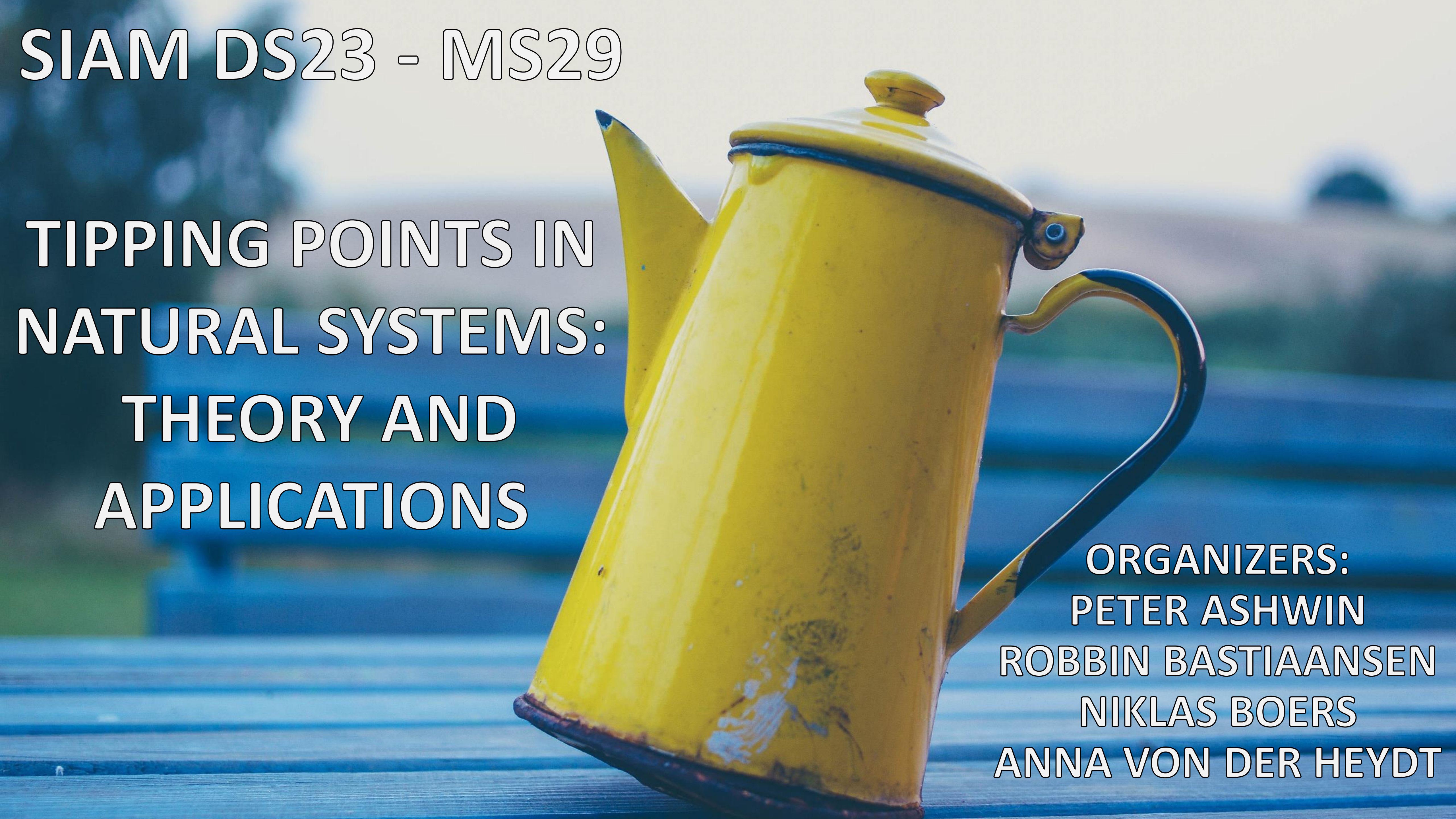


SIAM DS23 - MS29

TIPPING POINTS IN NATURAL SYSTEMS: THEORY AND APPLICATIONS

ORGANIZERS:
PETER ASHWIN
ROBBIN BASTIAANSEN
NIKLAS BOERS
ANNA VON DER HEYDT

MS29 Tipping Points in Natural Systems: Theory and applications

4:55 – 5:20	Robbin Bastiaansen	Tipping Phenomena and Time Scales
5:25 – 5:50	Kerstin Lux	Uncertainty Quantification for Tipping Points of the Atlantic Meridional Overturnin Circulation
5:55 – 6:20	Iacopo Longo	Rigorous Criteria for Tipping and Tracking of Concave Coercive ODEs
6:25 – 6:50	Sebastian Wieczorek	Rate-Induced Tipping to Zombie Fires

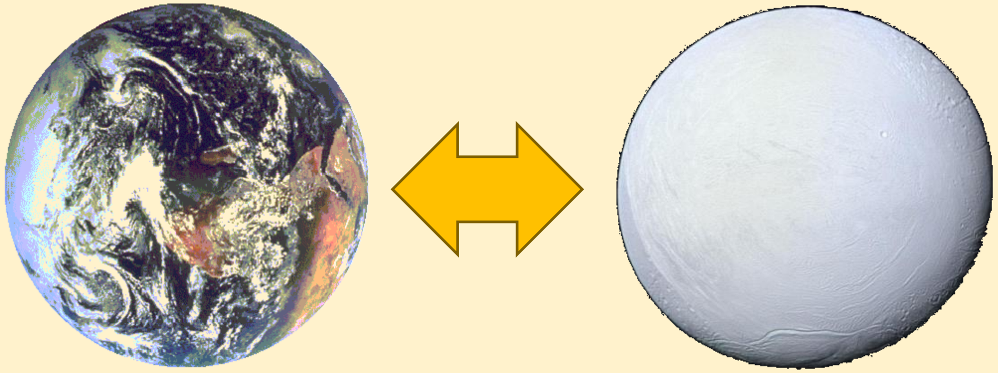
TIPPING PHENOMENA AND TIME SCALES

ROBBIN BASTIAANSEN
(R.BASTIAANSEN@UU.NL)

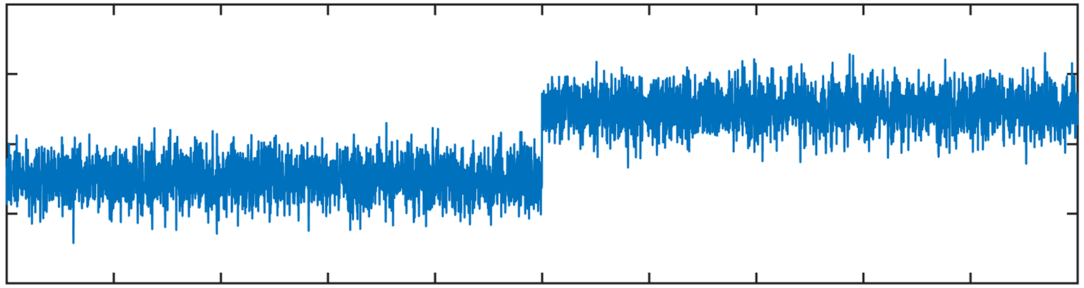
SIAM DS23, 2023-05-14

Tipping Points

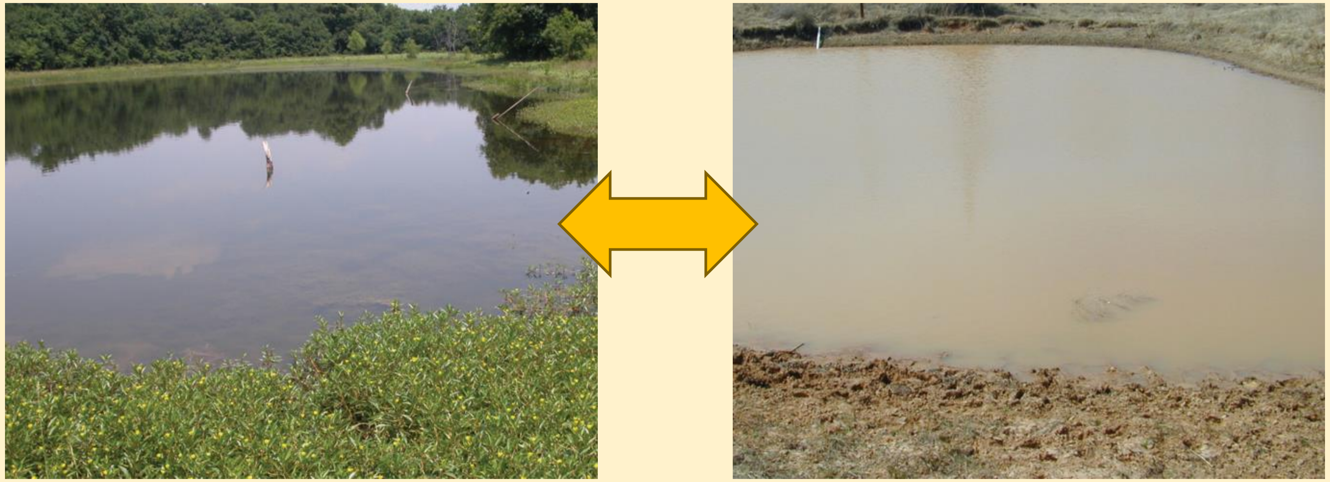
IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”



Planetary transitions

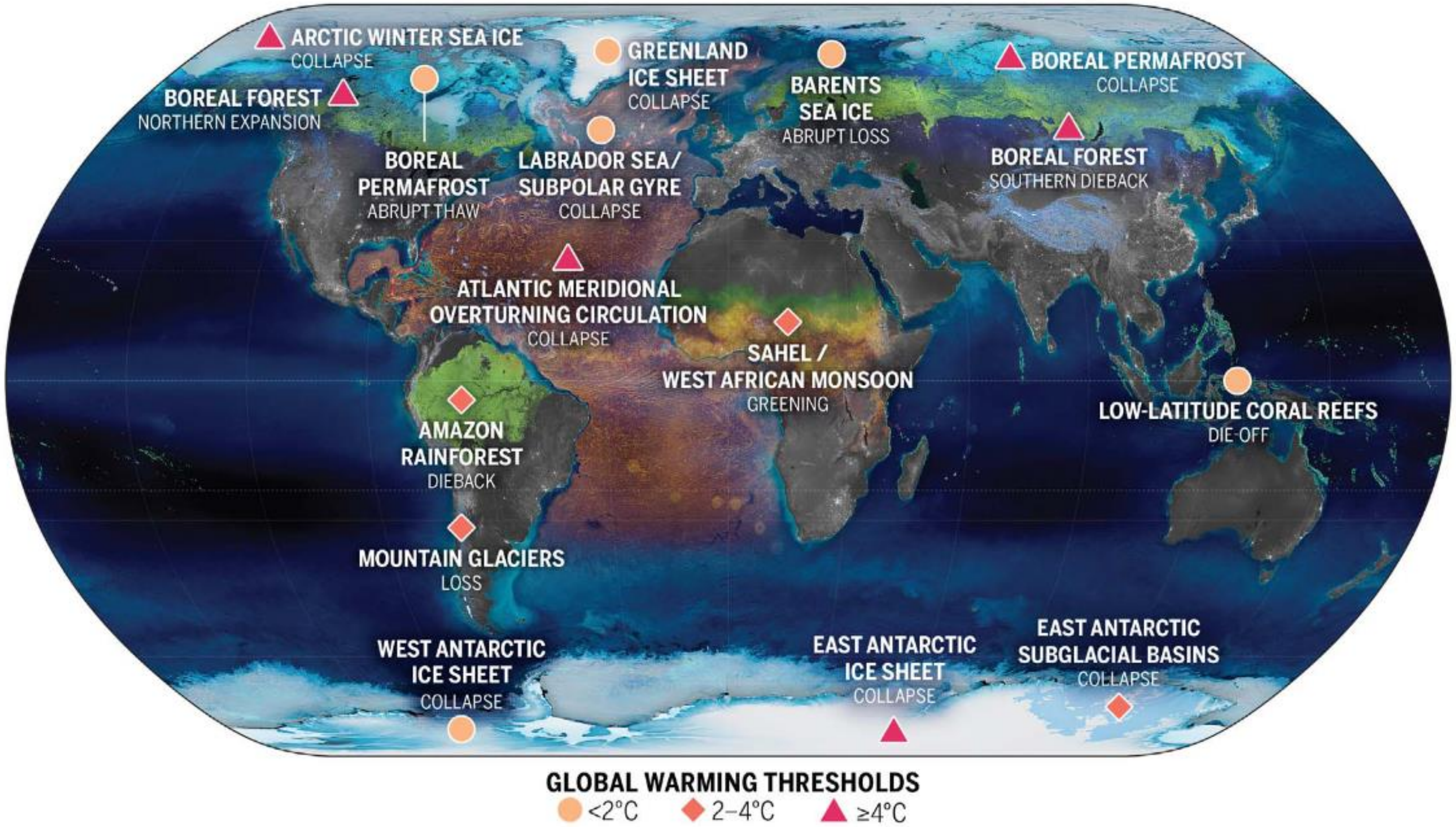


Ecosystem shifts



Tipping Points

IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”

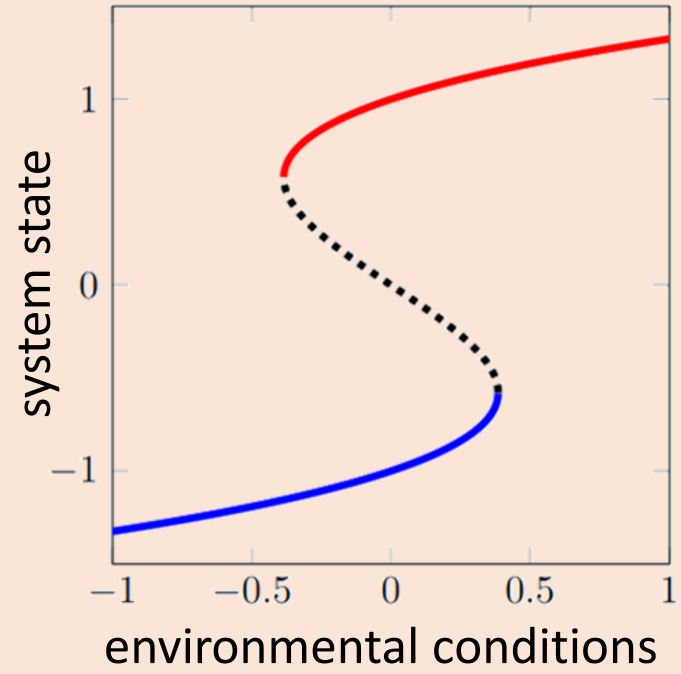


source: McKay et al, 2022

Mathematics

Tipping points ↔ Bifurcations

$$\frac{dy}{dt} = f(y, \mu)$$



Classic Theory of Tipping

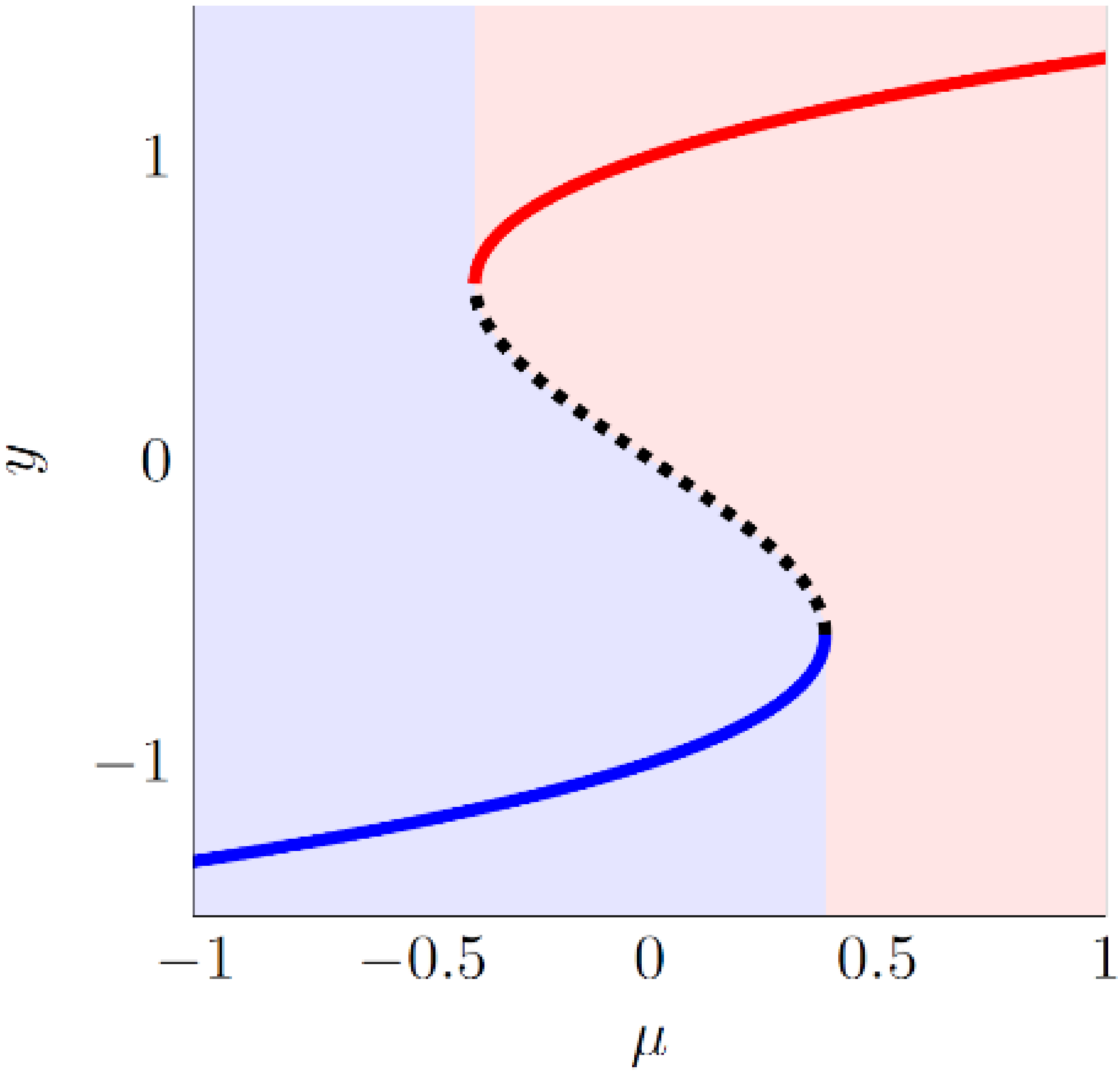
$$\frac{d\vec{y}}{dt} = f(\vec{y};\mu)$$

Canonical example:

$$\frac{dy}{dt} = y(1 - y^2) + \mu$$

Bifurcation structure and location of tipping points are hard to predict

→ Kerstin's talk @5:25 for progress in this direction



How does tipping work?

$$\frac{dy}{dt} = y(1 - y^2) + \mu$$

Internal Dynamics

$$\frac{d\mu}{dt} = \delta$$

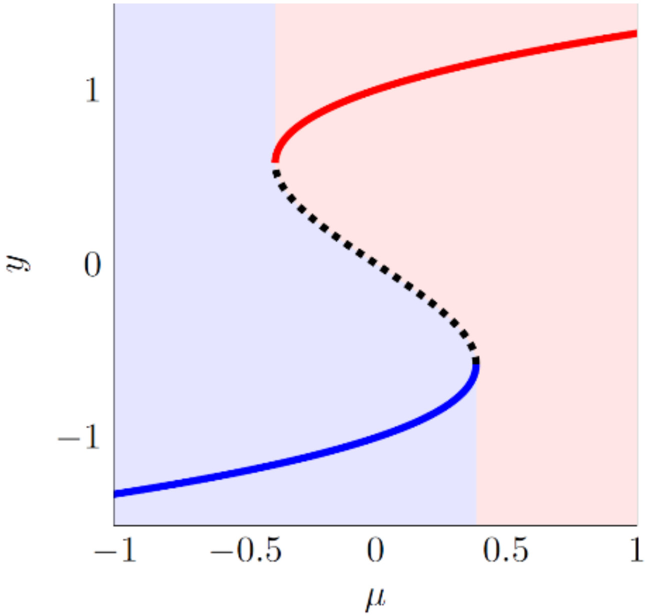
Parameter Drift

Time Scale Separation

$0 < \delta \ll 1$: Bifurcation-tipping (B-tipping)

$\delta \geq \mathcal{O}(1)$: more complicated stuff

→ Iacopo’s talk @ 5:55
→ Sebastian’s talk @ 6:25



Time Scales

INTERNAL TIME SCALES:

dynamics of tipping element

EXTERNAL TIME SCALE:

parameter drift

Previous slide: 1 internal time scale

Upcoming: examples with multiple internal time scales

EXAMPLE 1: Multiscale Global Energy Balance Model

$$C\frac{dT}{dt} = Q_0(1 - \alpha) - \epsilon(T)\sigma T^4 + \mu$$

Short-Wave

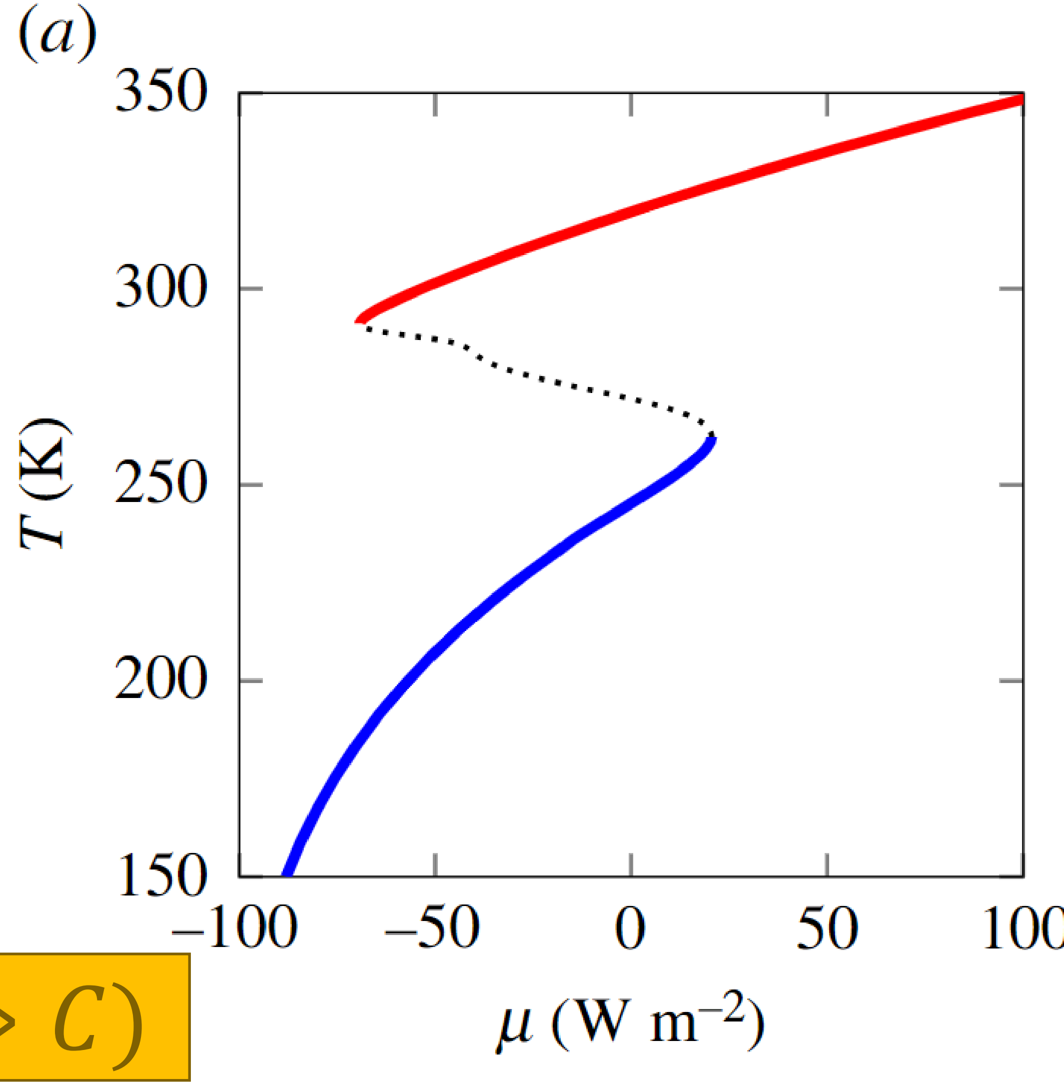
Long-Wave

CO2

$$\tau_\alpha \frac{d\alpha}{dt} = \alpha_0(T) - \alpha$$

Dynamic albedo

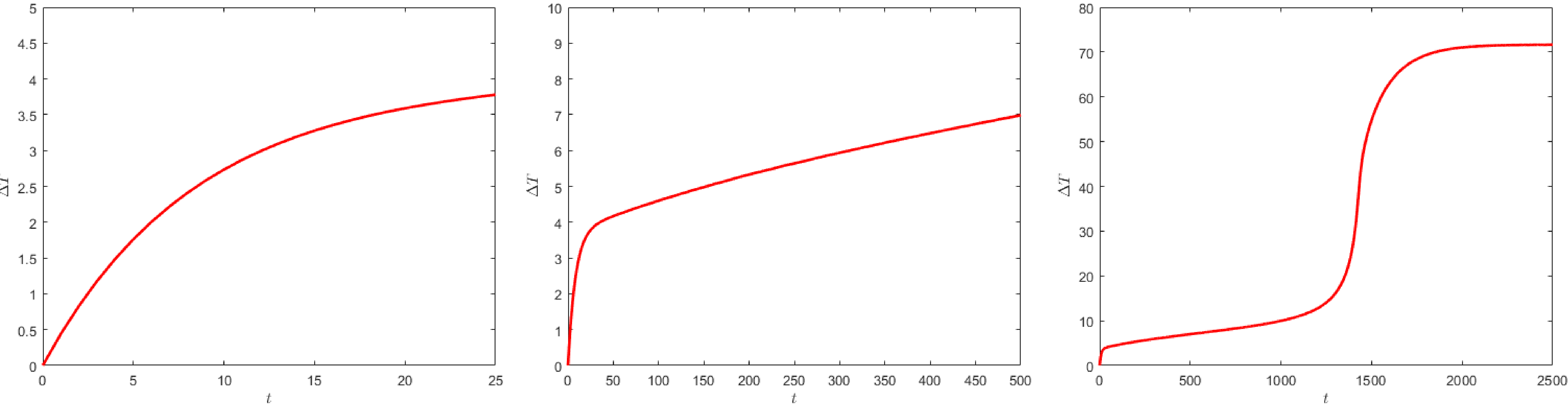
Internal Time Scale Separation ($\tau_\alpha \gg C$)



Abrupt 4xCO2 forcing experiment

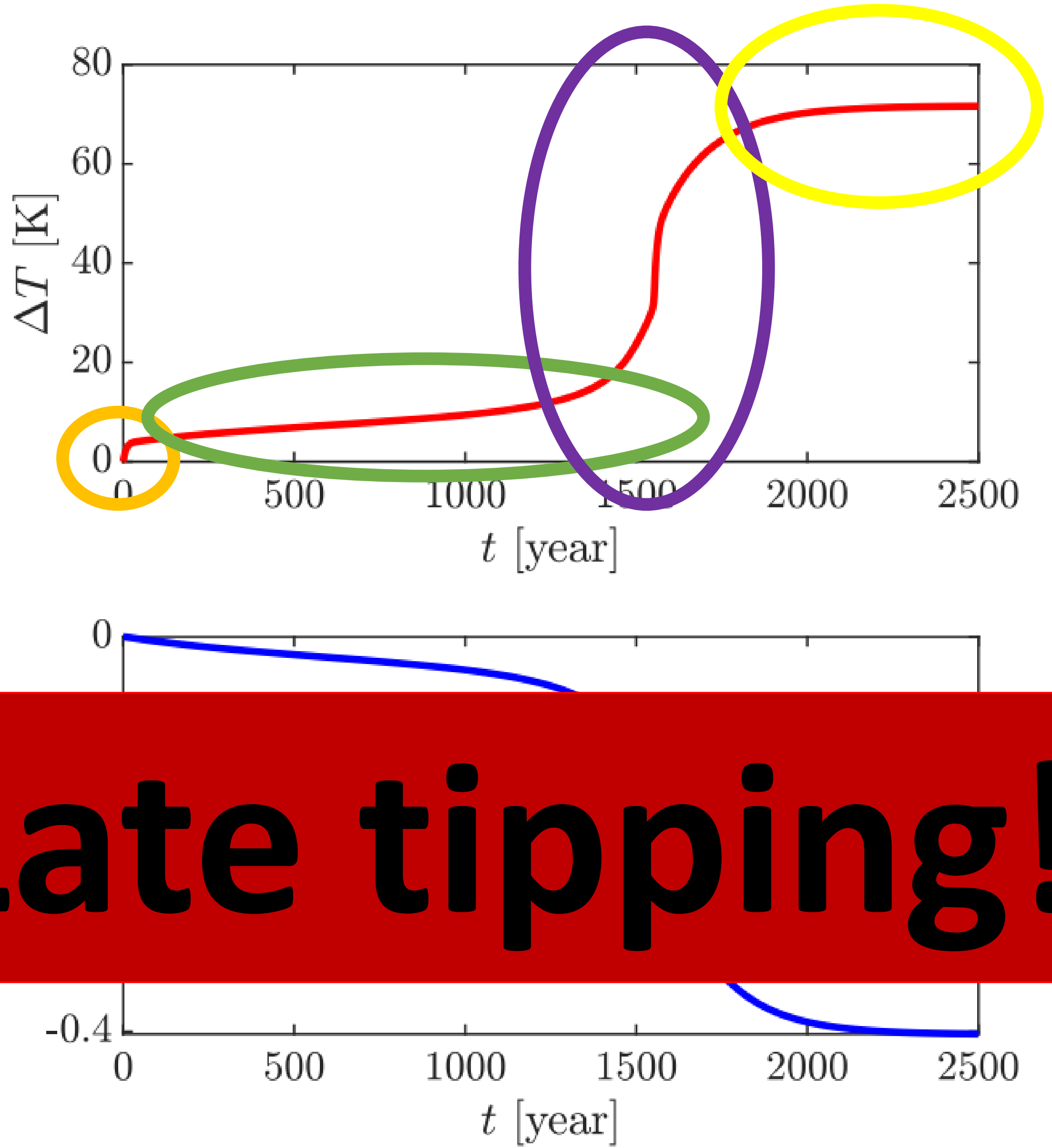
- Initialize for μ_0 (initial CO2-levels)
- Change to μ_1 (4xCO2 levels)

→ Look at dynamics

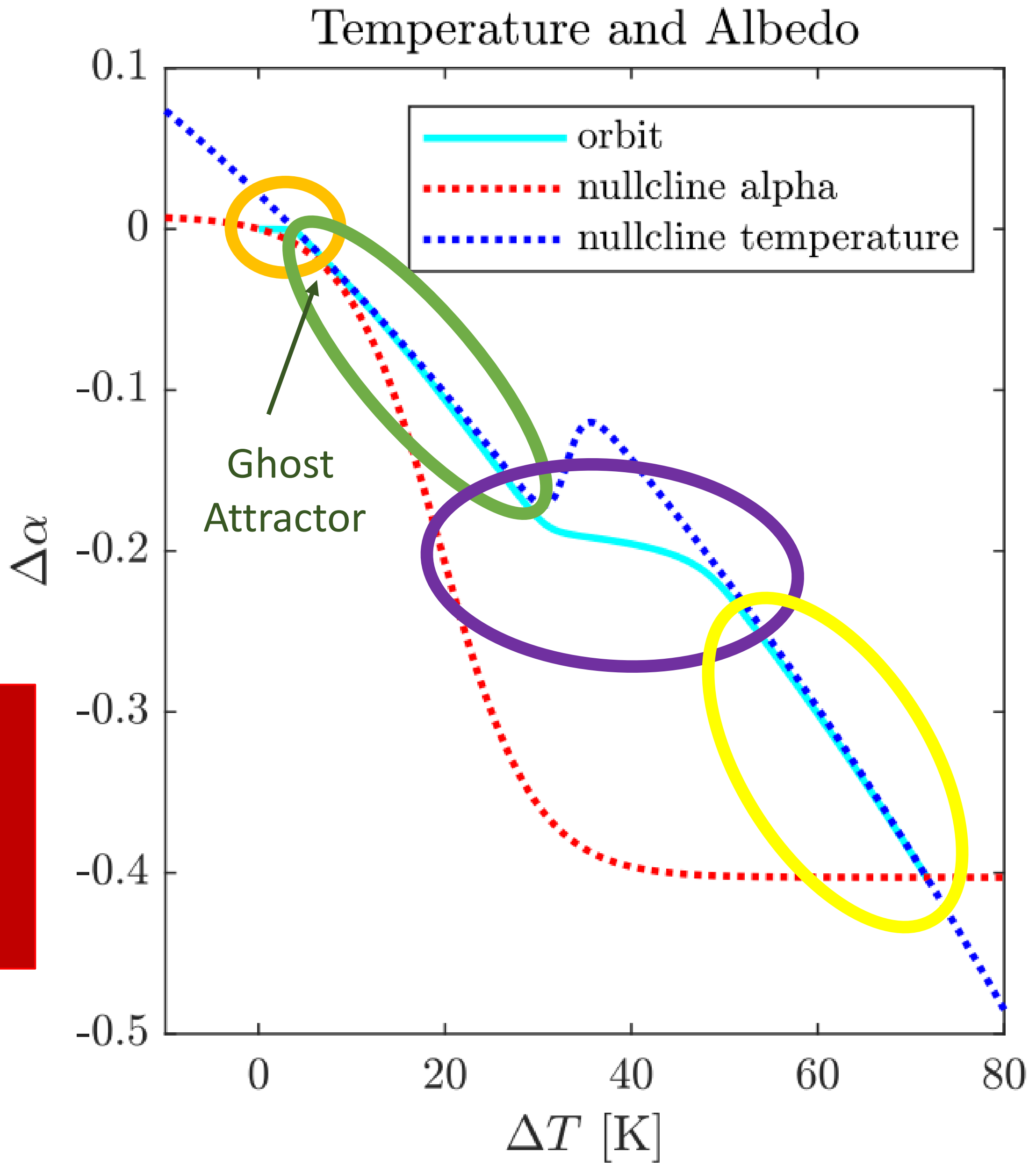


[Bastiaansen, Ashwin, Von der Heydt, 2023]

How does this work?



Late tipping!



[Bastiaansen, Ashwin, Von der Heydt, 2023]

EXAMPLE 2: AMOC ↔ ICE interaction

Tipping Element 1 (ICE)

$$\frac{dI}{dt} = f(I, R, T)$$

Energy balance model
[Eisenman & Wettlaufer, 2009]

Tipping Element 2 (AMOC)

$$\tau_O \frac{dT}{dt} = g_1(T, S, I)$$

$$\tau_O \frac{dS}{dt} = g_2(T, S)$$

2-Box Model
[Stommel, 1961]

$$\tau_O \gg 1$$

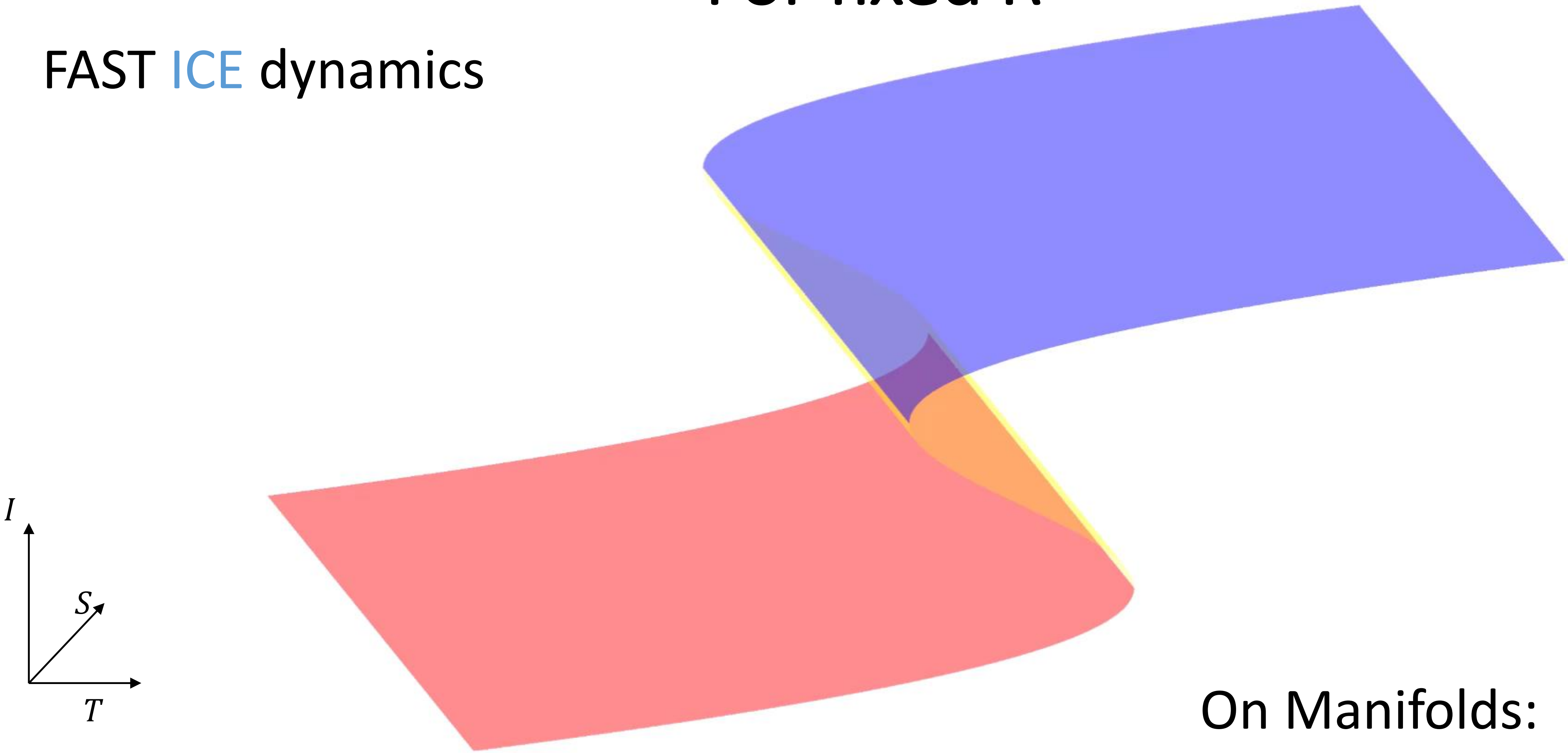
Parameter drift

$$\frac{dR}{dt} = \delta$$

[Lohman et al, 2021] & [David Hokken, MSc thesis, 2022]

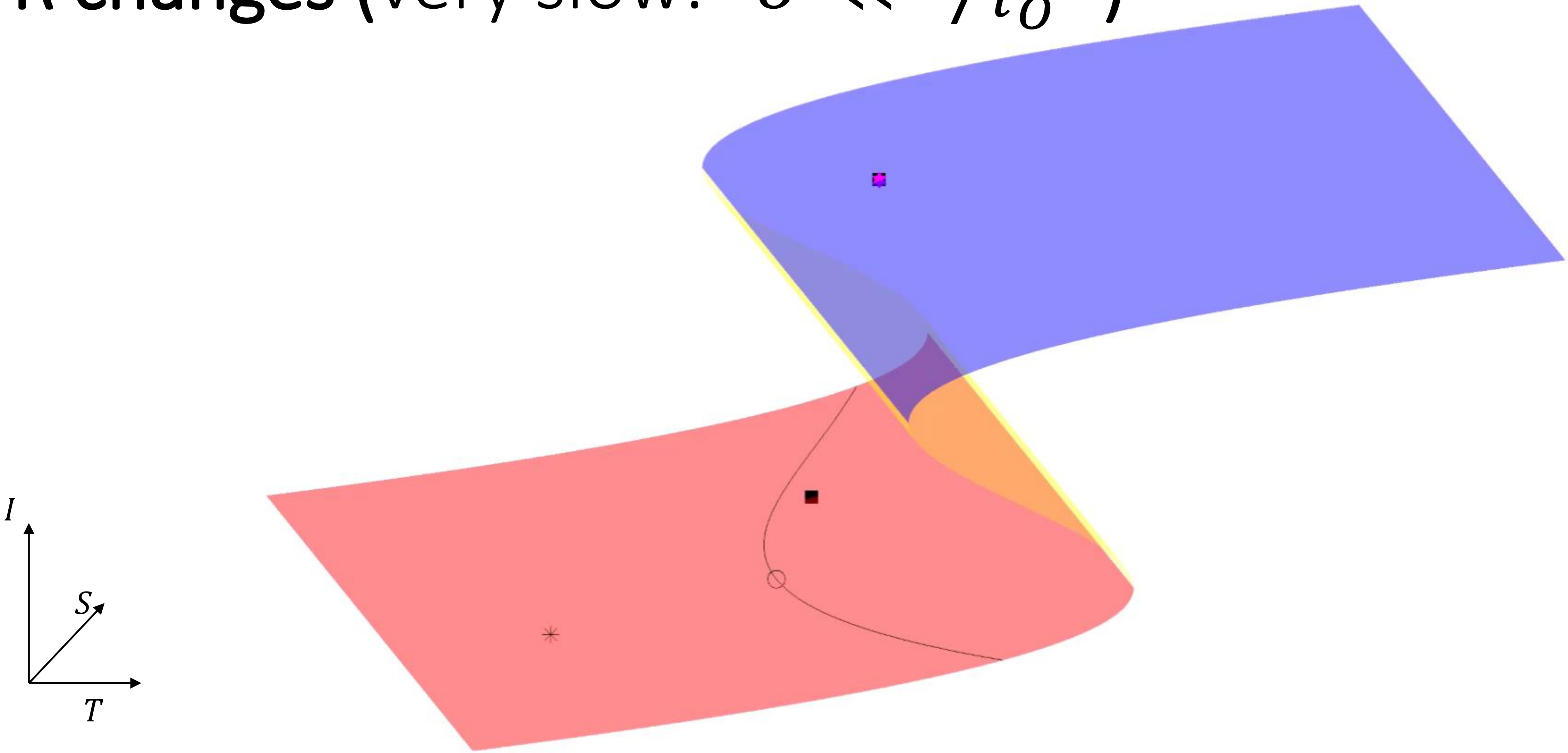
For fixed R

FAST ICE dynamics



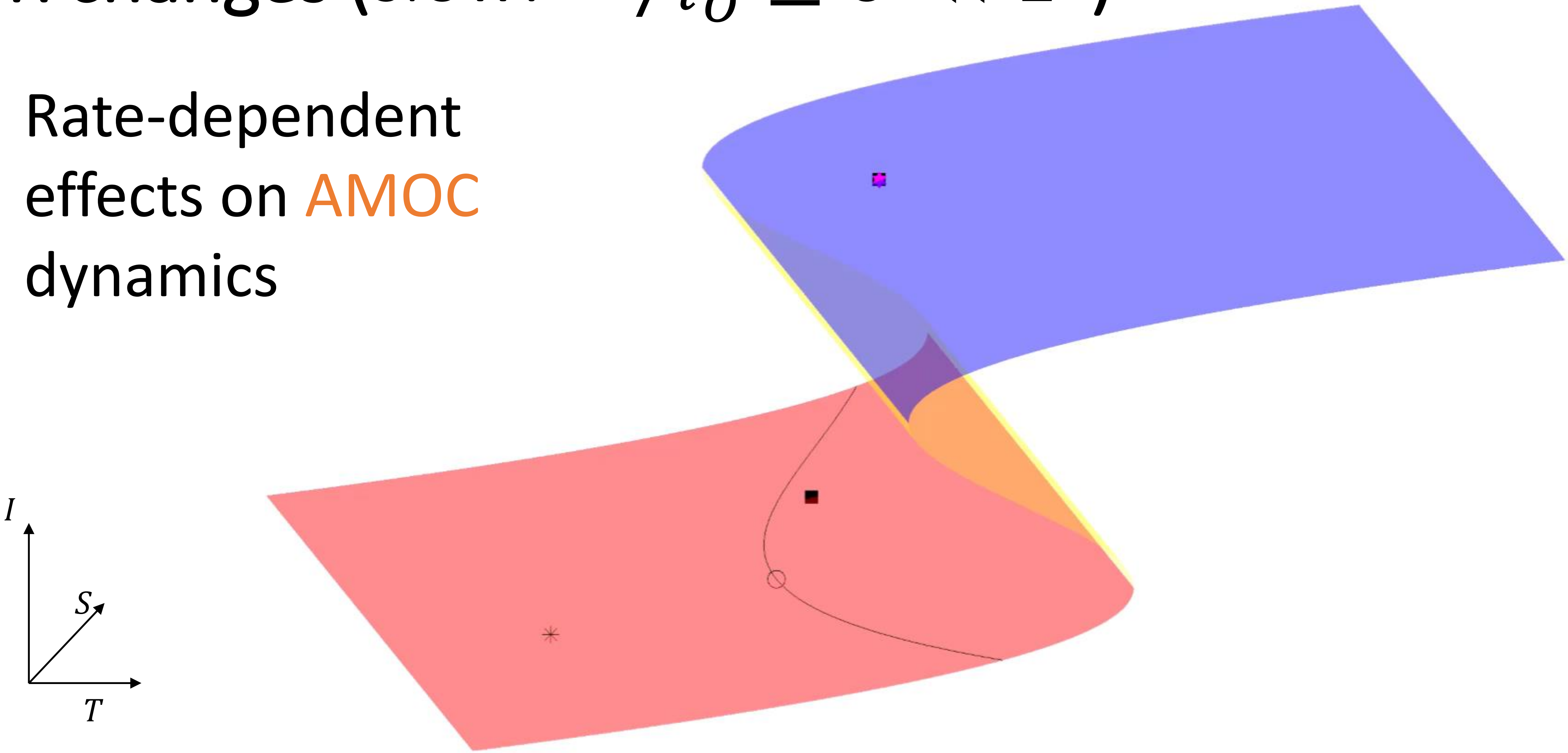
On Manifolds:
SLOW AMOC dynamics

R changes (very slow: "$\delta \ll {}^{1}/_{\tau_O}$")



R changes (slow: "$^1/_{\tau_O} \leq \delta \ll 1$")

Rate-dependent effects on AMOC dynamics



R changes (fast: "$\delta \gg 1$")

- Whole structure breaks down
- Rate-dependent effects on AMOC and ICE dynamics

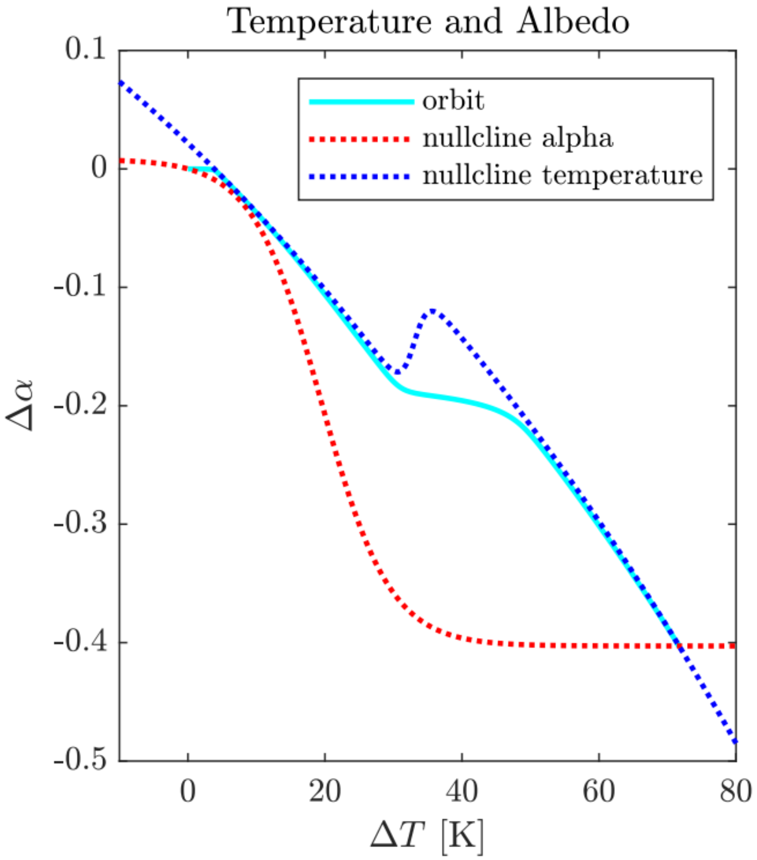
Conclusions

- Tipping DYNAMICS also important

TIME SCALES !

In multiscale systems:

- Late tipping possible
- FAST-SLOW analysis possible
- Rate-induced effects depend on time scales



slides at bastiaansen.github.io

Thanks to:
Peter Ashwin, Anna von der Heydt, David Hokken