tipping and tipping cascades in systems with multiple time scales

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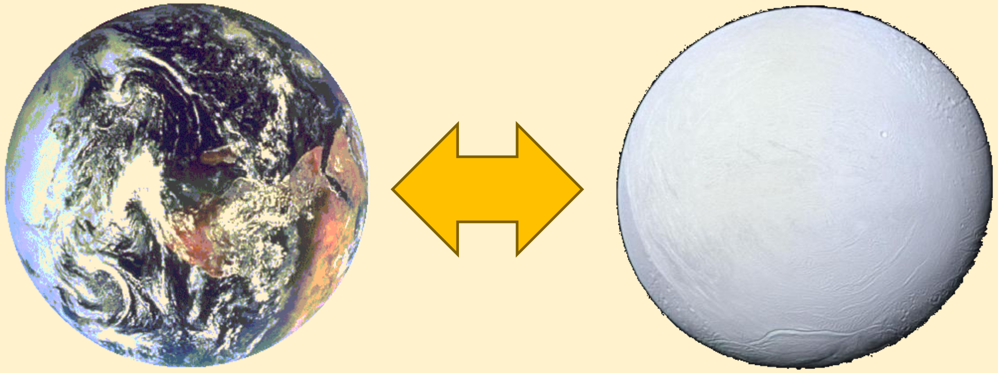
Lorentz workshop
Multiple Scales: Theory & Applications
2024-07-11

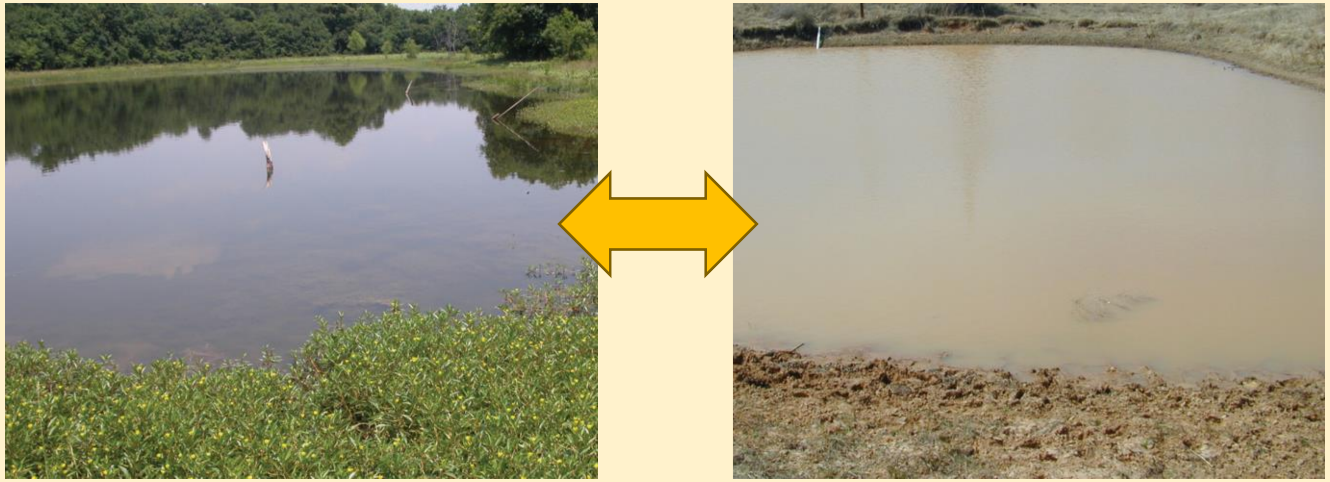
Tipping Points

IPCC AR6 (2021) : "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly"



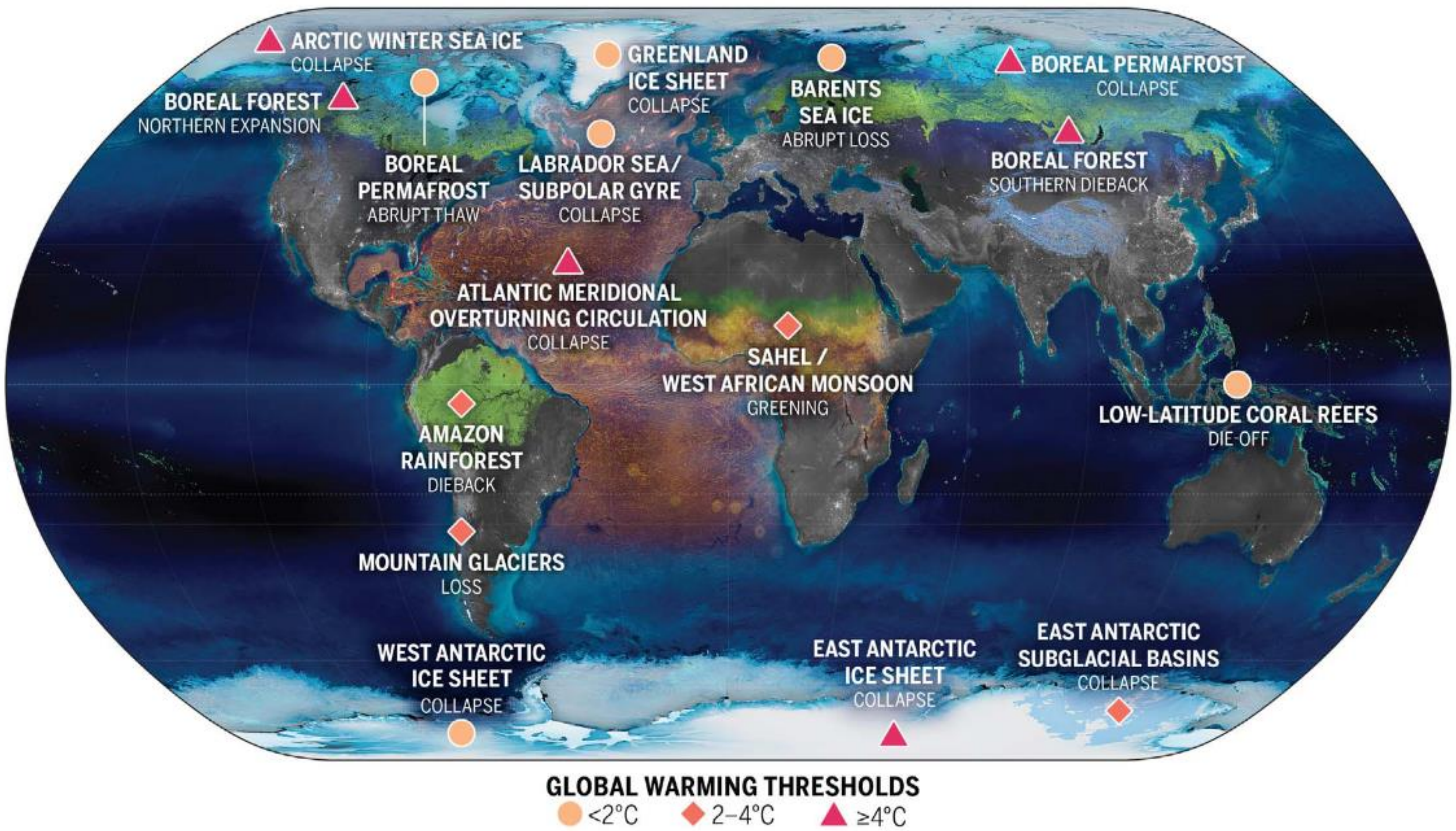
Planetary transitions

Ecosystem shifts



Tipping Points

IPCC AR6 (2021) : "a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly"

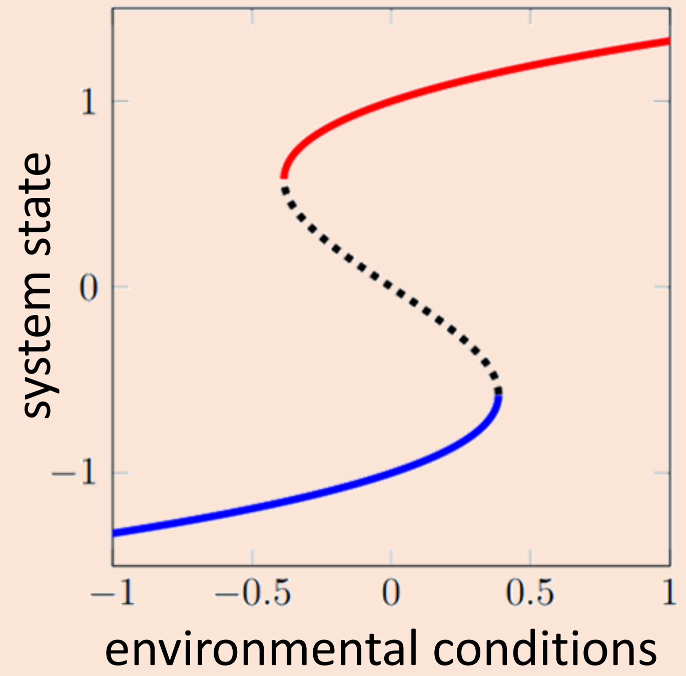


source: Armstrong McKay et al, 2022

Mathematics

Tipping points ↔ Bifurcations

$$\frac{dy}{dt} = f(y, \mu)$$



How does tipping work?

$$\frac{dx}{dt} = f(x; \mu)$$

Internal Dynamics

$$\frac{d\mu}{dt} = \tau g(\tau t)$$

Parameter Drift

Time Scale Separation

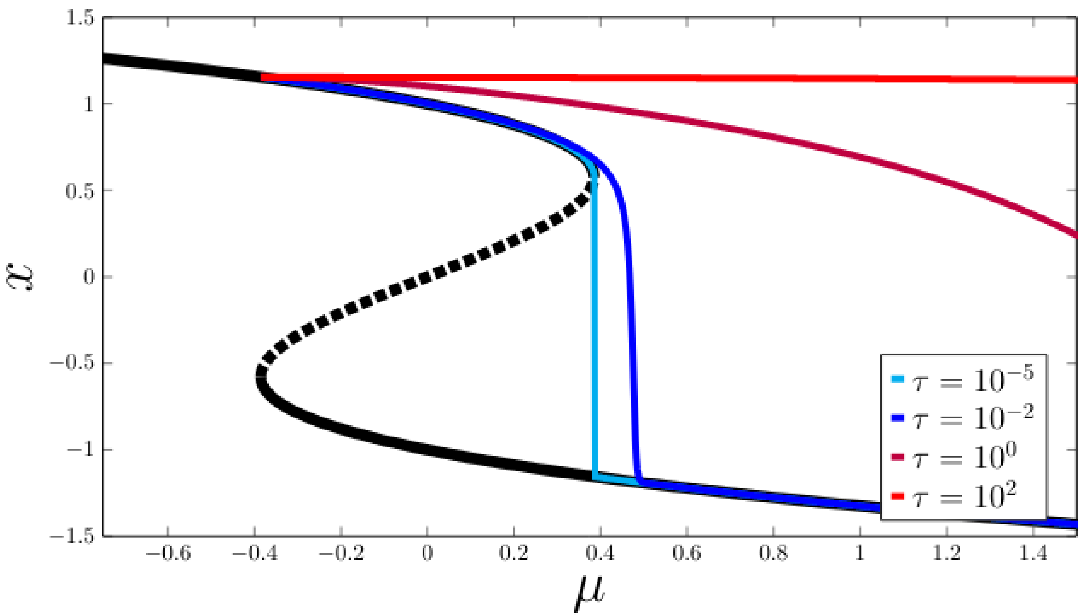
$\tau \ll 1$: forcing slow compared to system dynamics → B-tipping

$\tau \gg 1$: forcing fast compared to system dynamics → S-tipping

$\tau = \mathcal{O}(1)$: forcing comparable to system dynamics →R-tipping

Example 1:

$$\frac{dx}{dt} = (x - a\mu) - (x - a\mu)^3 - \mu$$
$$\frac{d\mu}{dt} = \tau$$



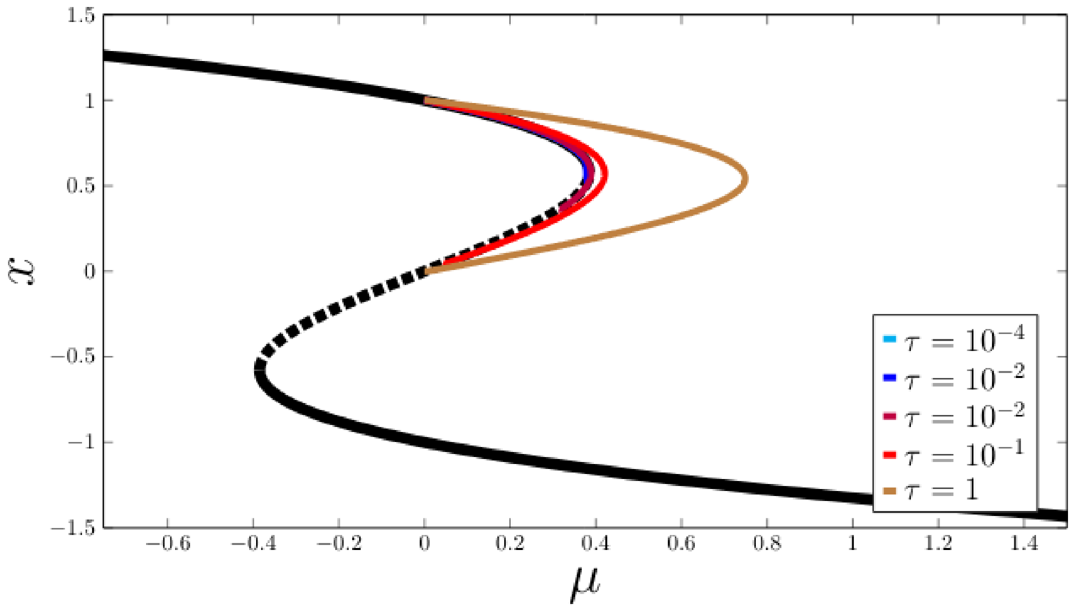
Safe Overshoots

Example 1:

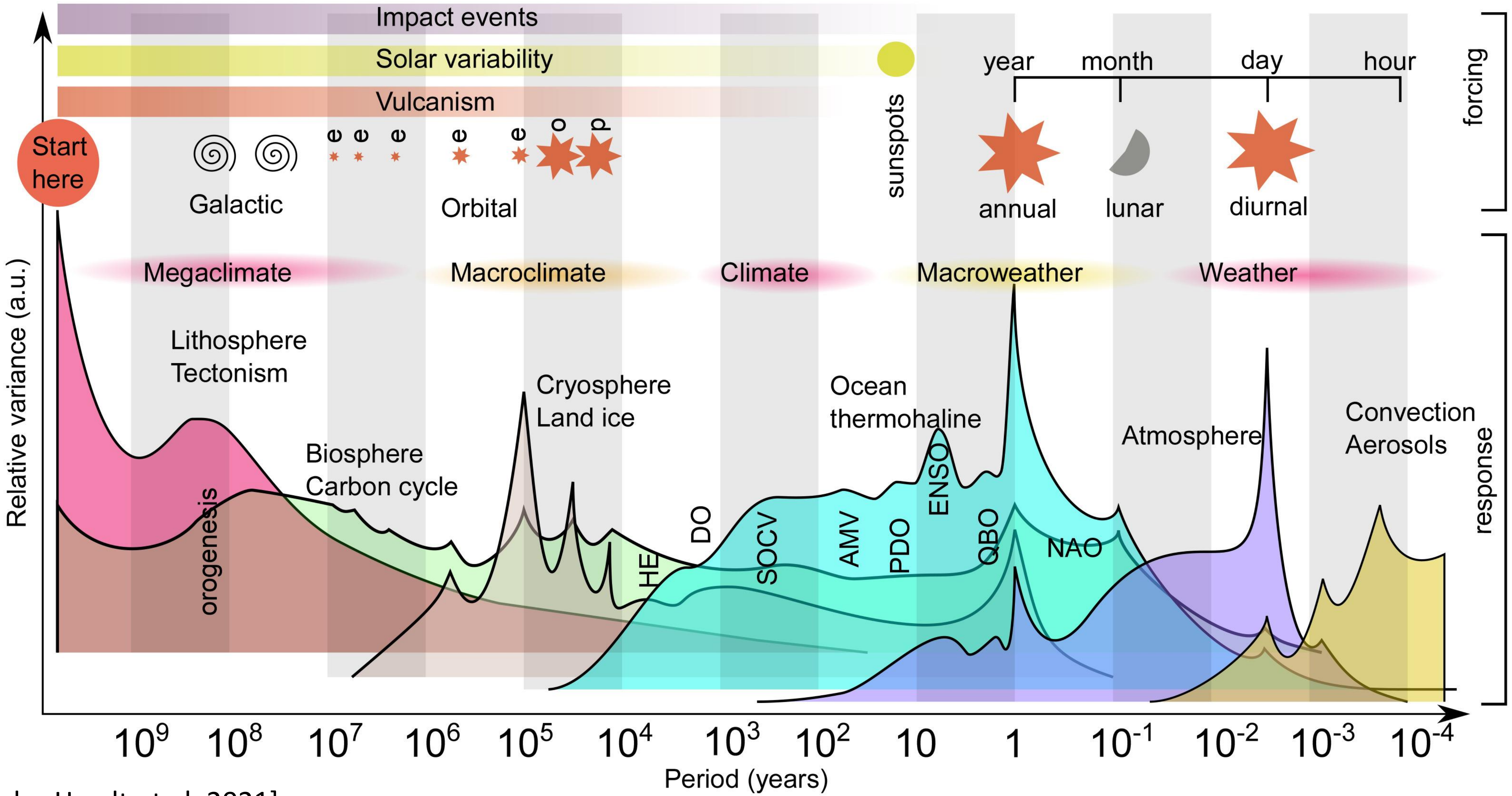
$$\frac{dx}{dt} = (x - a\mu) - (x - a\mu)^3 - \mu$$
$$\frac{d\mu}{dt} = \tau g(\tau t)$$

Pulse-like overshoot scenario:

$$g(s) = -\alpha \tanh(s)\,\mathrm{sech}(s)$$



Climate Timescales



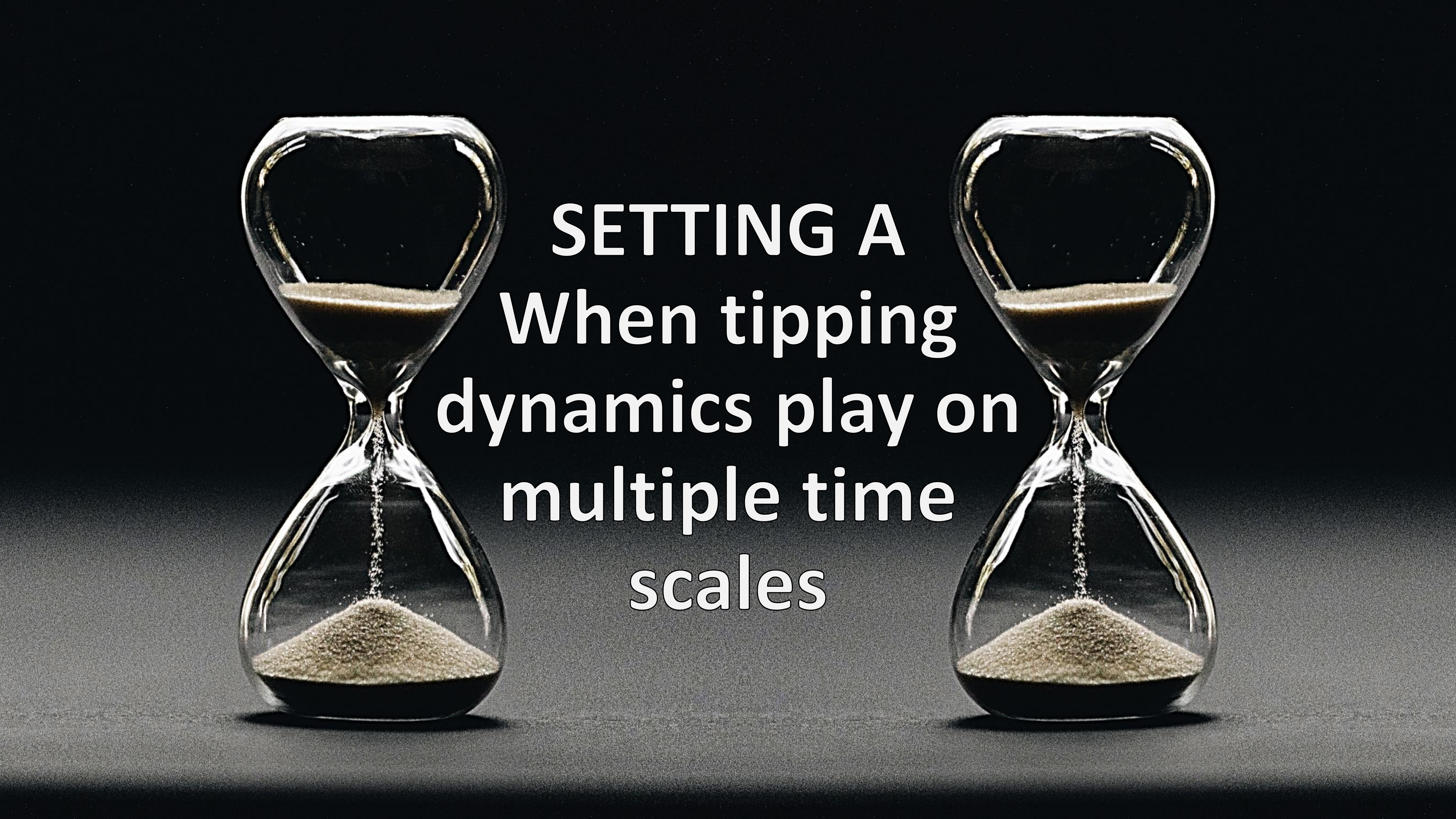
[Von der Heydt et al, 2021]

How does tipping work?

$$\frac{dx}{dt} = f(x;\mu)$$ ← Internal Dynamics

$$\frac{d\mu}{dt} = \tau g(\tau t)$$ ← Parameter Drift

Time Scale Separation (points to both τ)


SETTING A
When tipping dynamics play on multiple time scales

EXAMPLE 2: Multiscale Global Energy Balance Model

$$C\frac{dT}{dt} = Q_0(1 - \alpha) - \epsilon(T)\sigma T^4 + \mu$$

Short-Wave

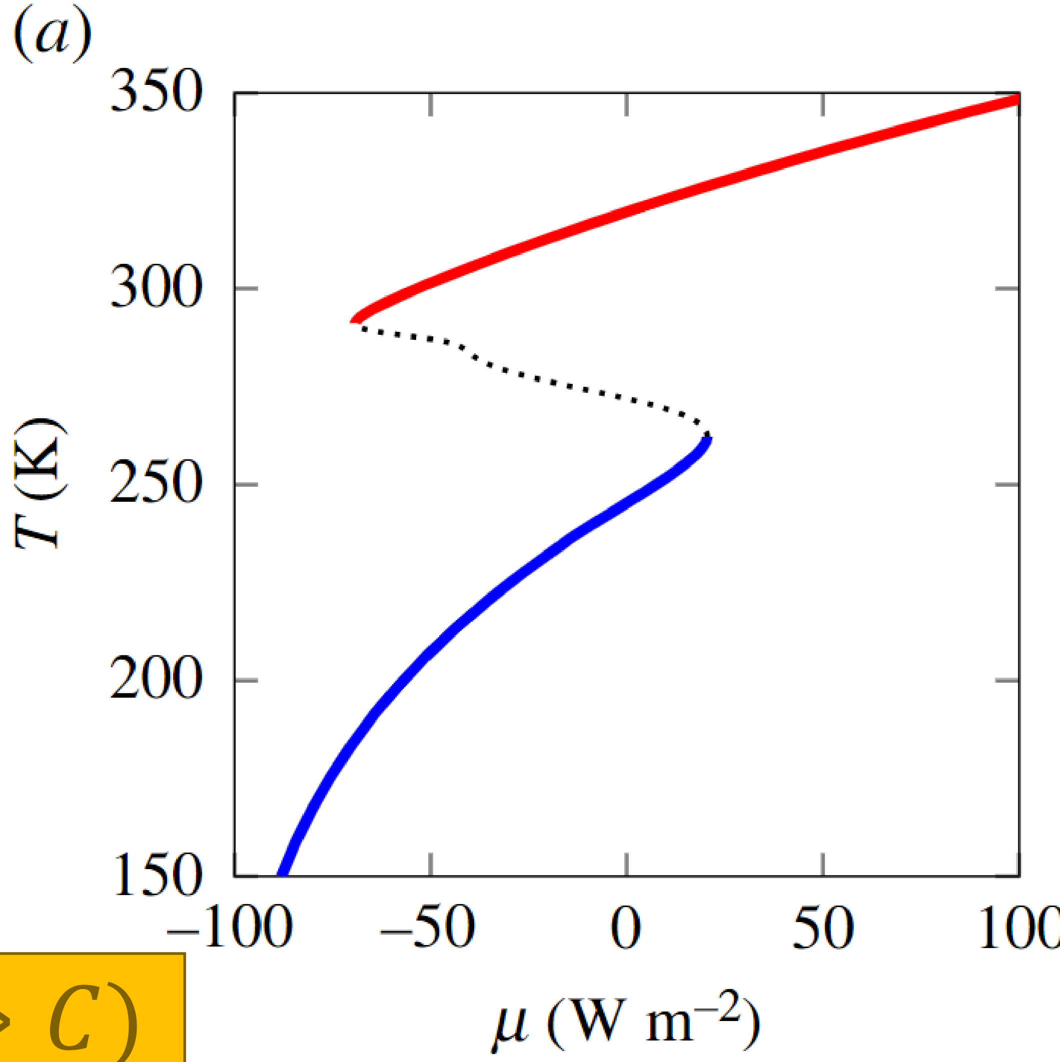
Long-Wave

CO2

$$\tau_\alpha \frac{d\alpha}{dt} = \alpha_0(T) - \alpha$$

Dynamic albedo

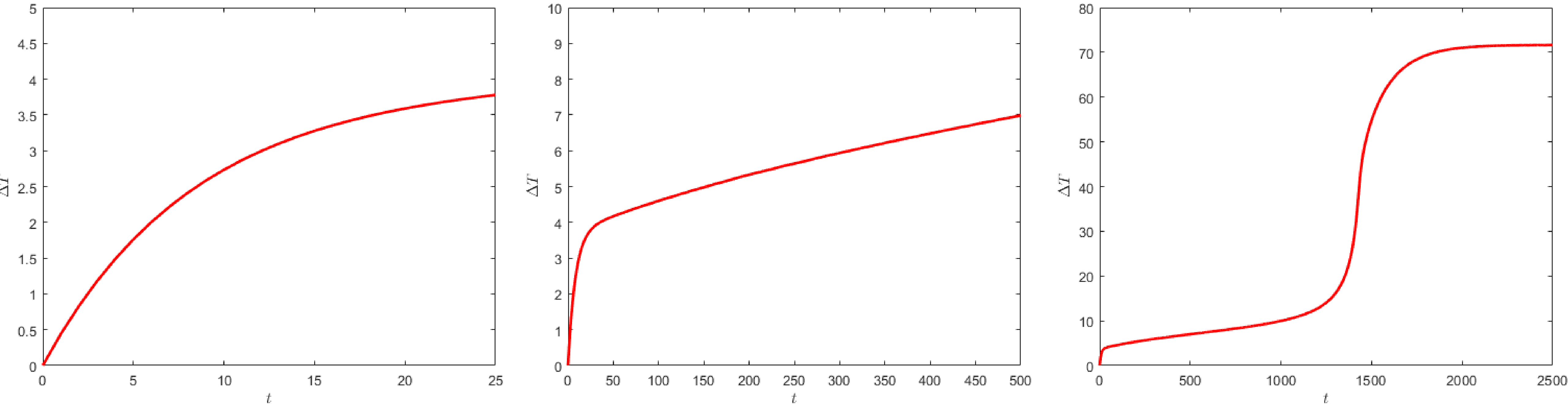
Internal Time Scale Separation ($\tau_\alpha \gg C$)



Abrupt 4xCO2 forcing experiment

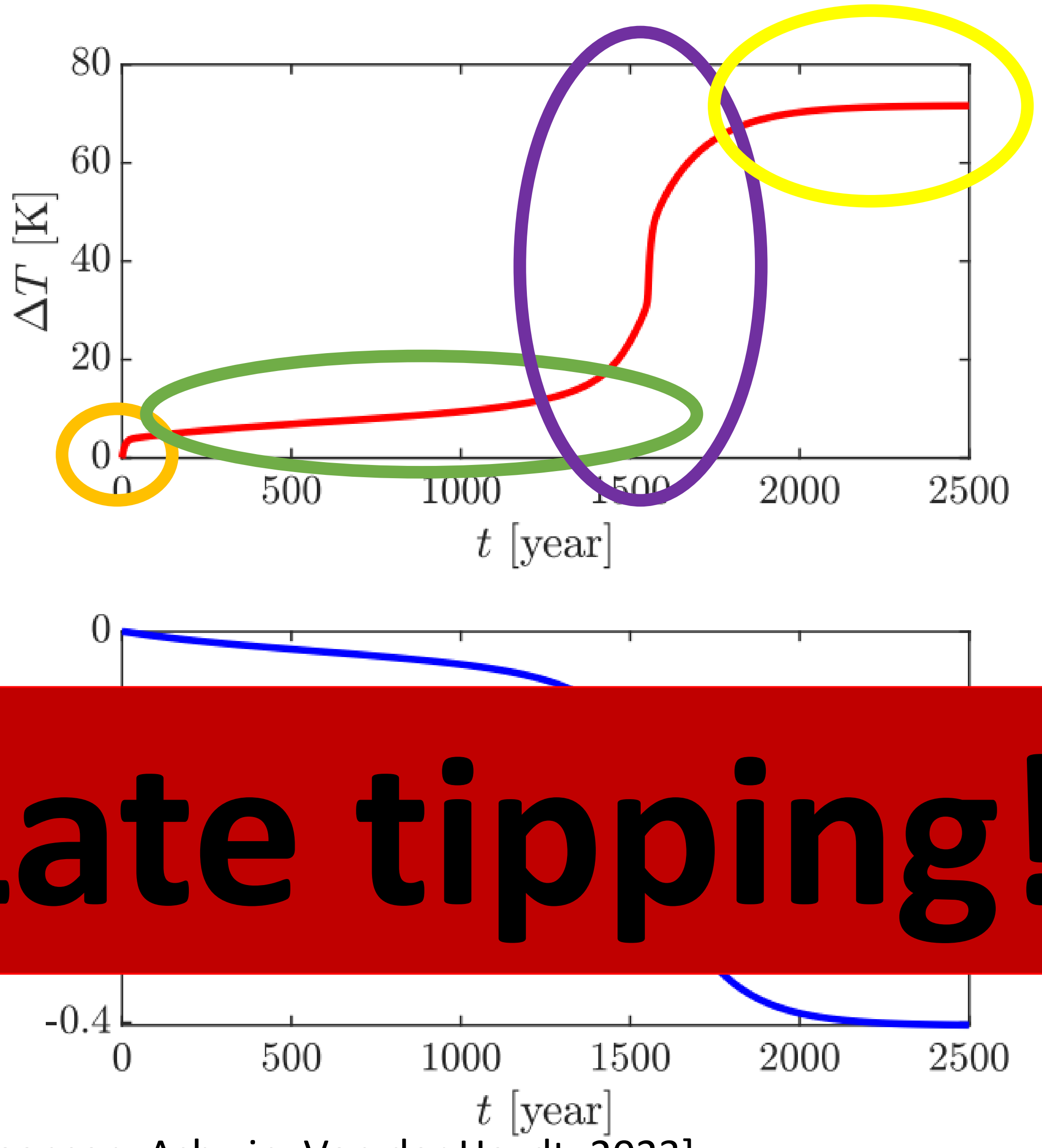
- Initialize for μ_0 (initial CO2-levels)
- Change to μ_1 (4xCO2 levels)

→ Look at dynamics

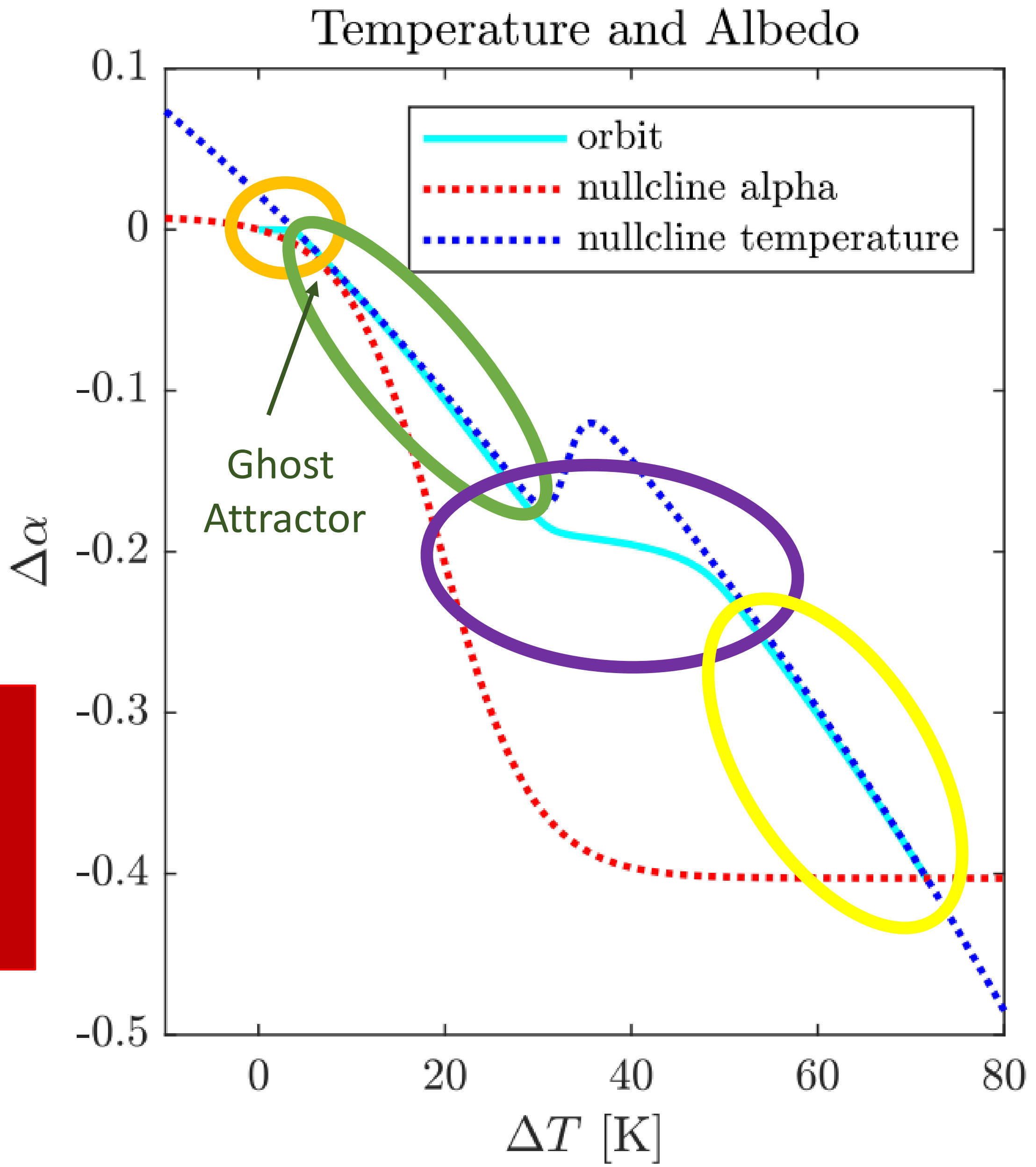


[Bastiaansen, Ashwin, Von der Heydt, 2023]

How does this work?



Late tipping!

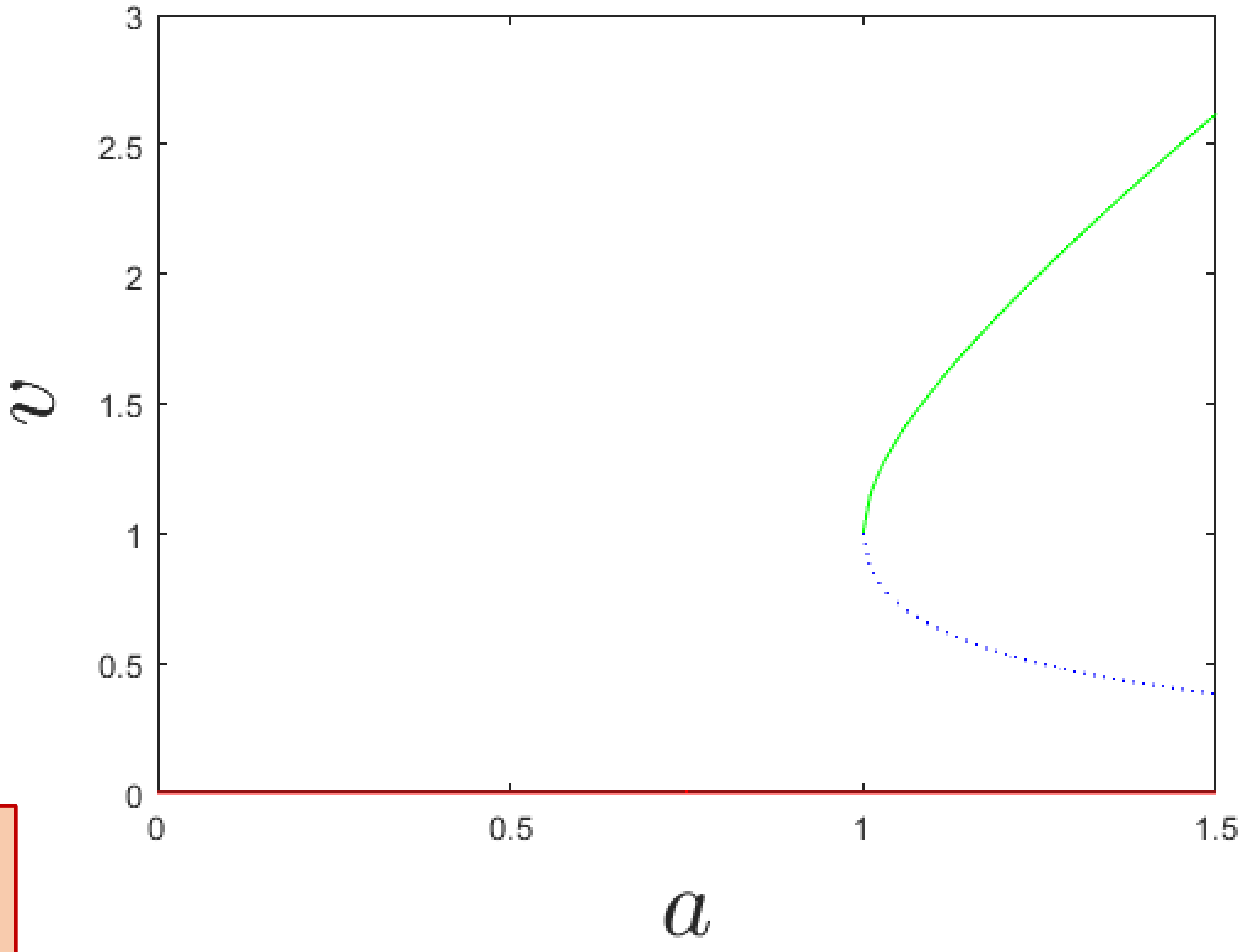


[Bastiaansen, Ashwin, Von der Heydt, 2023]

EXAMPLE 3: Time scale of feedback

$$\frac{du}{dt} = a - u - uv^2$$

$$\frac{dv}{dt} = uv^2 - mv$$



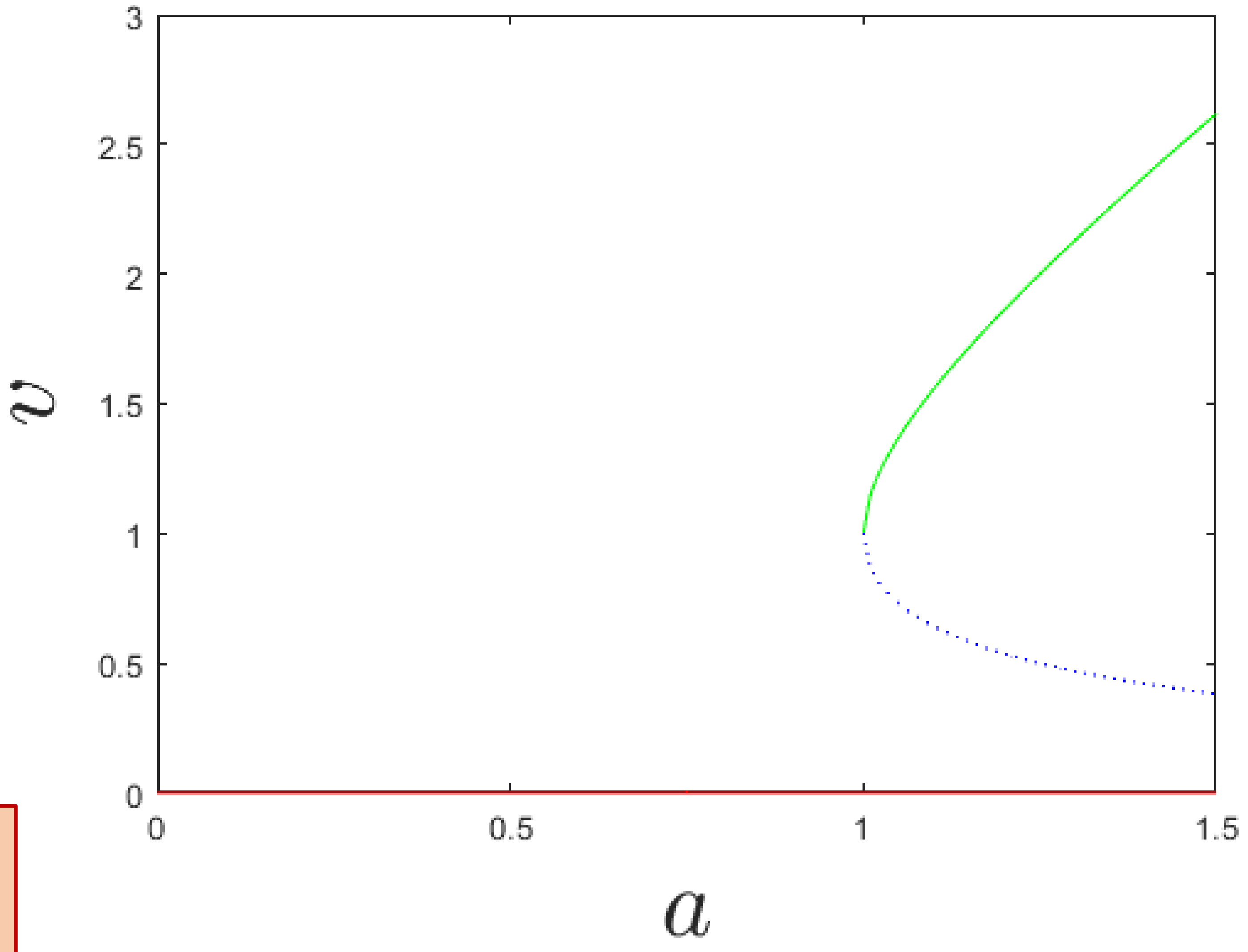
Parameters:
$m = 0.5$

EXAMPLE 3: Time scale of feedback

$$\frac{du}{dt} = a - u - uvs$$

$$\frac{dv}{dt} = uvs - mv$$

$$\frac{ds}{dt} = \tau_{INT}(v - s)$$



Parameters:
$m = 0.5$

EXAMPLE 3: Time scale of feedback

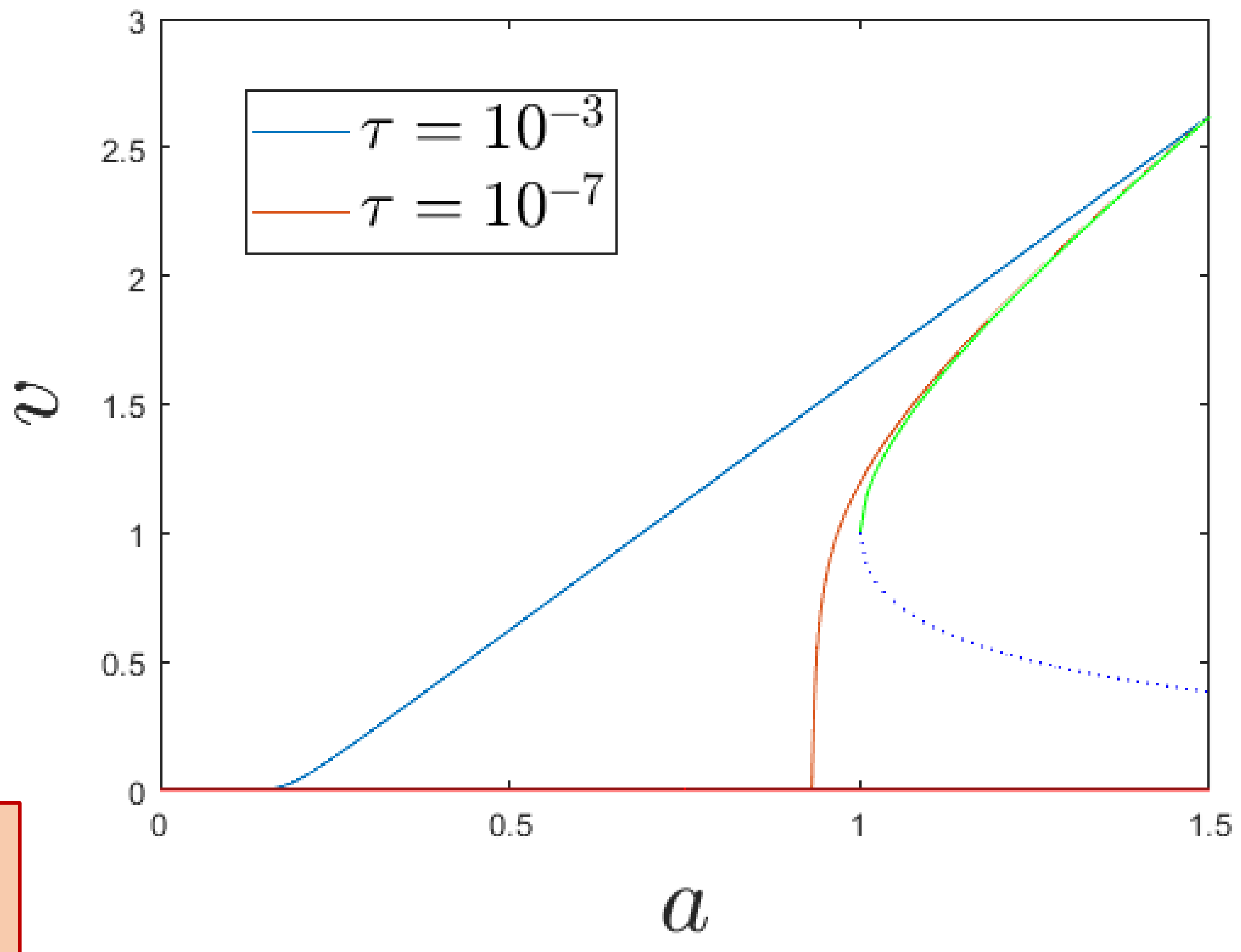
$$\frac{du}{dt} = a - u - uvs$$

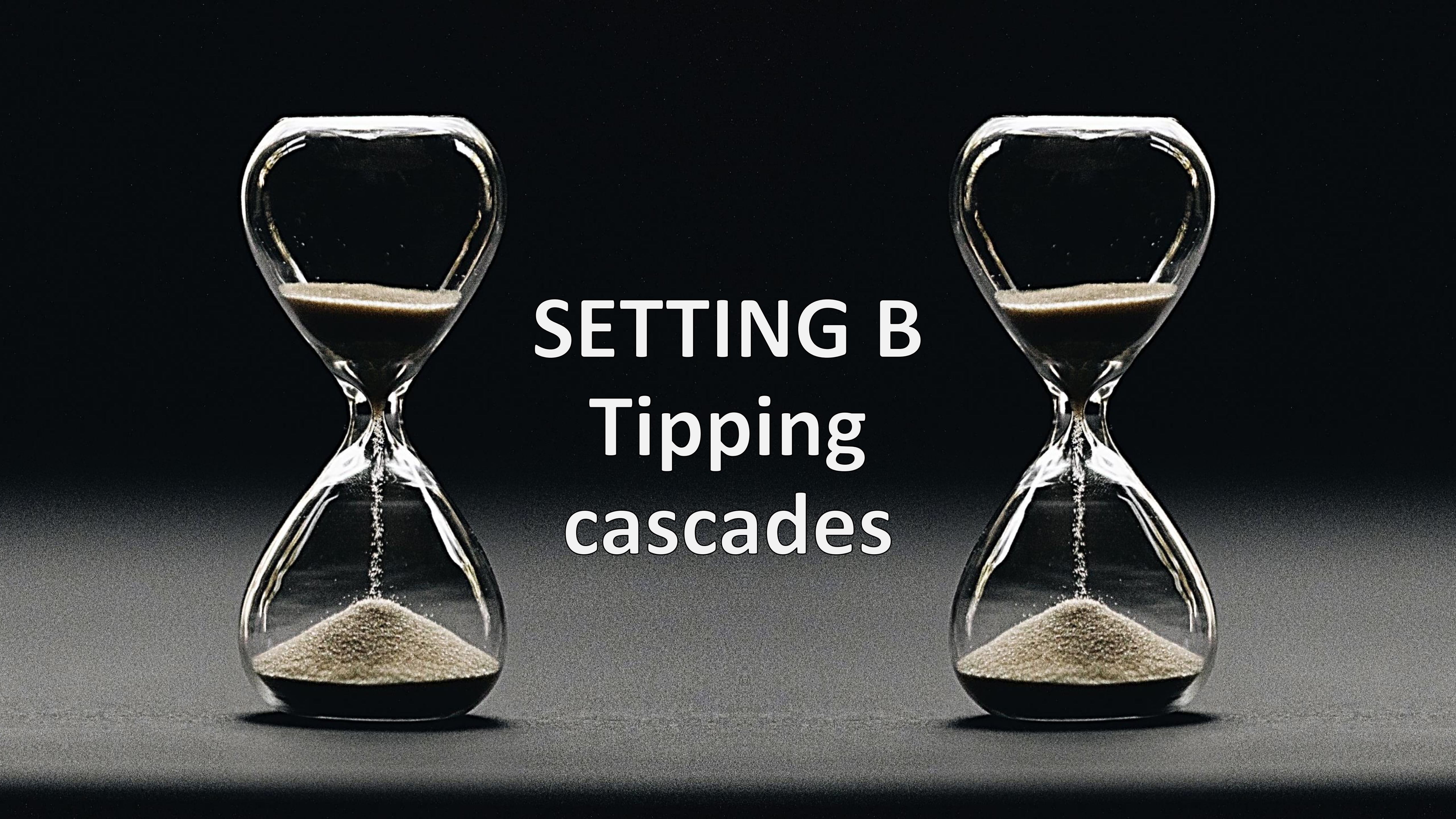
$$\frac{dv}{dt} = uvs - mv$$

$$\frac{ds}{dt} = \tau_{INT}\,(v - s)$$

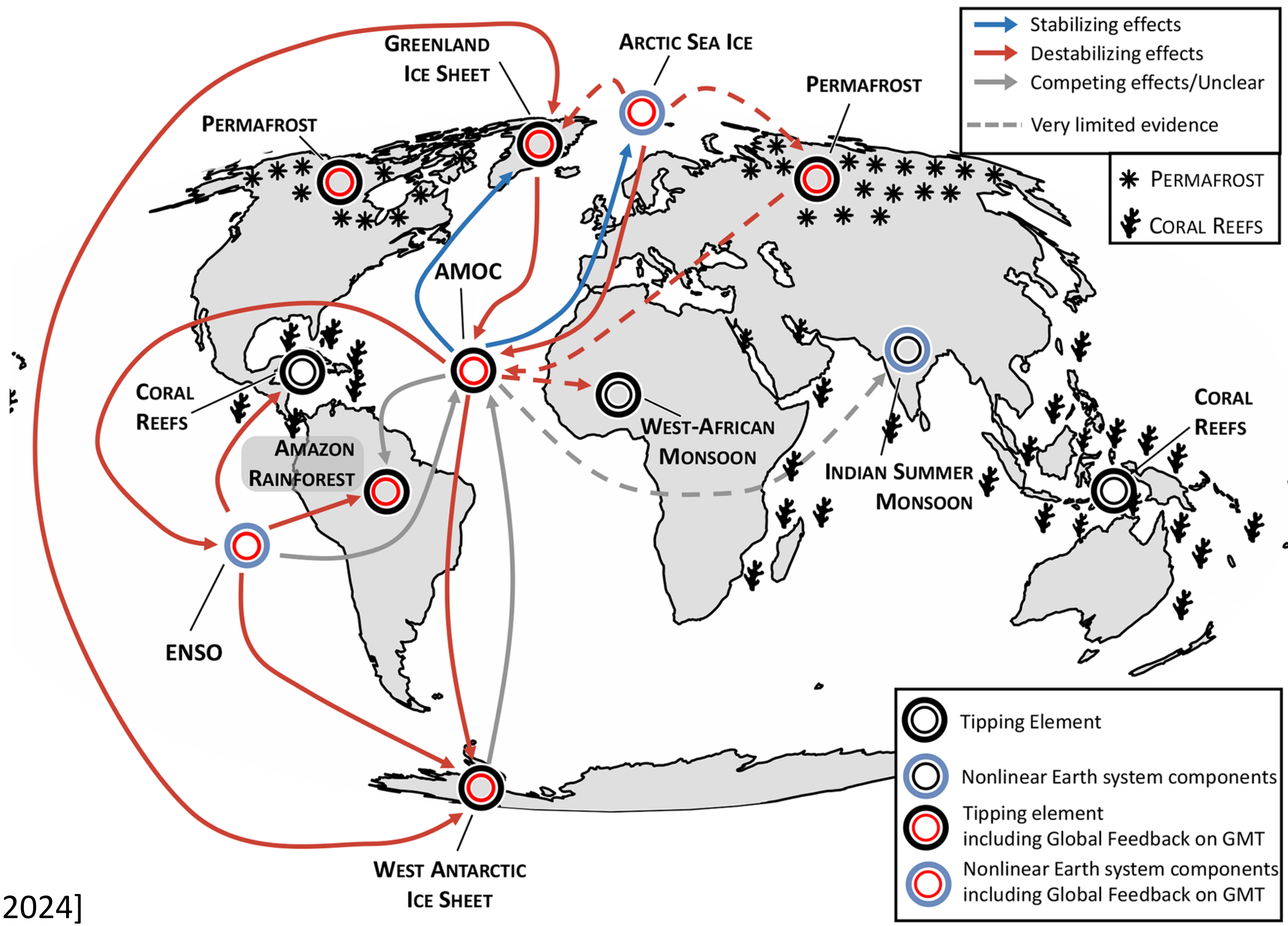
$$\frac{da}{dt} = -\tau$$

Parameters:
$m = 0.5$
$\tau_{INT} = 10^{-5}$



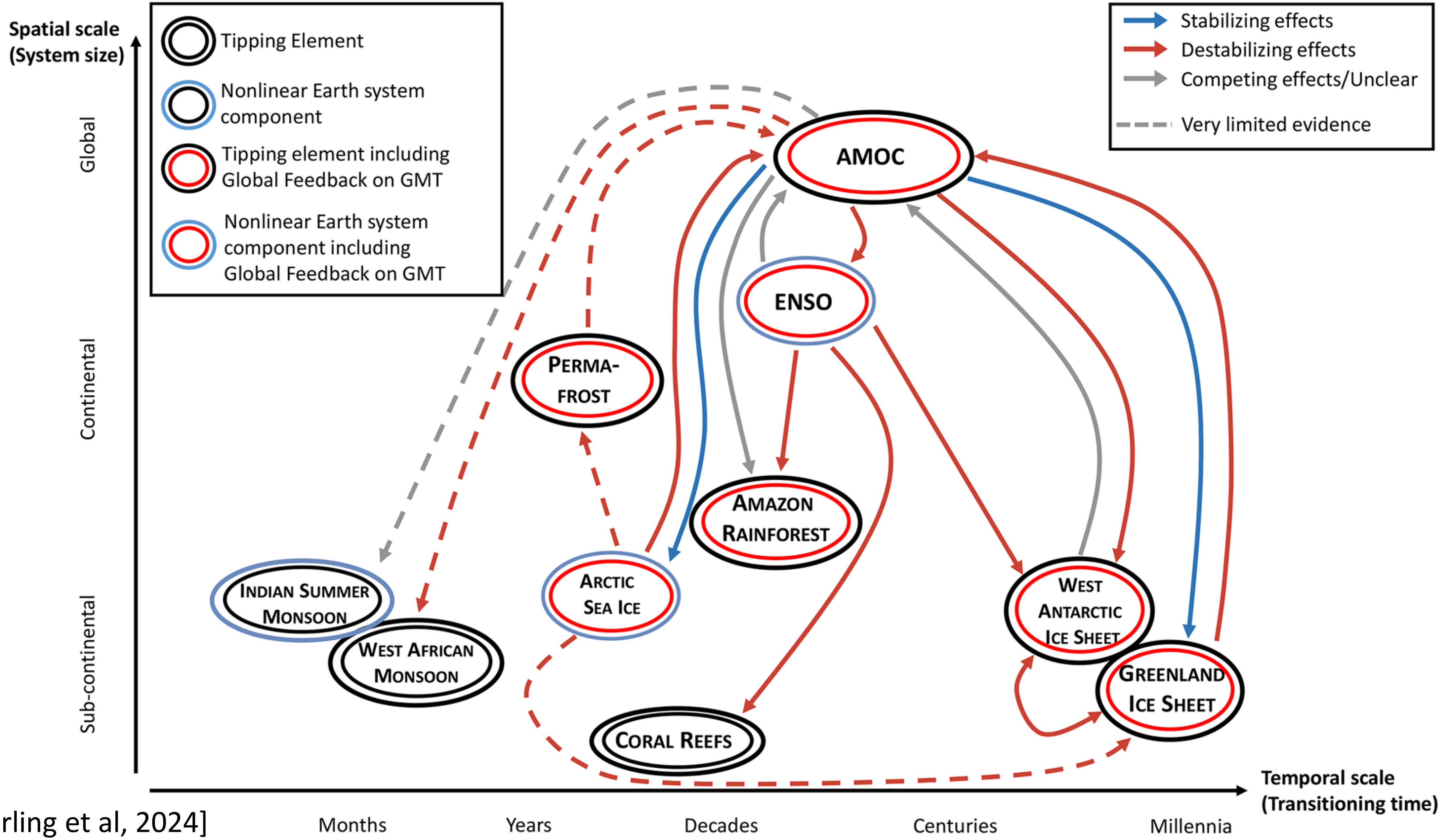

SETTING B
Tipping
cascades

Interactions between tipping elements



[Wunderling et al, 2024]

Interactions between tipping elements



[Wunderling et al, 2024]

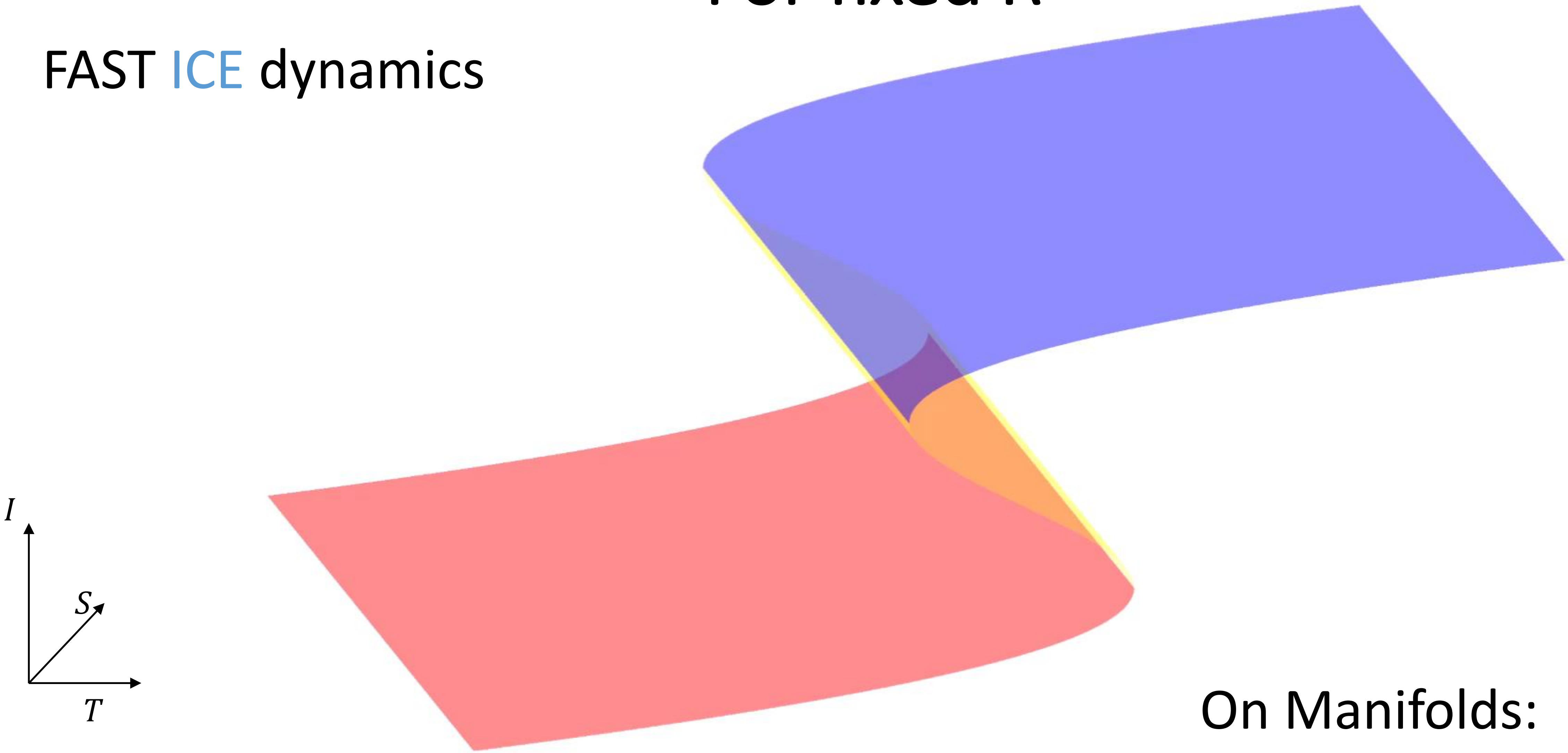
EXAMPLE 4: AMOC $\leftrightarrow$ ICE interaction

Tipping Element 1 (ICE)	$\frac{dI}{dt} = f(I, R, T)$	Energy balance model [Eisenman & Wettlaufer, 2009]
Tipping Element 2 (AMOC)	$\tau_O \frac{dT}{dt} = g_1(T, S, I)$ $\tau_O \frac{dS}{dt} = g_2(T, S)$	2-Box Model [Stommel, 1961] $\tau_O \gg 1$
Parameter drift	$\frac{dR}{dt} = \tau$	

[Lohman et al, 2021] & [David Hokken, MSc thesis, 2022]

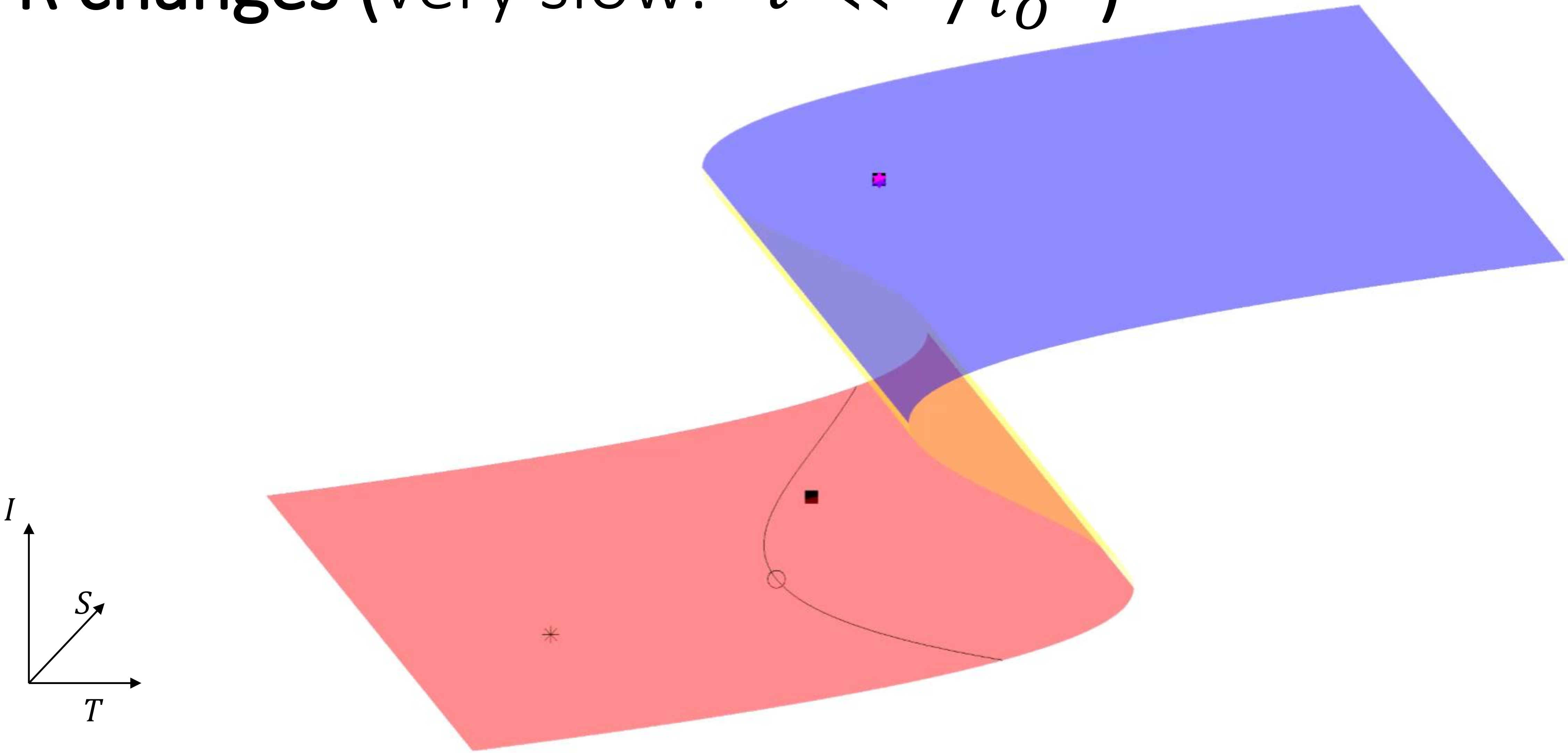
For fixed R

FAST ICE dynamics



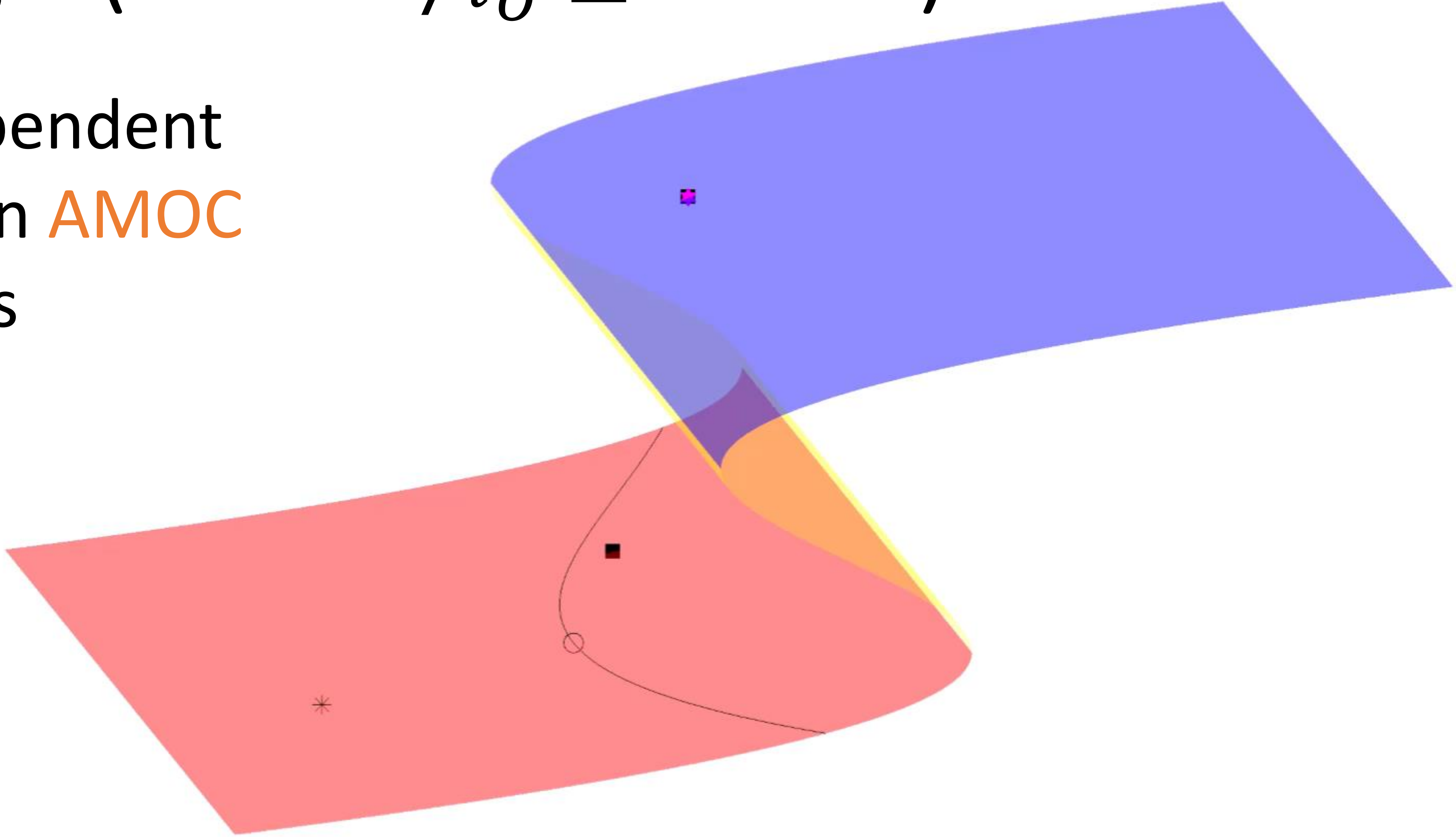
On Manifolds:
SLOW AMOC dynamics

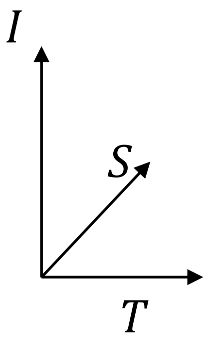
R changes (very slow: "$\tau \ll {}^{1}/_{\tau_O}$")



R changes (slow: "$^1/_{\tau_O} \leq \tau \ll 1$")

Rate-dependent effects on AMOC dynamics



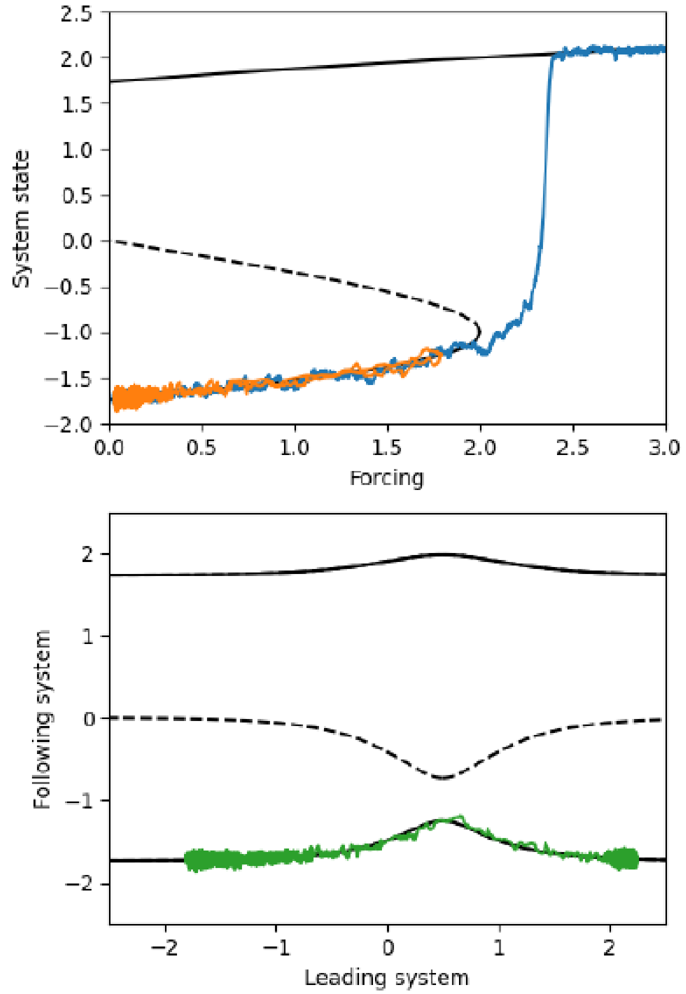


EXAMPLE 5: Conceptual Model for tipping cascades

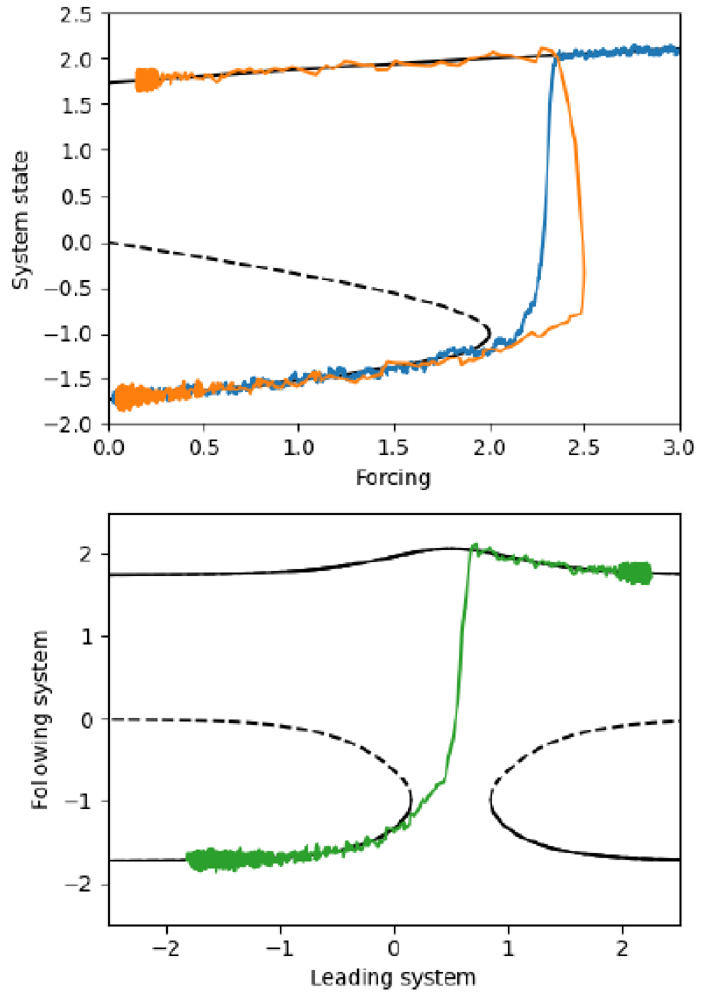
Tipping Element 1	$$\frac{dx}{dt} = f(x, \Lambda(r))$$	
Tipping Element 2	$$\varepsilon \frac{dy}{dt} = f(y, M(x))$$	$\varepsilon \ll 1$
Parameter drift	$$\frac{dr}{dt} = \tau$$	

[Peter Ashwin, Robbin Bastiaansen, Anna von der Heydt, Paul Ritchie, *in progress*]

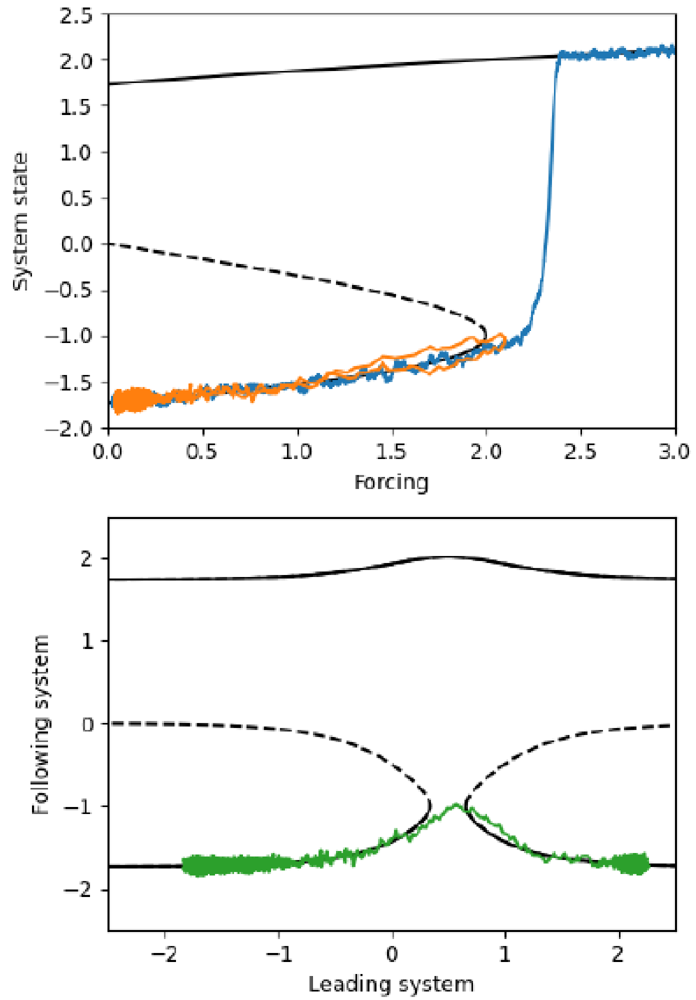
EXAMPLE 5: Conceptual Model for tipping cascades



No cascade



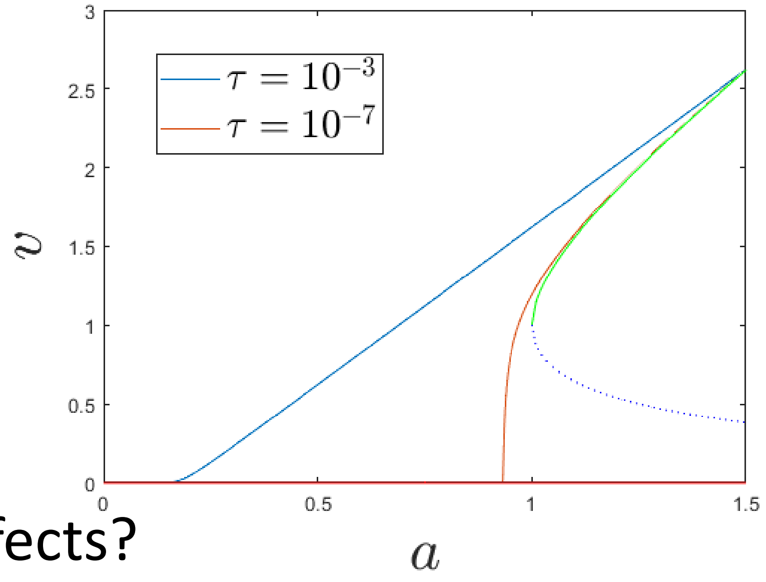
Cascade



No cascade – but overshoot

[Peter Ashwin, Robbin Bastiaansen, Anna von der Heydt, Paul Ritchie, *in progress*]

Discussion Points



- Tipping DYNAMICS also important

A: Tipping in systems with multiple time scales

- Late tipping possible → predictable?
- Rate-induced effects depend on time scales
  → Is this a way to get better grip on non-autonomous effects?
- Response to faster changes might look less abrupt

B: Tipping cascades

- Can fast-slow analysis help?
  → Should we study tipping elements *in isolation*?
  → Do current interactions between tipping elements tell us enough?

slides at bastiaansen.github.io

Thanks to:
Peter Ashwin, Anna von der Heydt, David Hokken,
Anna van der Kaaden, Rita Mak, Paul Ritchie