The role of timescales for tipping behaviour

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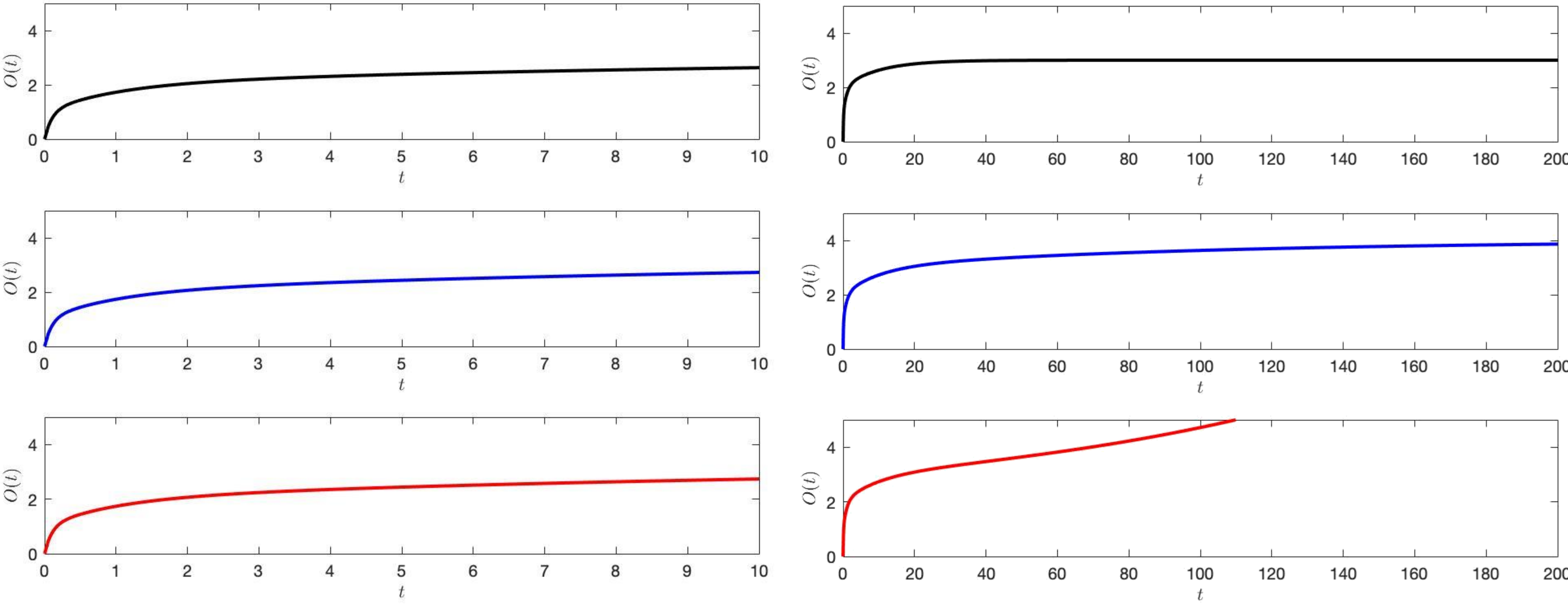
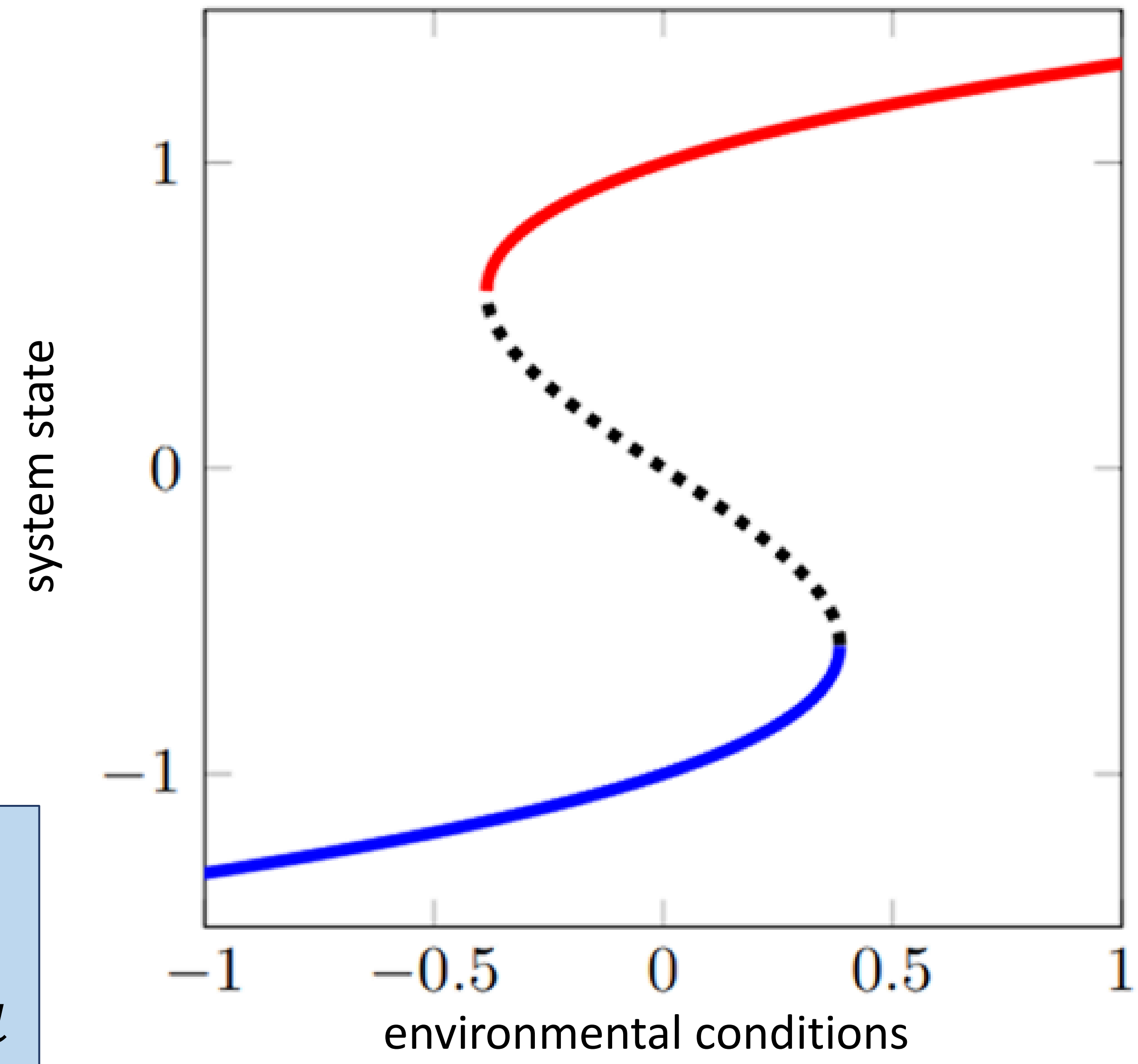
Importance of timescales



[Bastiaansen, Ashwin, Von der Heydt, 2023]

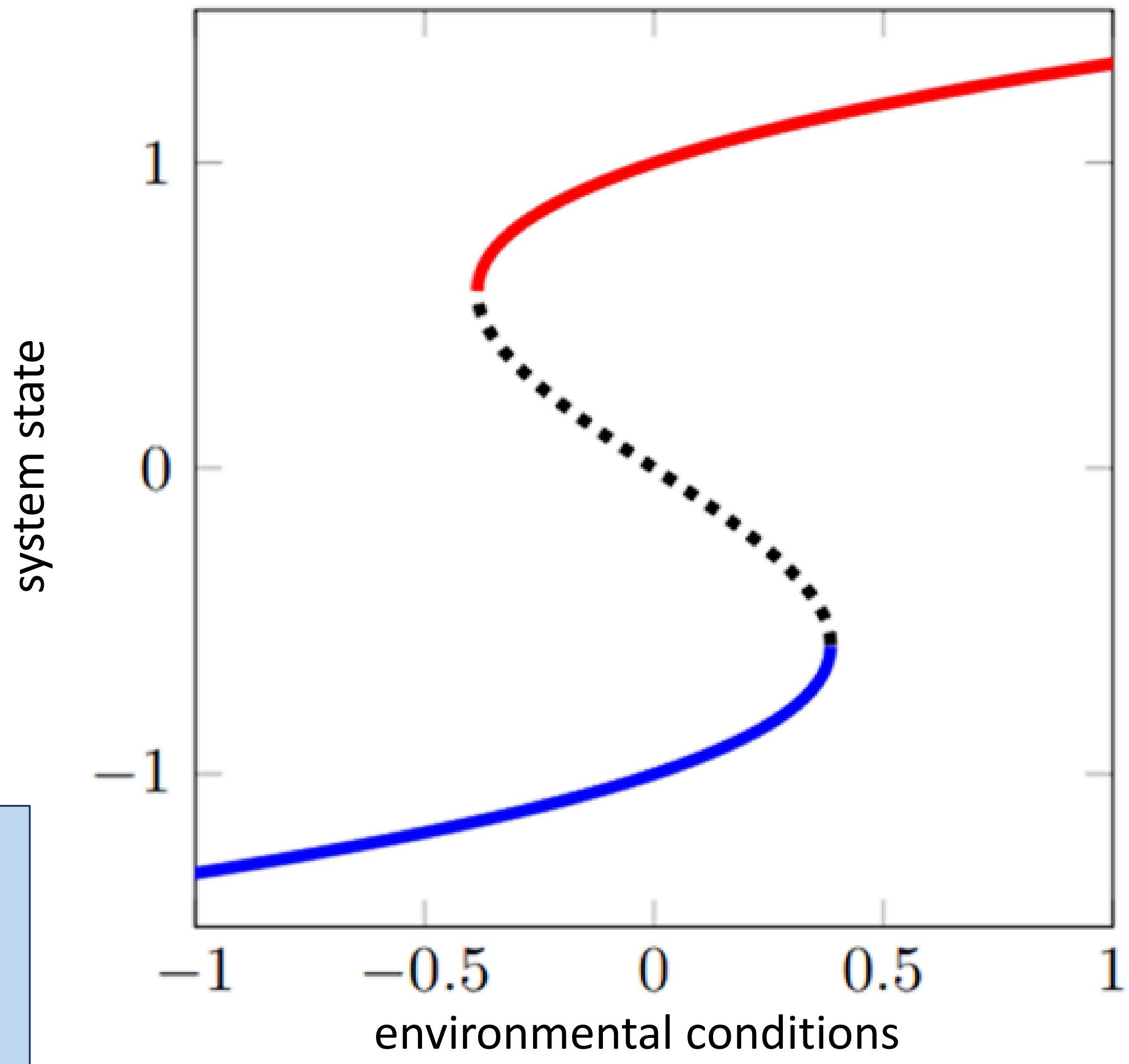
Tipping points ↔ Bifurcations

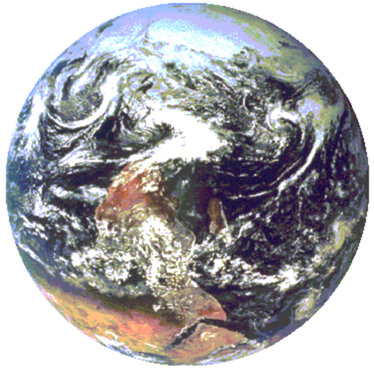


Example System:

$$\frac{dx}{dt} = x - x^3 + \mu$$

Tipping points ↔ Bifurcations

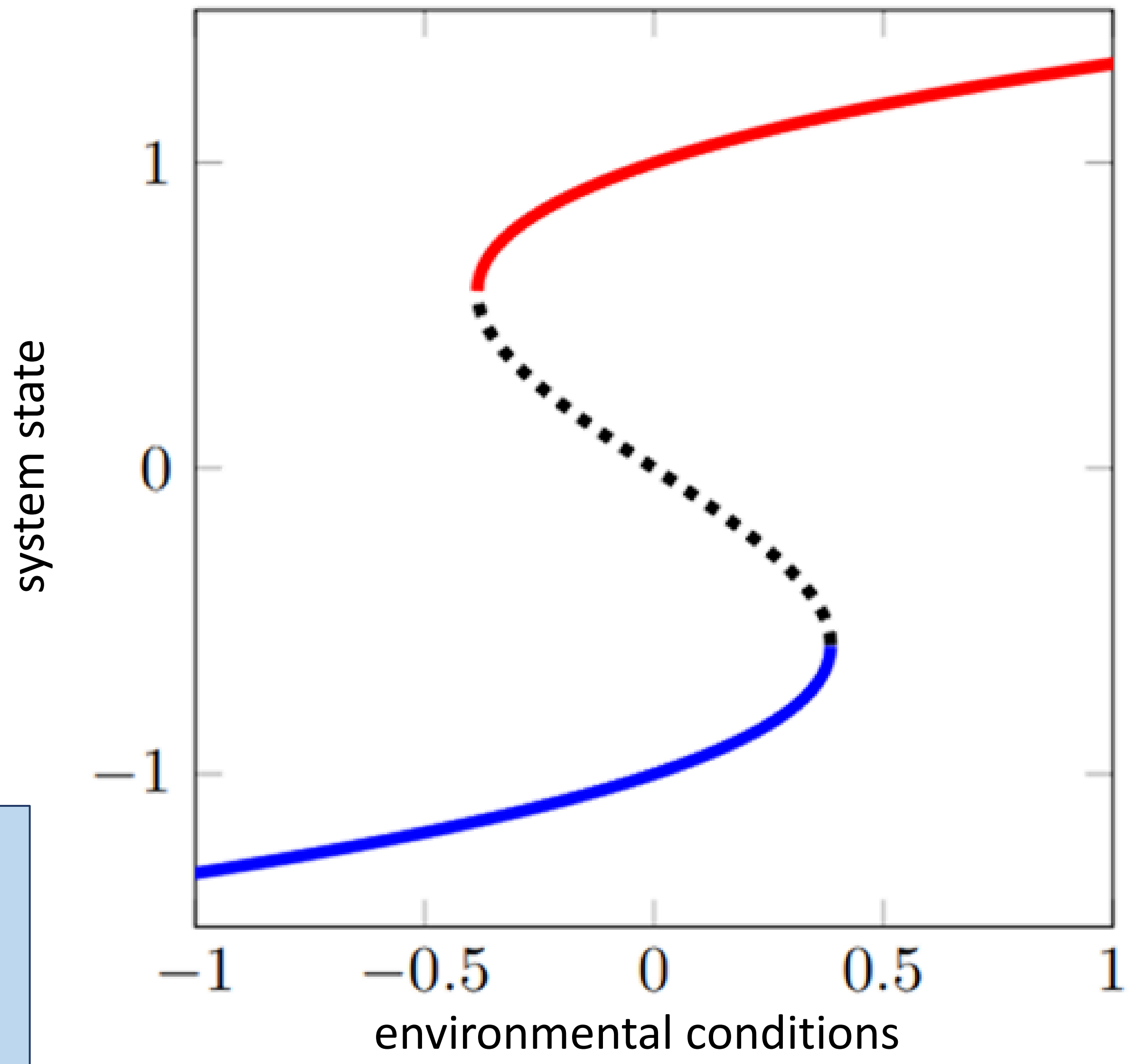




Example System:

$$\frac{dx}{dt} = x - x^3 + \mu$$

Tipping points ⟷ Bifurcations



Example System:

$$\frac{dx}{dt} = x - x^3 + \mu$$

How does tipping work?

$$\frac{dx}{dt} = f(x;\mu)$$ ← Internal Dynamics

$$\frac{d\mu}{dt} = \tau g(\tau t)$$ ← Parameter Drift

Time Scale Separation (both τ)

$\tau \ll 1$: forcing slow compared to system dynamics → B-tipping

$\tau \gg 1$: forcing fast compared to system dynamics → S-tipping

$\tau = \mathcal{O}(1)$: forcing comparable to system dynamics → R-tipping

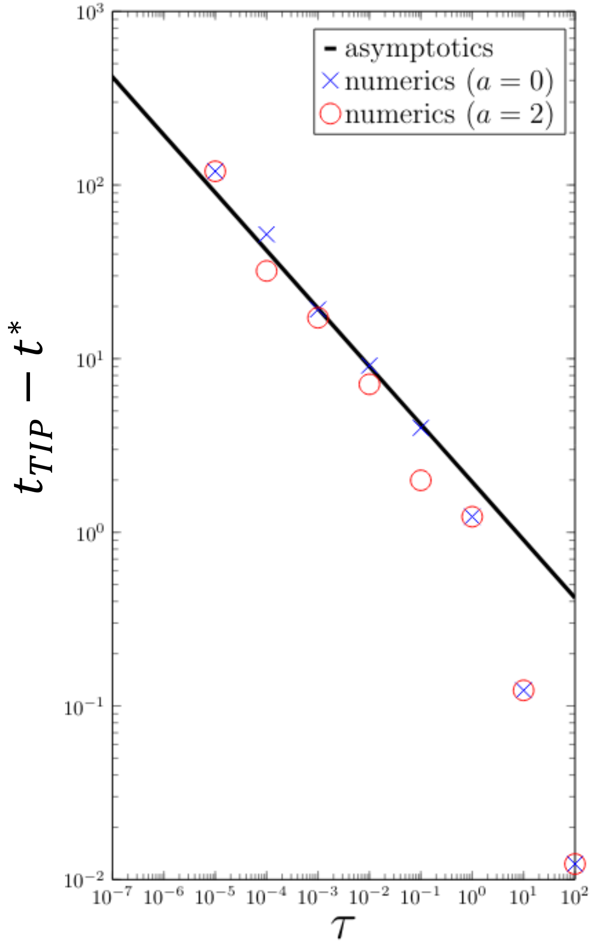
Example 1:

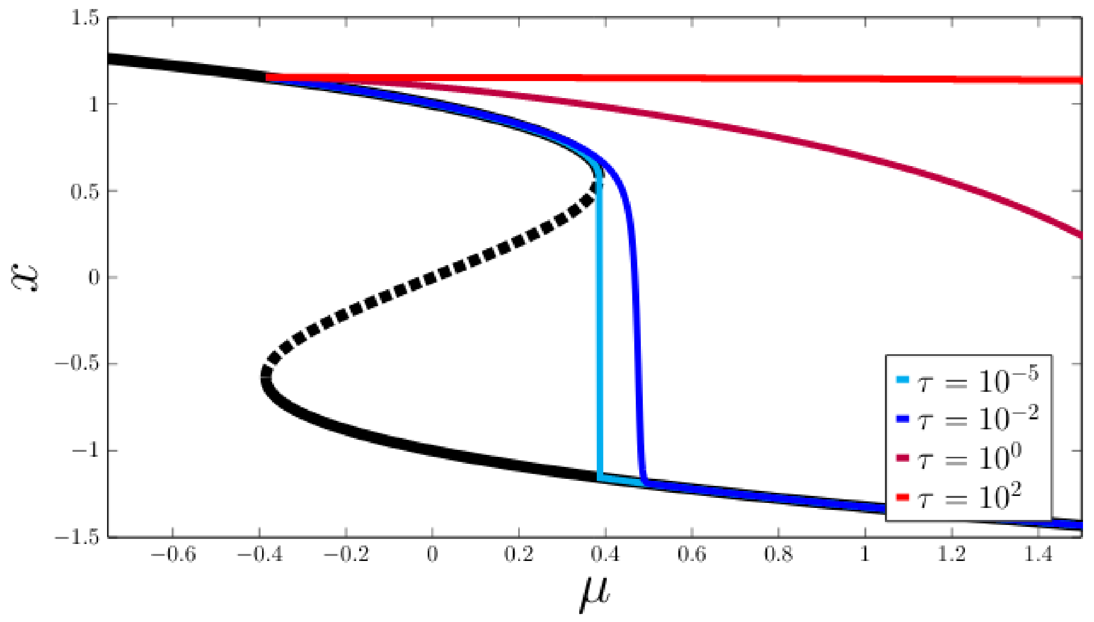
$$\frac{dx}{dt} = (x - a\mu) - (x - a\mu)^3 - \mu$$

$$\frac{d\mu}{dt} = \tau$$

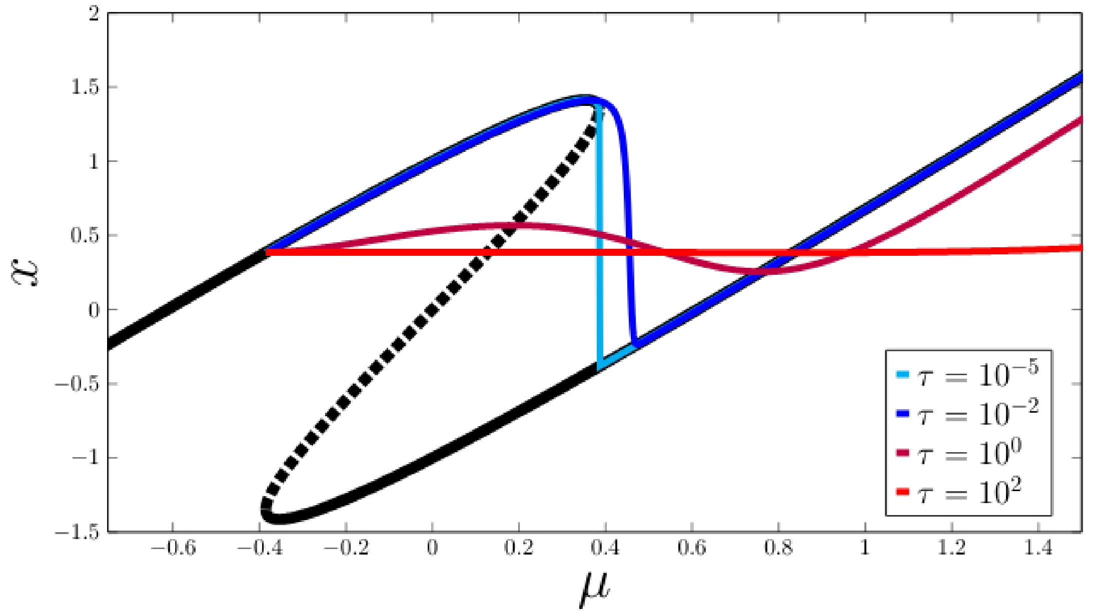
Overshoot timing approximation:

$$t_{TIP} = t^* + (1.946)\,\tau^{-1/3}$$





(a) $a = 0$



(b) $a = 2$

Safe Overshoots

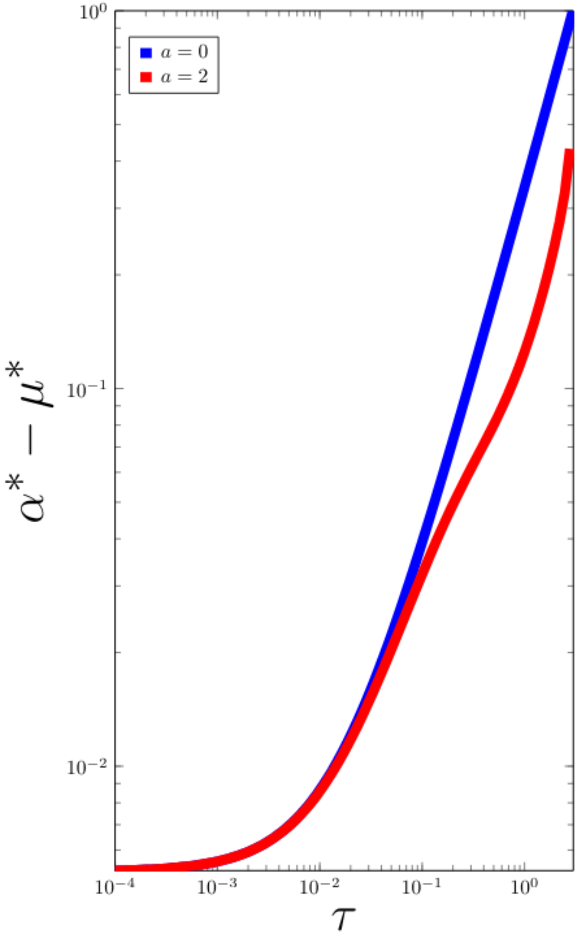
Example 1:

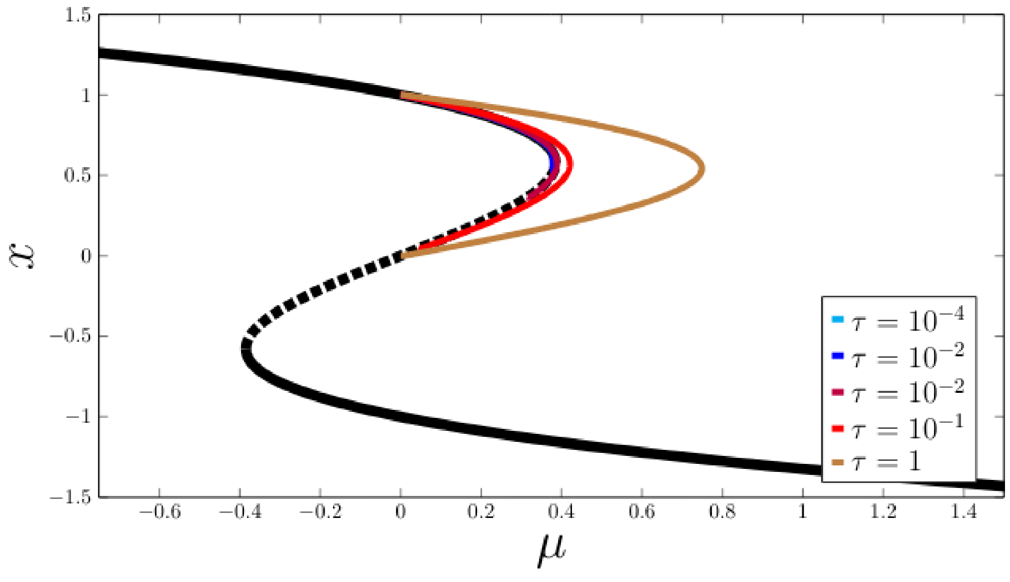
$$\frac{dx}{dt} = (x - a\mu) - (x - a\mu)^3 - \mu$$

$$\frac{d\mu}{dt} = \tau g(\tau t)$$

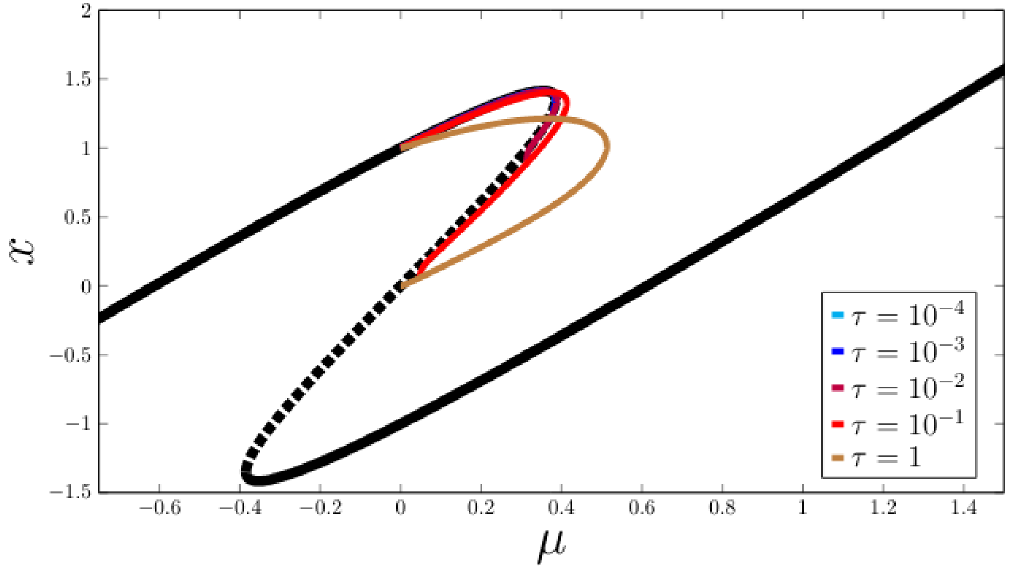
Pulse-like overshoot scenario:

$$g(s) = -\alpha \tanh(s)\,\mathrm{sech}(s)$$





(a) $a = 0$



(b) $a = 2$

EXAMPLE 2: Multiscale Global Energy Balance Model

$$C\frac{dT}{dt} = Q_0(1 - \alpha) - \epsilon(T)\sigma T^4 + \mu$$

Short-Wave

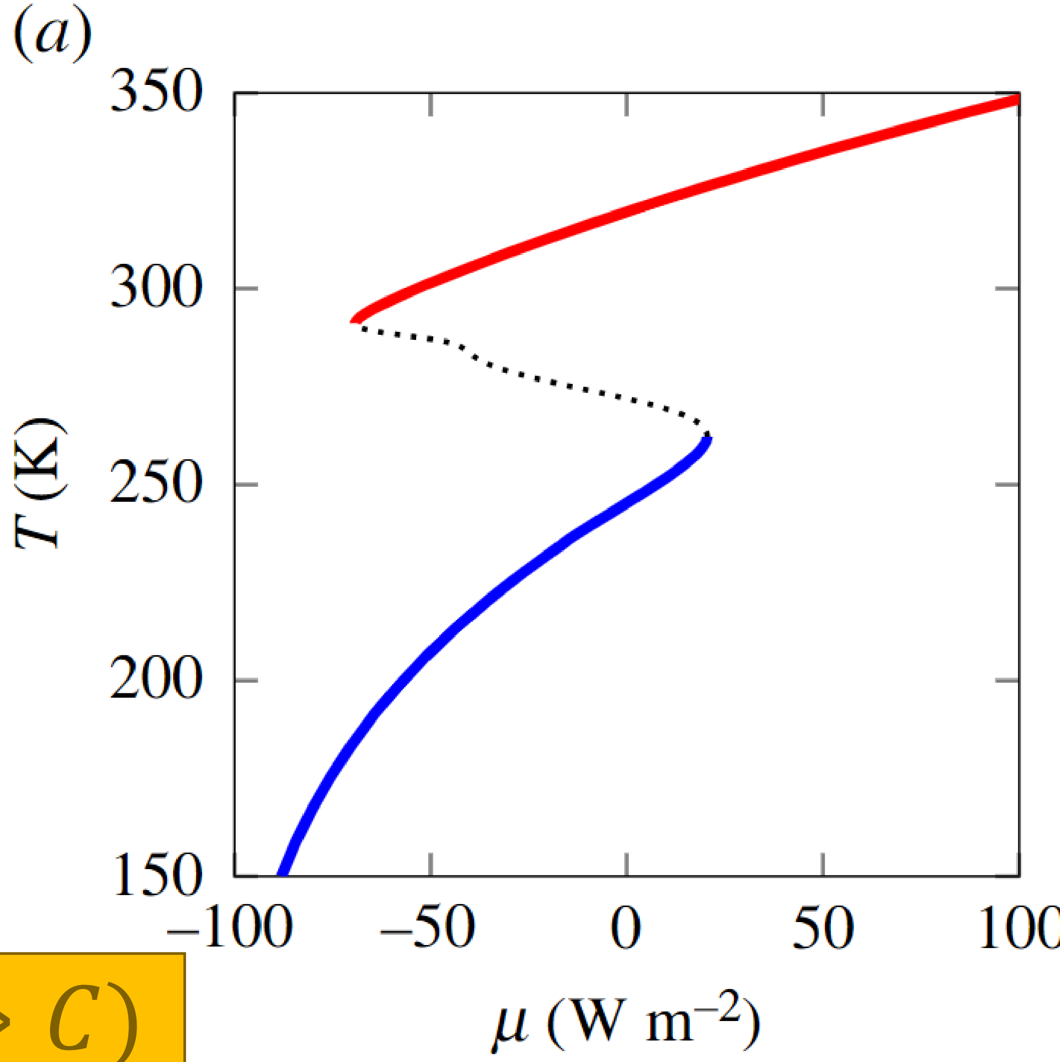
Long-Wave

CO2

$$\tau_\alpha \frac{d\alpha}{dt} = \alpha_0(T) - \alpha$$

Dynamic albedo

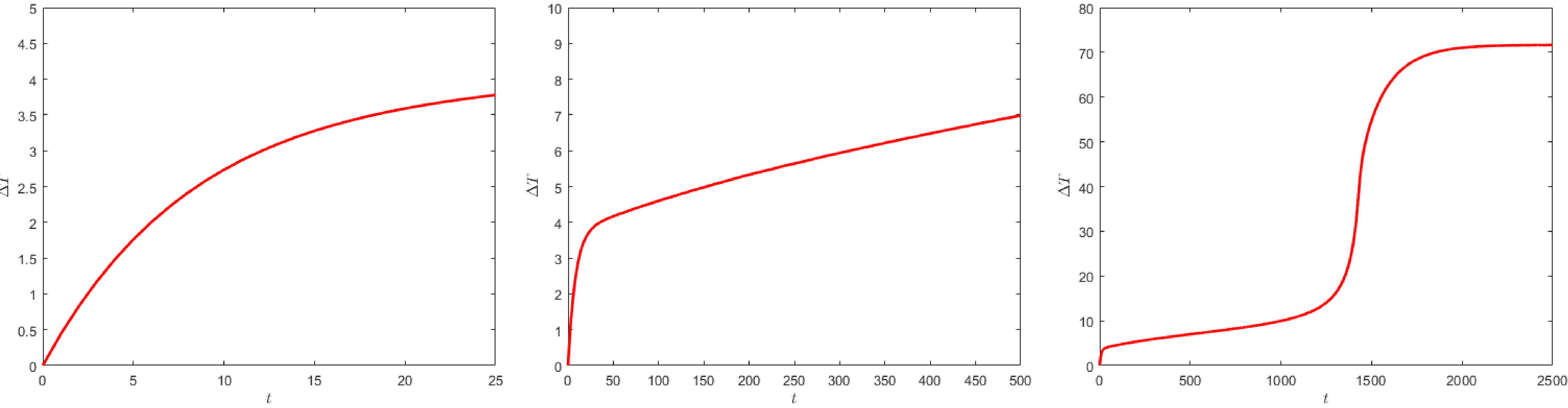
Internal Time Scale Separation ($\tau_\alpha \gg C$)



Abrupt 4xCO2 forcing experiment

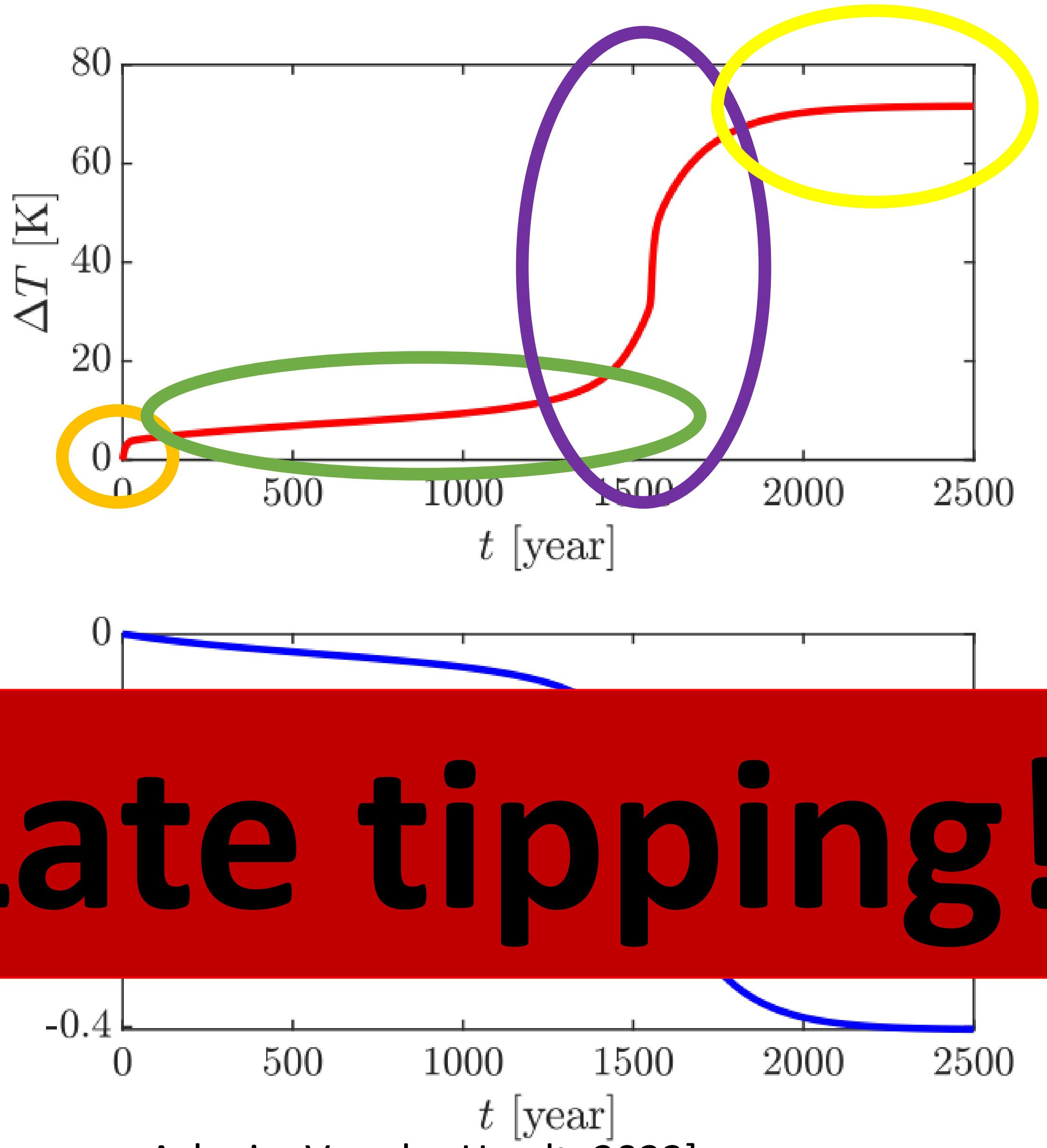
- Initialize for μ_0 (initial CO2-levels)
- Change to μ_1 (4xCO2 levels)

→ Look at dynamics

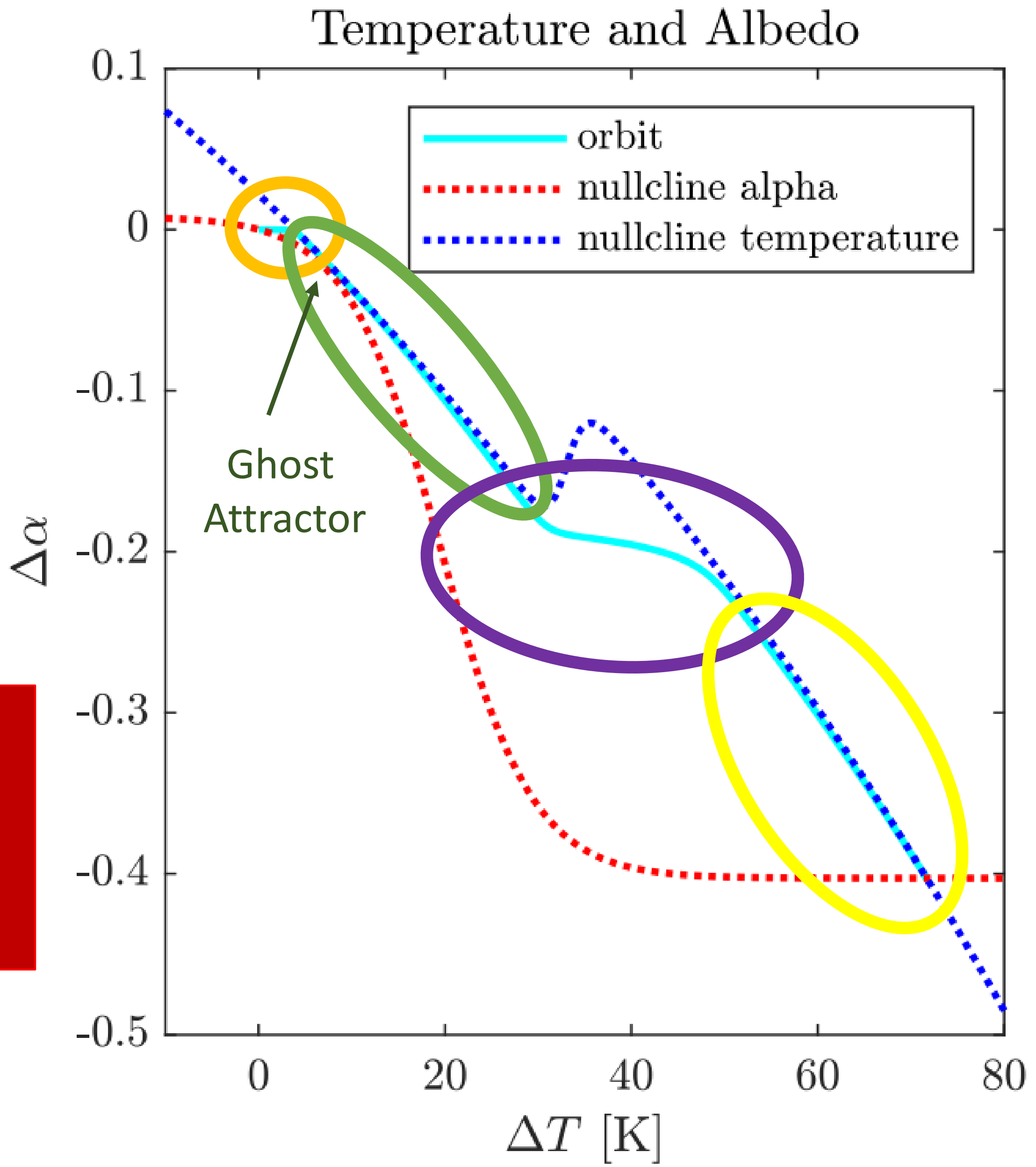


[Bastiaansen, Ashwin, Von der Heydt, 2023]

How does this work?



Late tipping!

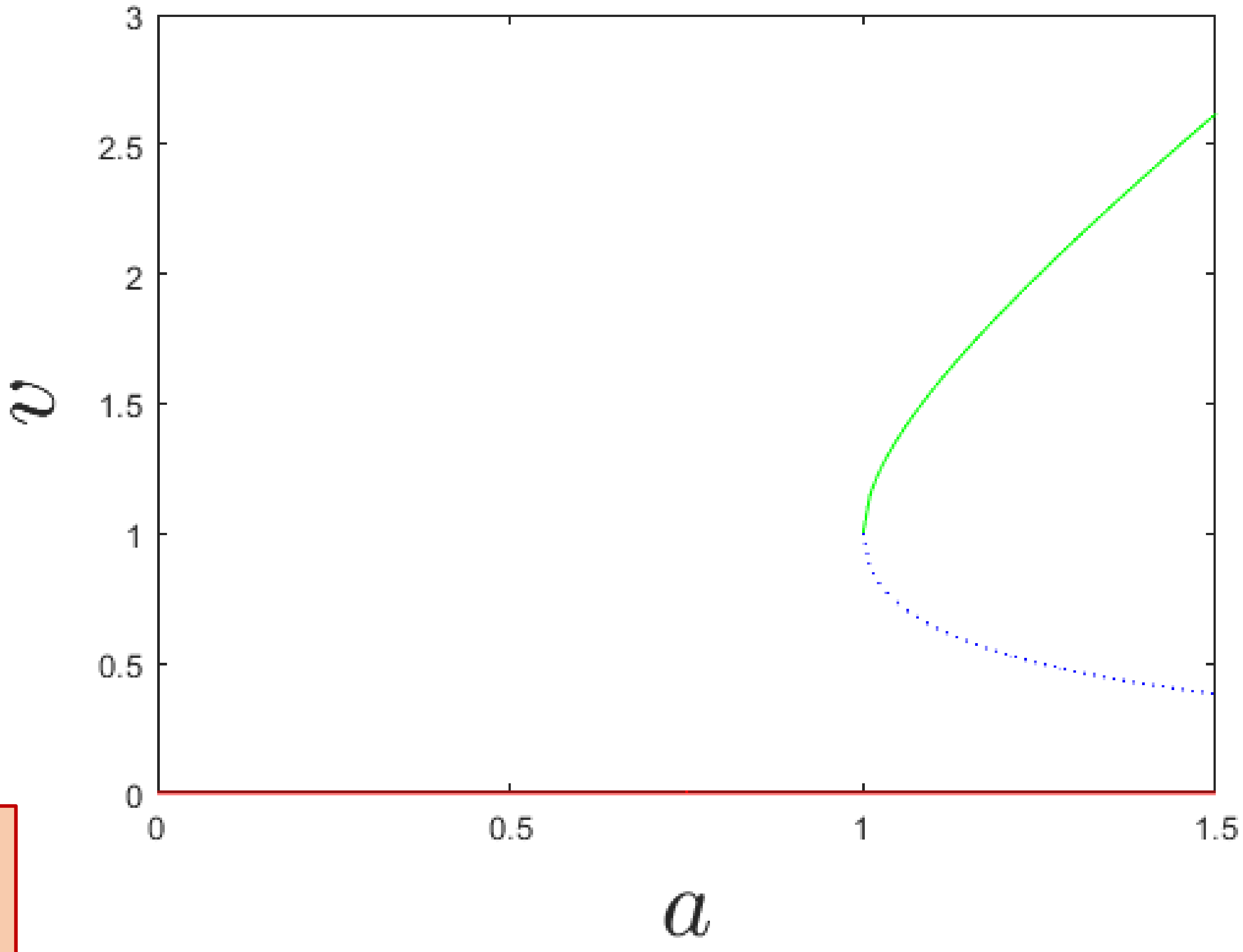


[Bastiaansen, Ashwin, Von der Heydt, 2023]

EXAMPLE 3: Time scale of feedback

$$\frac{du}{dt} = a - u - uv^2$$

$$\frac{dv}{dt} = uv^2 - mv$$



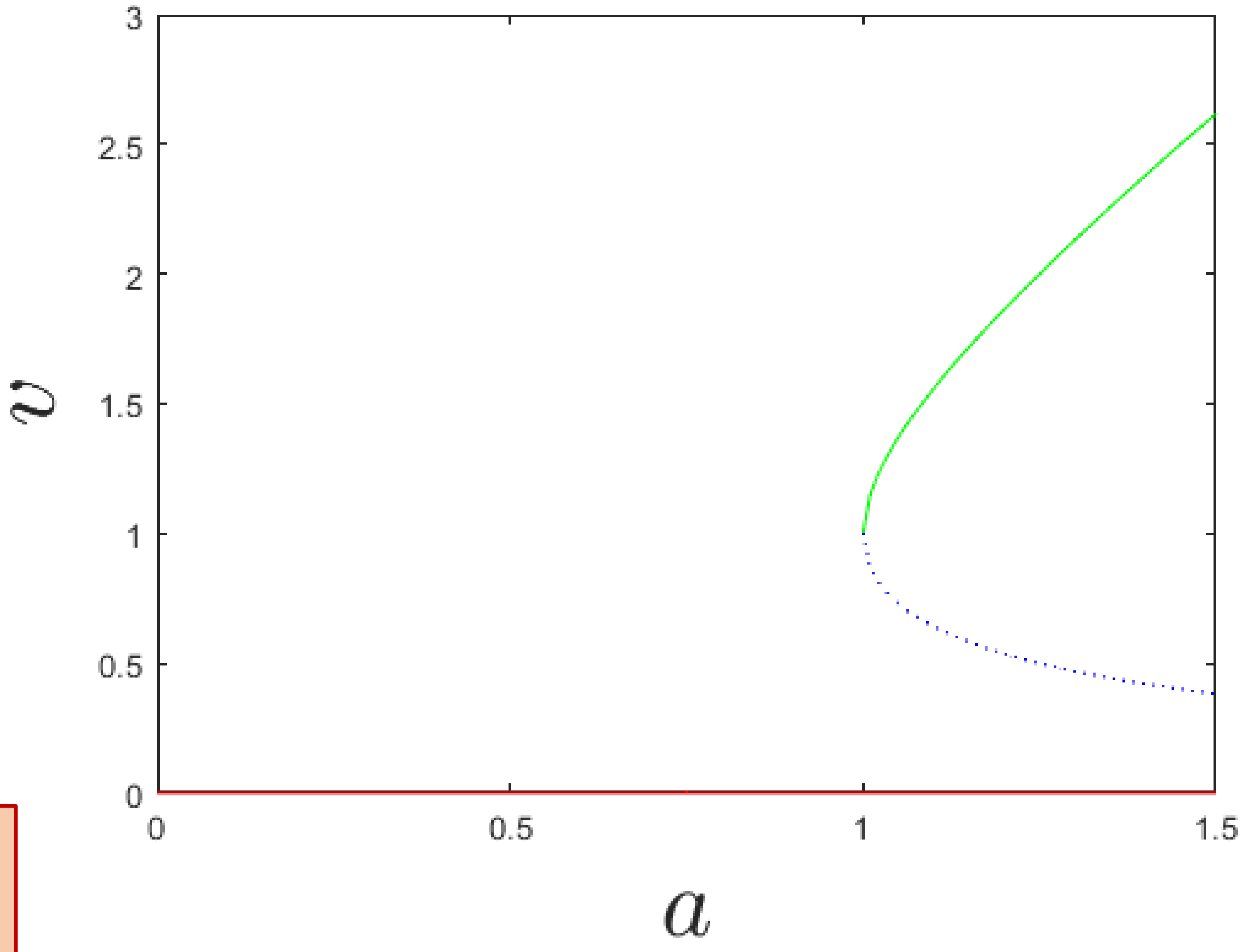
Parameters:
$m = 0.5$

EXAMPLE 3: Time scale of feedback

$$\frac{du}{dt} = a - u - uvs$$

$$\frac{dv}{dt} = uvs - mv$$

$$\frac{ds}{dt} = \tau_{INT}\,(v - s)$$



Parameters:
$m = 0.5$

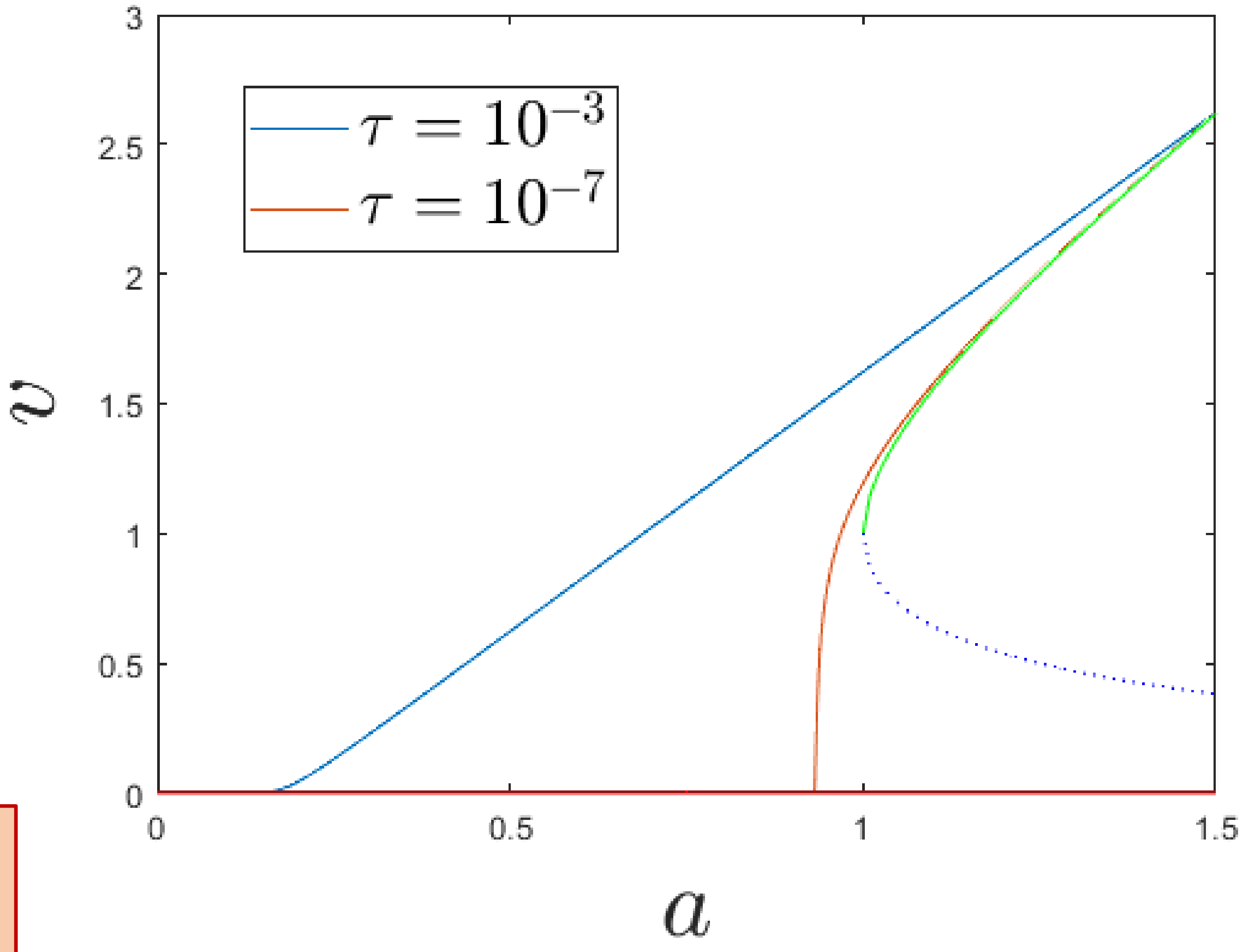
EXAMPLE 3: Time scale of feedback

$$\frac{du}{dt} = a - u - uvs$$

$$\frac{dv}{dt} = uvs - mv$$

$$\frac{ds}{dt} = \tau_{INT}\,(v - s)$$

$$\frac{da}{dt} = -\tau$$



Parameters:

$m = 0.5$

$\tau_{INT} = 10^{-5}$

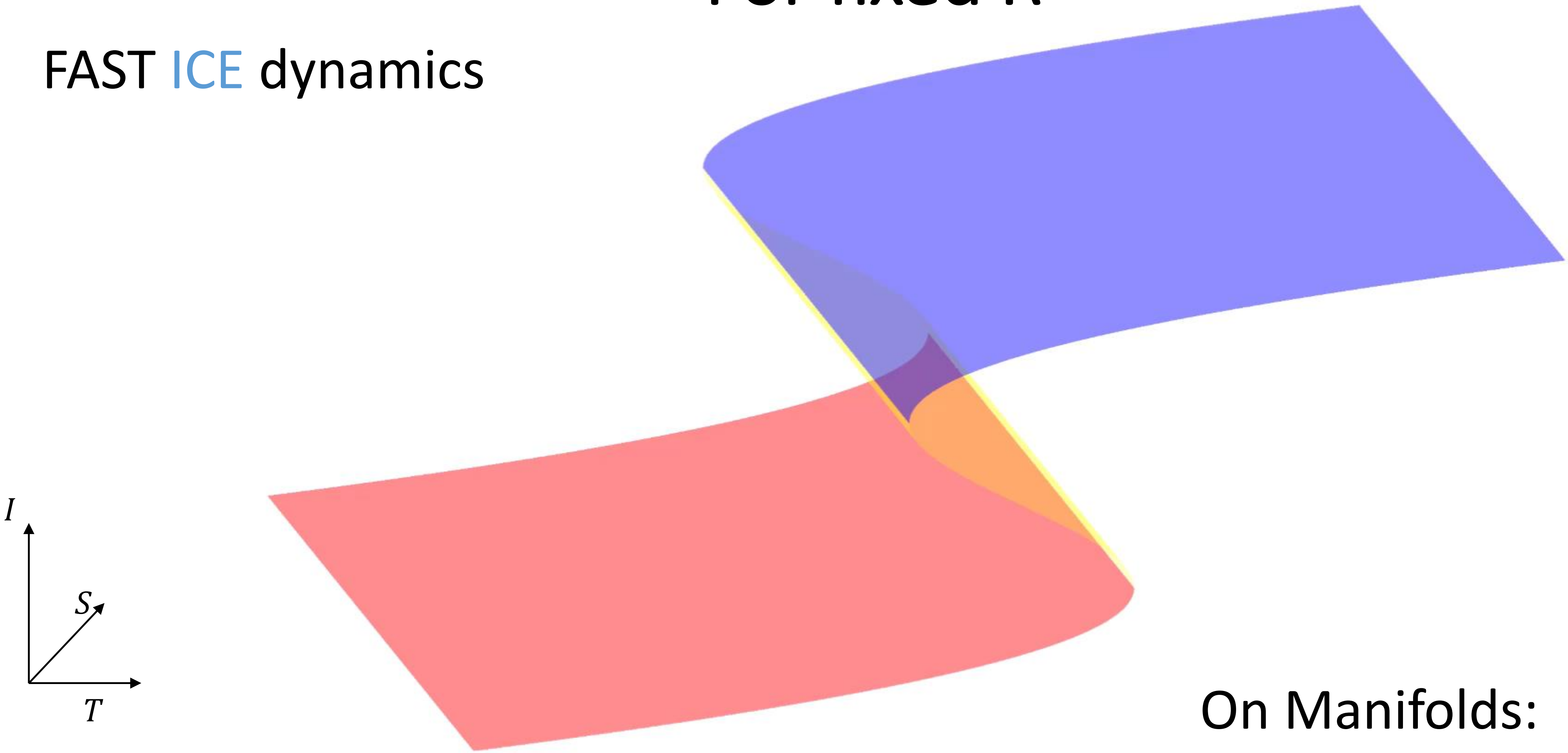
EXAMPLE 4: AMOC $\leftrightarrow$ ICE interaction

Tipping Element 1 (ICE)	$$\frac{dI}{dt} = f(I, R, T)$$	Energy balance model [Eisenman & Wettlaufer, 2009]
Tipping Element 2 (AMOC)	$$\tau_O \frac{dT}{dt} = g_1(T, S, I)$$ $$\tau_O \frac{dS}{dt} = g_2(T, S)$$	2-Box Model [Stommel, 1961] $\tau_O \gg 1$
Parameter drift	$$\frac{dR}{dt} = \tau$$	

[Lohman et al, 2021] & [David Hokken, MSc thesis, 2022]

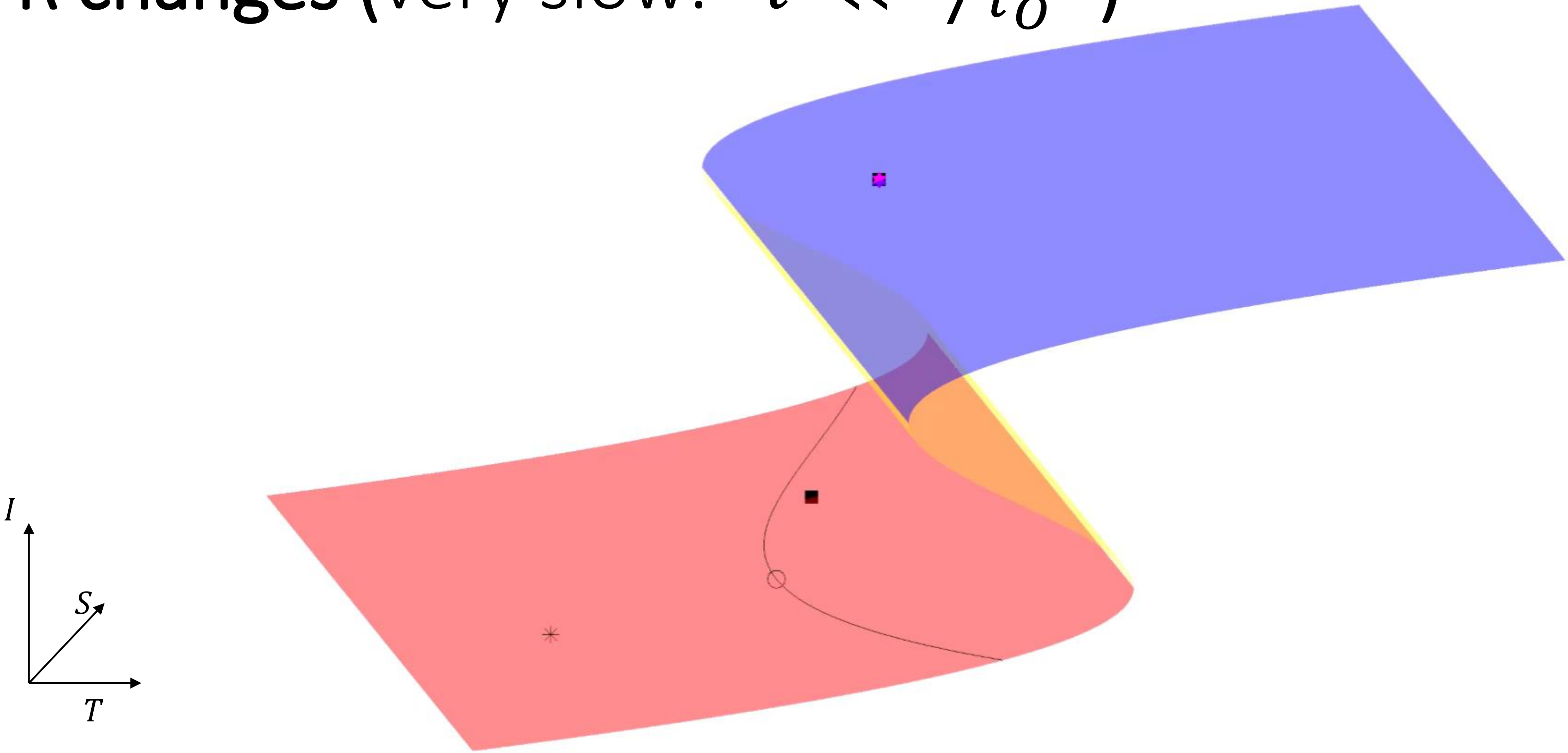
For fixed R

FAST ICE dynamics



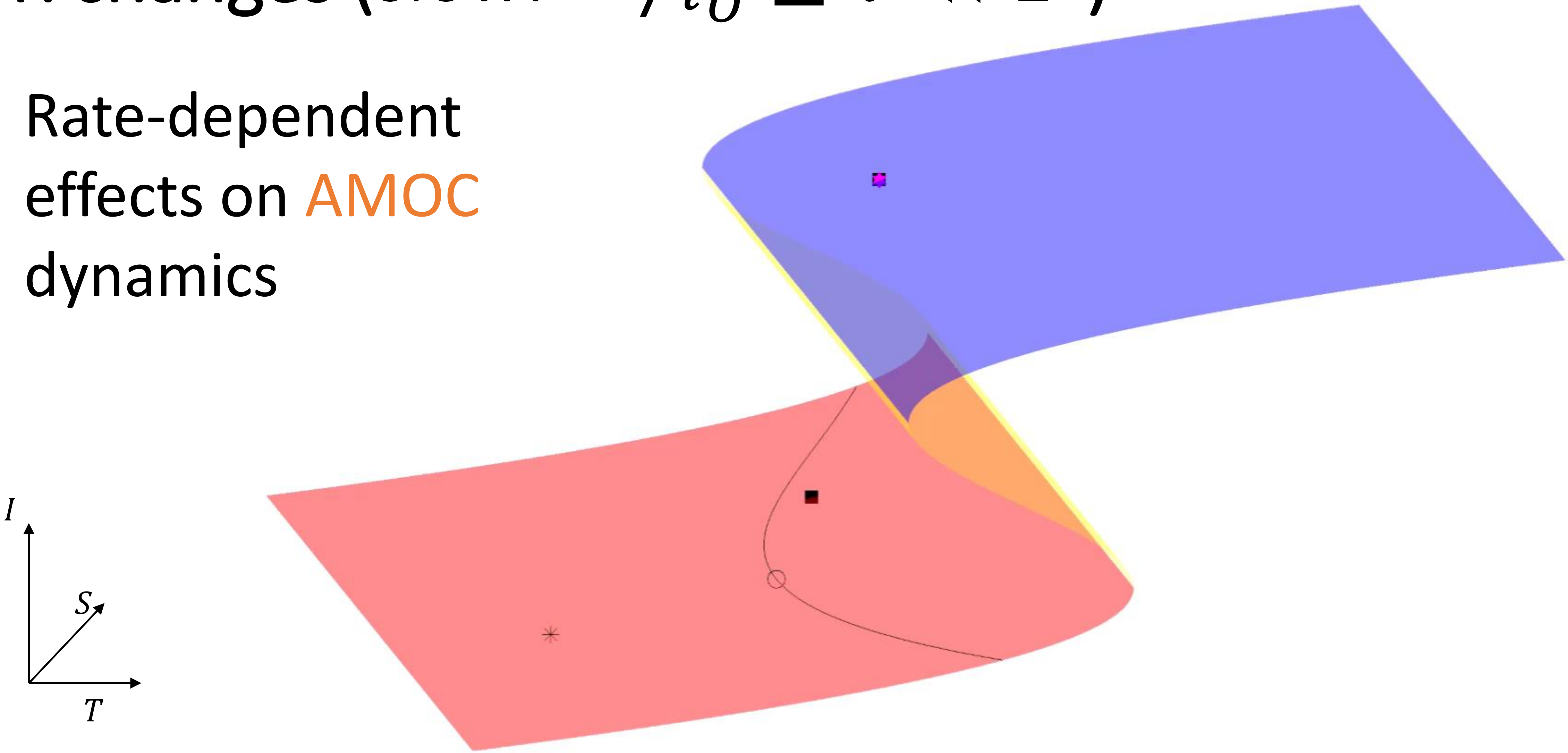
On Manifolds:
SLOW AMOC dynamics

R changes (very slow: "$\tau \ll {}^{1}/_{\tau_O}$")



R changes (slow: " $^{1}/_{\tau_O} \leq \tau \ll 1$")

Rate-dependent effects on AMOC dynamics



EXAMPLE 5: Dryland Ecosystem

$$w_t = w_{xx} - w + a - wv^2$$
$$v_t = D^2 v_{xx} - mv + wv^2$$

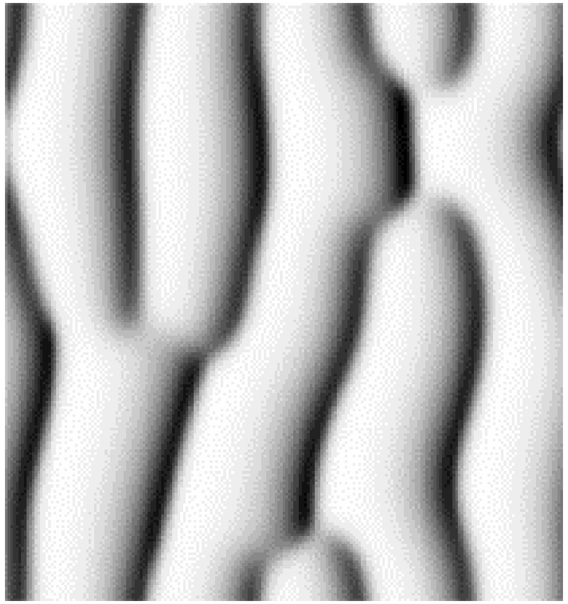
w : water

v : vegetation

D : ratio of diffusion

a : rainfall

m : mortality



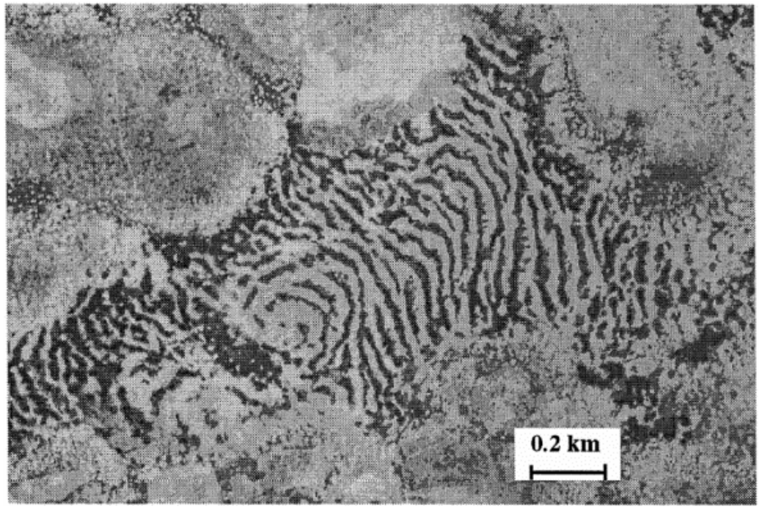
[Klausmeier, 1999]

SLOW pattern adaptation

Somaliland, 1948 [Macfadyen, 1950]

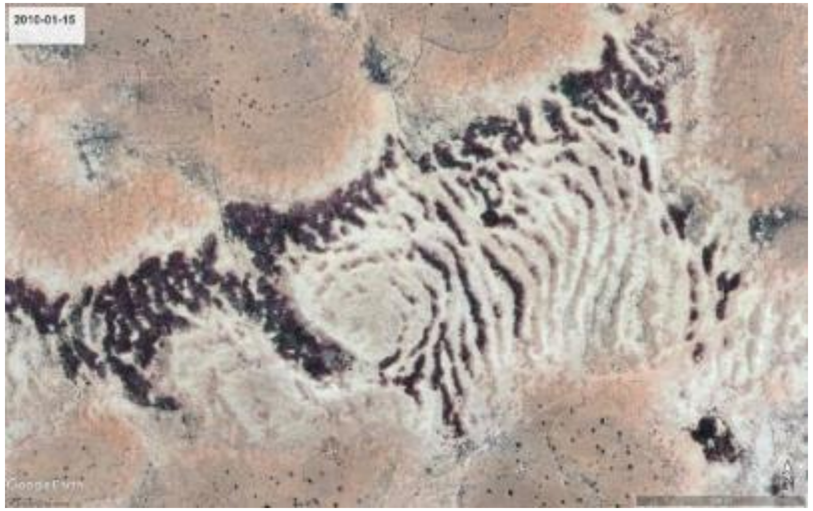
Somaliland, 2008

FAST Pattern Degradation

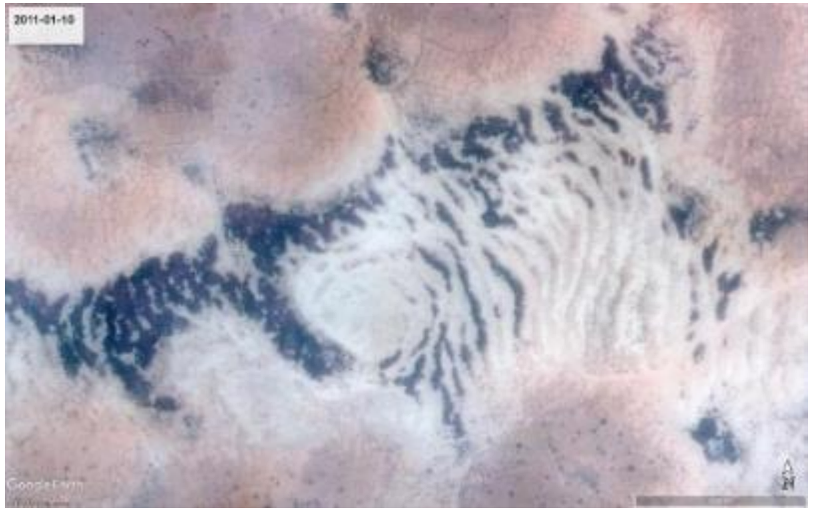


Niger, 1950 [Valentin, 1999]

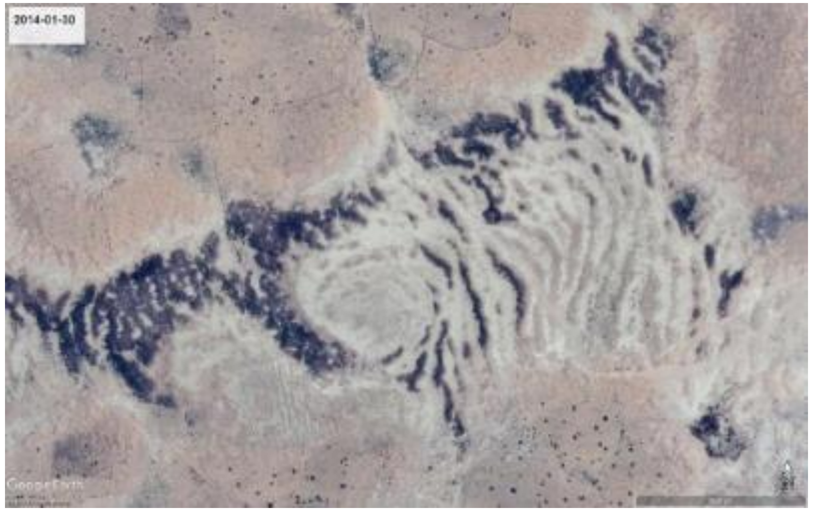
Niger, 2008



Niger, 2010



Niger, 2011



Niger, 2014

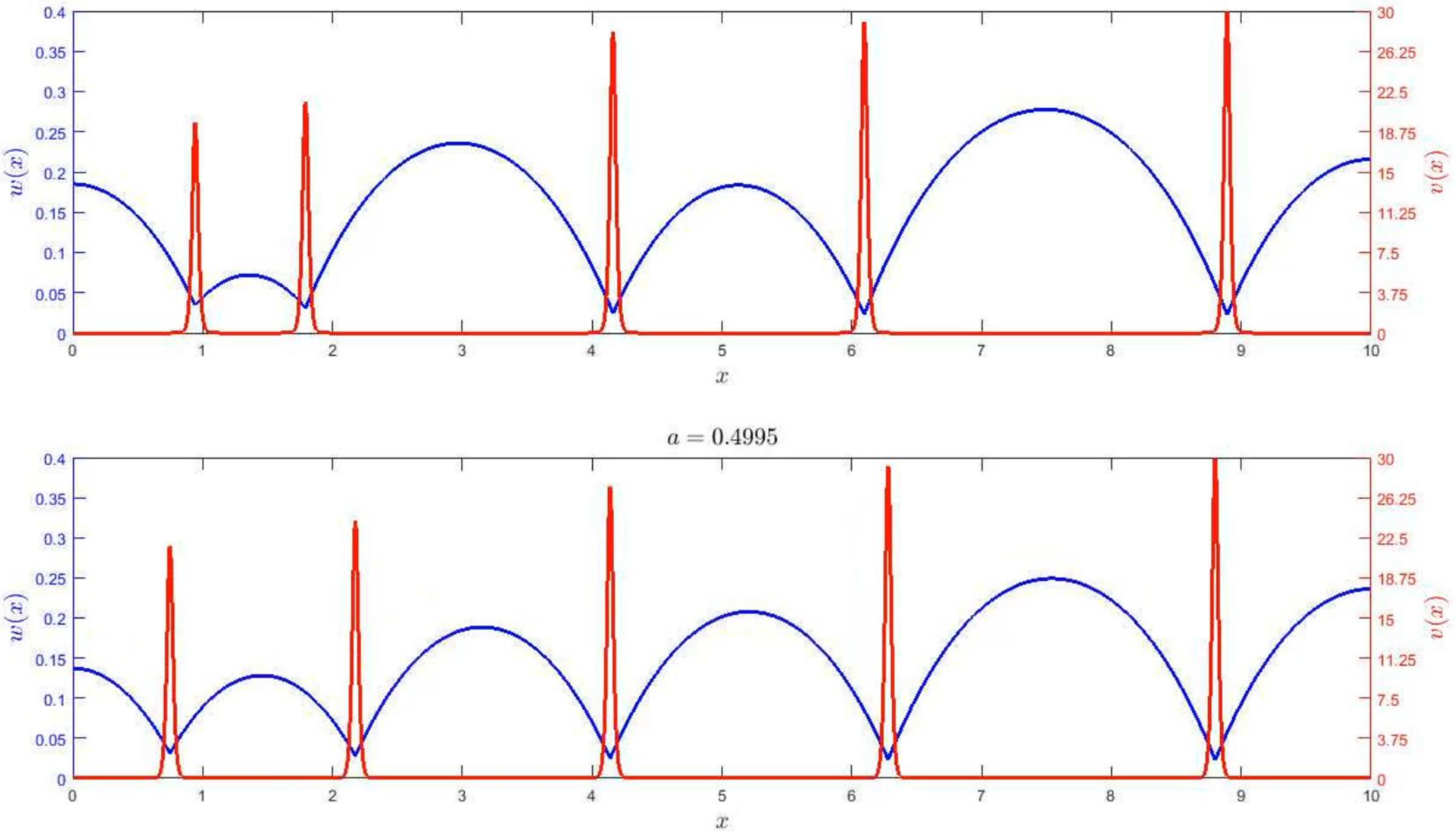
Niger, 2016

Dynamics of vegetation patches

Rate of climate change

FAST

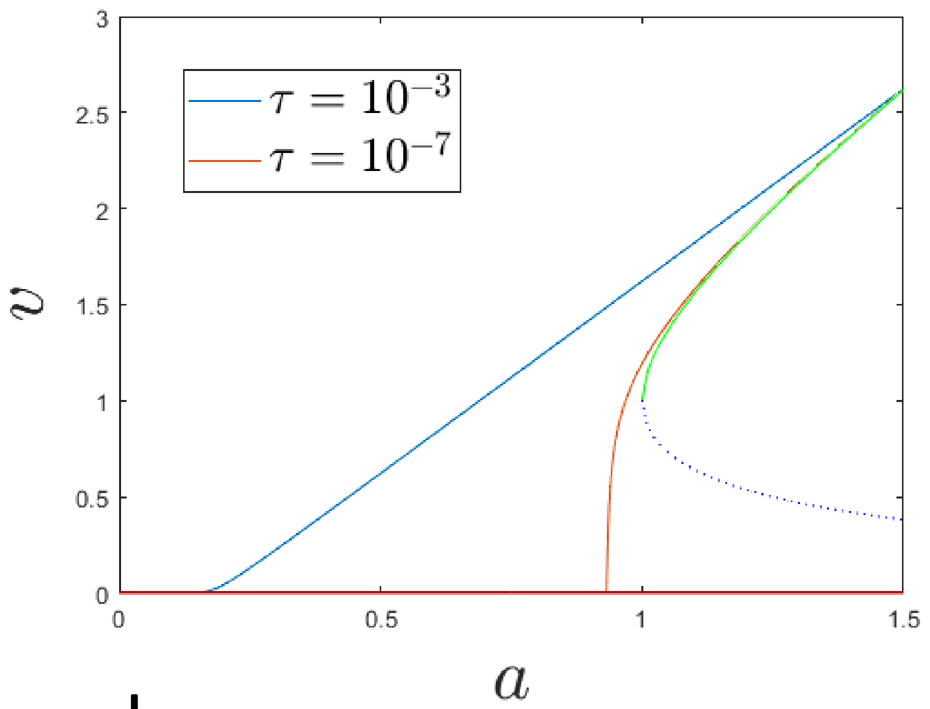
SLOW



Conclusions

- Tipping DYNAMICS also important

TIME SCALES !



In multiscale systems:

- Late tipping possible
- Rate-induced effects depend on time scales
- Response to faster changes might look less abrupt

slides at bastiaansen.github.io

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Peter Ashwin, Anna von der Heydt, David Hokken, Max Rietkerk, Arjen Doelman, Anna van der Kaaden, Maarten Eppinga