

Kantelpunten in het klimaat



dr. Robbin Bastiaansen
Docentennetwerk wiskunde (14 april 2025)



COP21 • CMP11
PARIS 2015
UN CLIMATE CHANGE CONFERENCE

Klimaatconferentie Parijs 2015

Akkoord van Parijs:

→ Beperk opwarming tot 2°C



COP21 • CMP11
PARIS 2015
UN CLIMATE CHANGE CONFERENCE

Klimaatconferentie Parijs 2015

Akkoord van Parijs:

→ Beperk opwarming tot **1.5°C**

A photograph of a massive glacier wall, likely the Perito Moreno Glacier. A large, jagged chunk of ice is falling from the face of the glacier into the dark water below, creating a large splash. The glacier's surface is highly textured with numerous vertical crevasses and ridges. The water in the foreground is dark and choppy, with some smaller ice floes visible.

PLANNING

Planning

1. Context – klimaatverandering en modellen
2. Analyse van modellen - bifurcatiediagram
3. Voorbeelden klimaatmodellen
4. Kantelpunten
5. (Analyse van ingewikkeldere modellen)
6. Context – kantelpunten in het klimaat
7. Uitdagingen analyse kantelpunten
8. Mogelijke verdiepingen



dr. Robbin Bastiaansen

Sinds 2022	Universitair Docent @	Mathematisch Instituut (MI) & Instituut voor Marien en Atmosferisch onderzoek Utrecht (IMAU), Universiteit Utrecht (UU)
2020-2022	Postdoc @	Instituut voor Marien en Atmosferisch onderzoek Utrecht (IMAU), Universiteit Utrecht (UU)
2015-2019	PhD @	Mathematisch Instituut (MI), Universiteit Leiden (UL)

Onderzoeksonderwerpen:

Tipping Points, Pattern Formation, Asymptotic Analysis, Response Theory, Climate Sensitivity, Climate Dynamics, Dynamical Systems, Mathematical Modelling, Bifurcation Theory, Differential Equations, Climate Change, Nonlinear Dynamics, Singular Perturbation Theory, Matched Asymptotics, Spatial Patterns, Complex Systems, Ecosystem Dynamics, Resilience, Fast-Slow dynamics



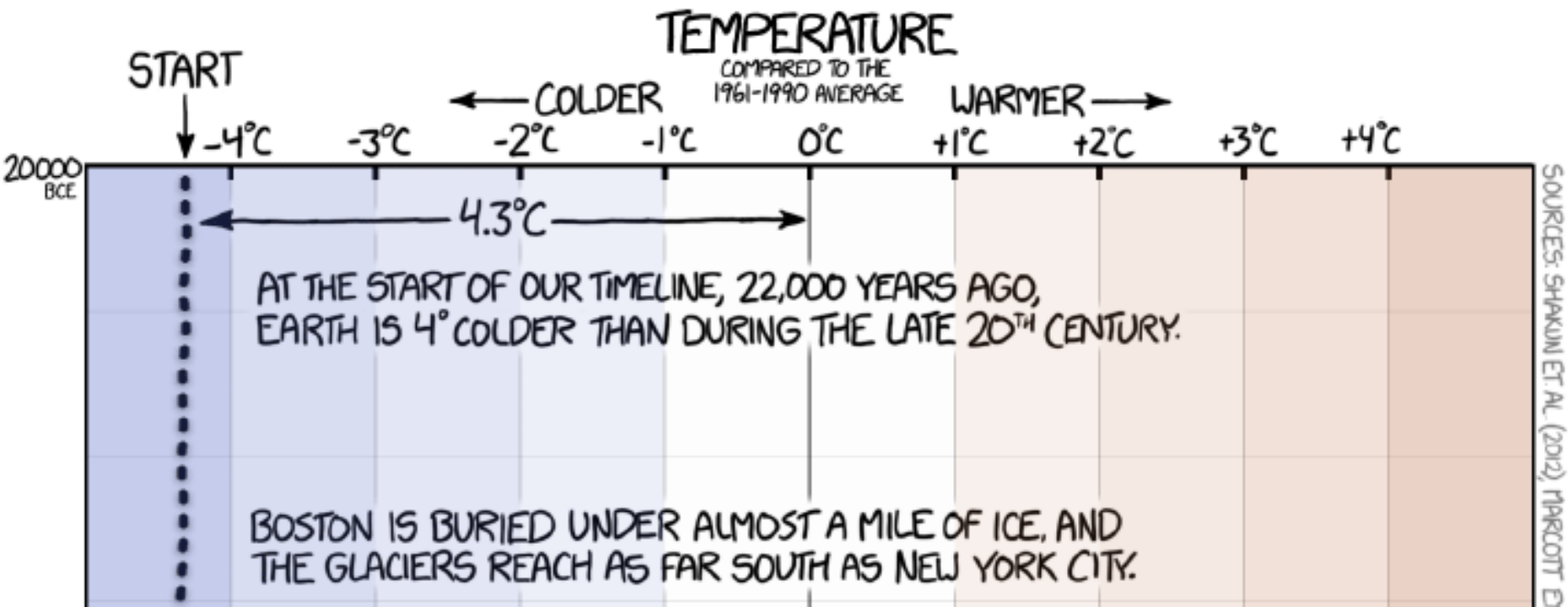
A dramatic landscape photograph featuring an industrial scene at sunset. In the background, several tall smokestacks release thick, dark plumes of smoke that rise into the sky. The sky is a deep orange and yellow, with large, billowing clouds that are illuminated from below by the setting sun. In the foreground and middle ground, a network of high-voltage power lines and their lattice towers stretches across the frame. The overall atmosphere is one of industrial activity and environmental impact.

KLIMAATVERANDERING

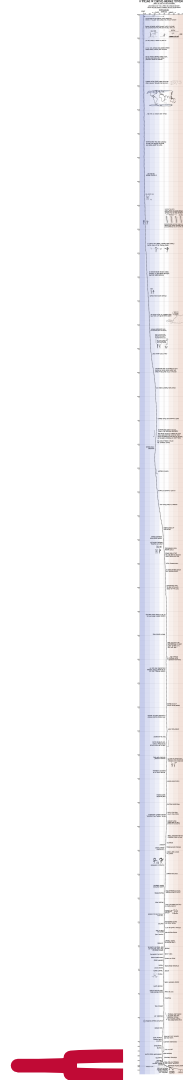
A TIMELINE OF EARTH'S AVERAGE TEMPERATURE

SINCE THE LAST ICE AGE GLACIATION

WHEN PEOPLE SAY "THE CLIMATE HAS CHANGED BEFORE,"
THESE ARE THE KINDS OF CHANGES THEY'RE TALKING ABOUT.



1 graad opwarming in
150 jaar

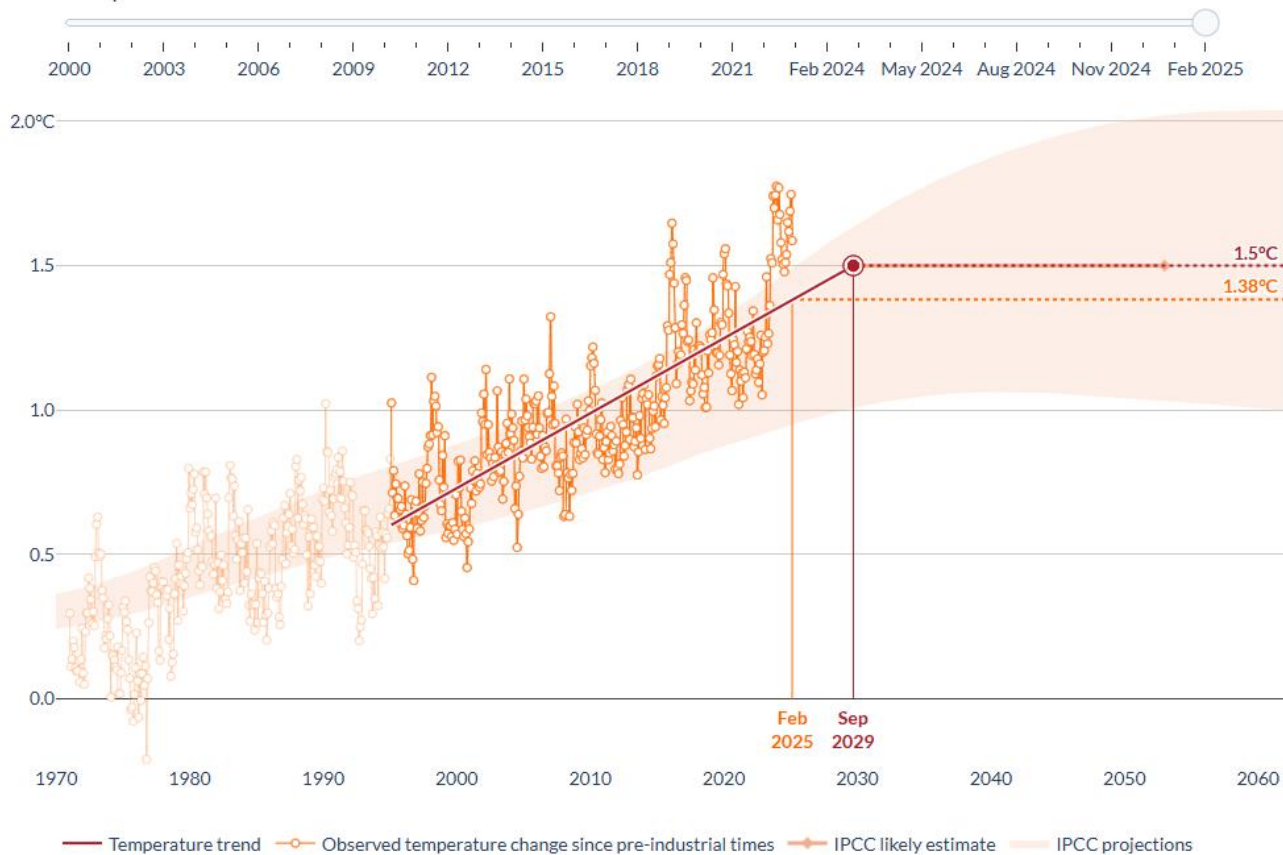


4 graden opwarming
in meer dan 20 000
jaar

Global warming reached an estimated **1.38°C** in **February 2025**.

If the 30-year warming trend leading up to then continued,
global warming would reach **1.5°C** by **September 2029**.

Extrapolate from: Feb 2025



A firefighter in a dark uniform and helmet is seen from behind, spraying a powerful stream of water from a hose. The scene is set at night in a forest, with a large, intense fire burning in the background, casting a bright orange glow over the entire scene. The water spray is captured in motion, creating a dense, white mist. The firefighter is positioned in the lower left foreground, facing away from the camera towards the fire. The background shows silhouettes of trees and a few vertical poles, possibly part of a fire suppression system. The overall atmosphere is one of urgency and danger.

GEVOLGEN VAN KLIMAATVERANDERING

Voorspellingen

- Meer zware neerslag
- Langere perioden van droogte
- Meer en intensere bosbranden
- Zeespiegelstijging
- Verzilting van water
- Meer stormen en orkanen
- Hogere landbouwopbrengsten

The Rodney & Otamatea Times
WAITEMATA & KAIPARA GAZETTE.
PRICE—10s per annum in advance
WARKWORTH, WEDNESDAY, AUGUST 14, 1912.
3d. per Copy.

Science Notes and News.

COAL CONSUMPTION AFFECT- ING CLIMATE.

The furnaces of the world are now burning about 2,000,000,000 tons of coal a year. When this is burned, uniting with oxygen, it adds about 7,000,000,000 tons of carbon dioxide to the atmosphere yearly. This tends to make the air a more effective blanket for the earth and to raise its temperature. The effect may be considerable in a few centuries.

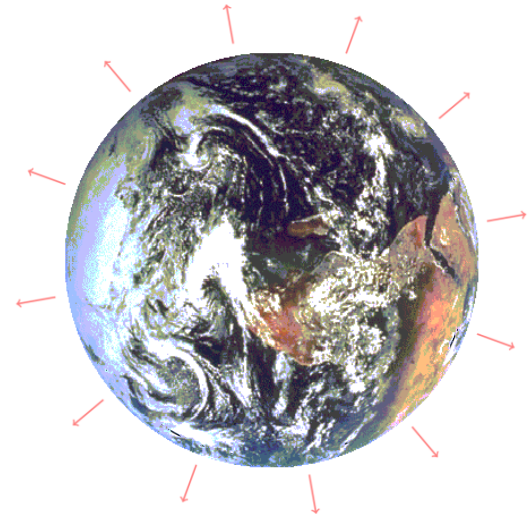
KERN VAN DE
MOEILIKHEID



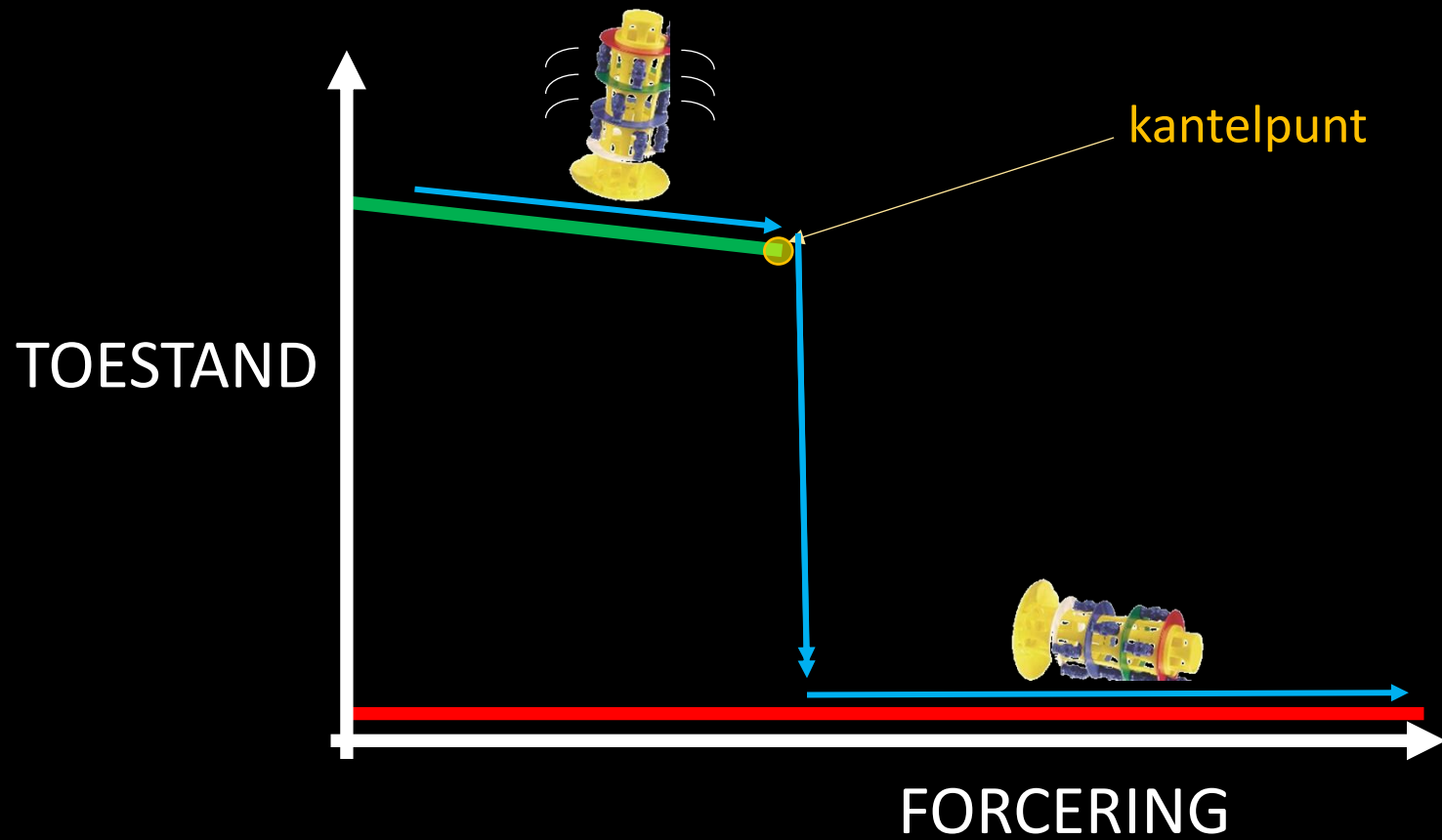
Niet-lineaire terugkoppeling



- veel ijs/sneeuw
- veel weerkaatsing zonlicht

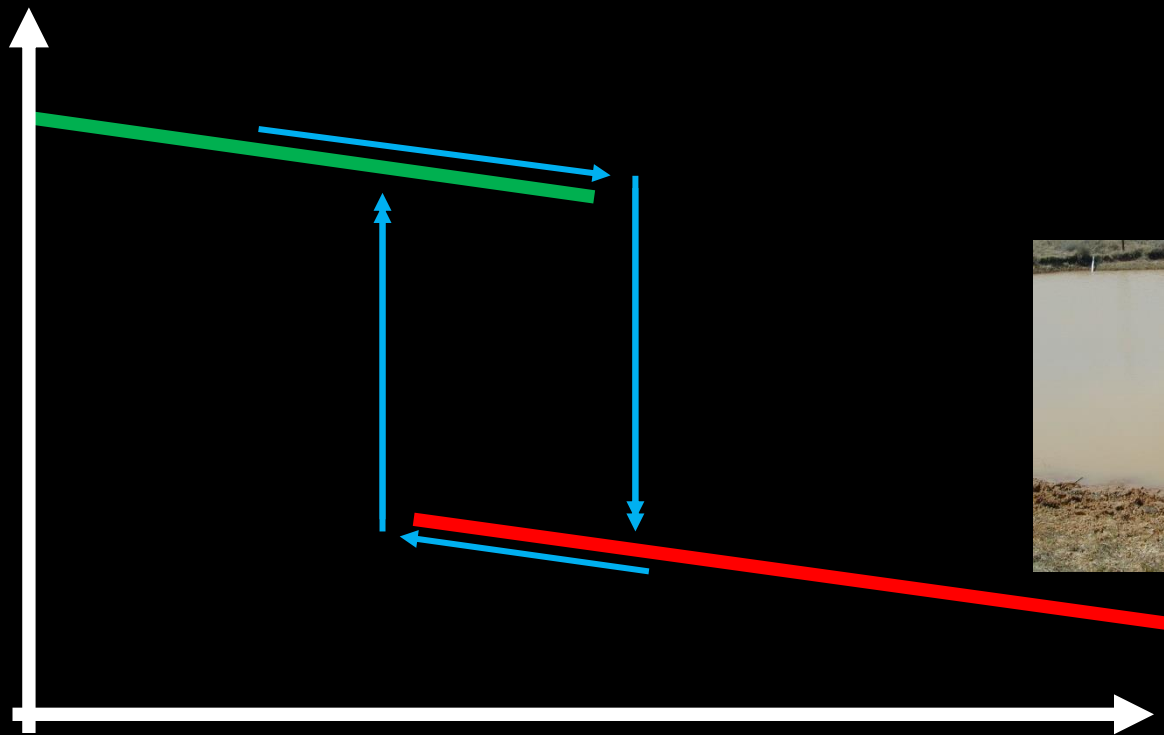


- weinig ijs/sneeuw
- weinig weerkaatsing zonlicht

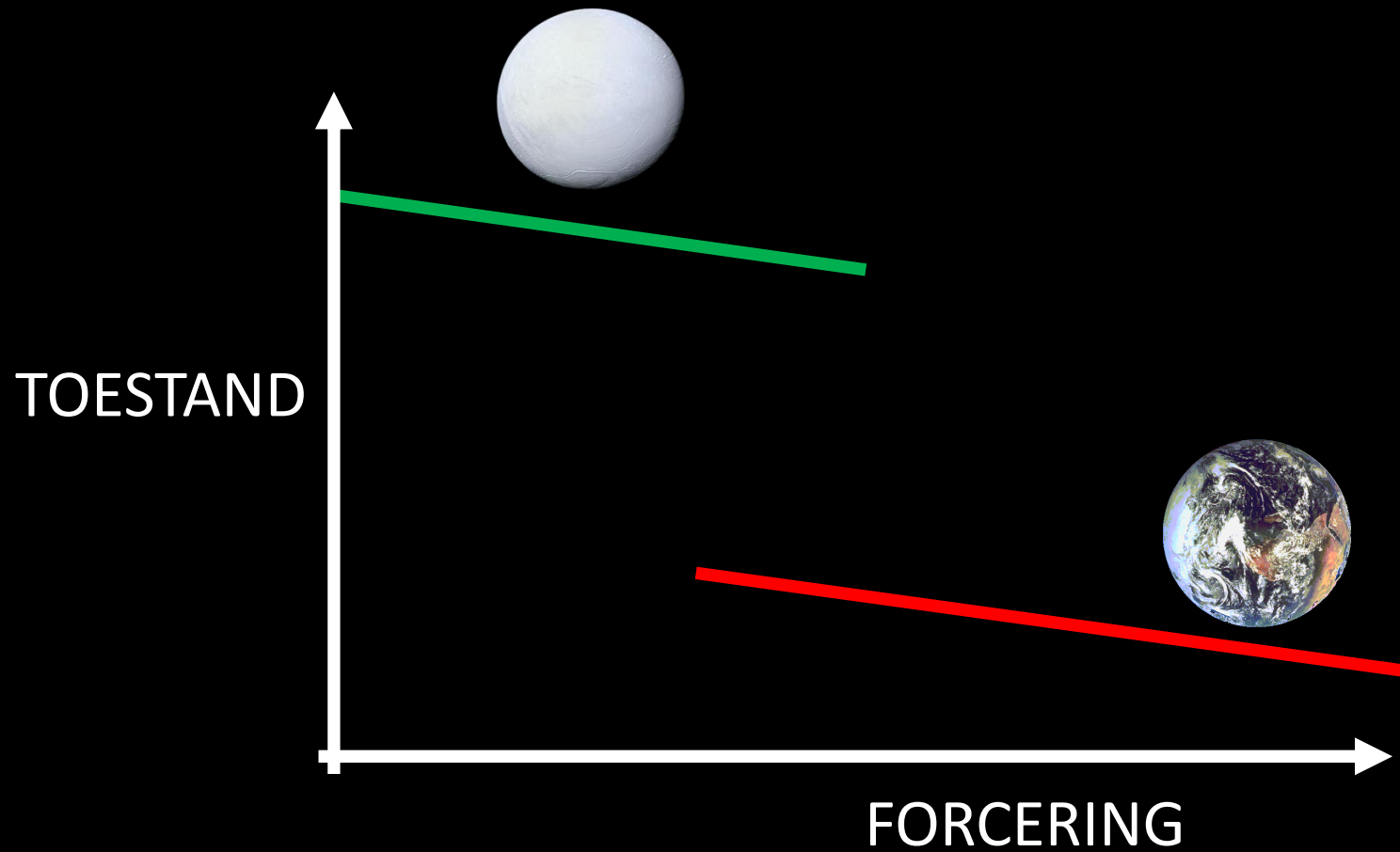




TOESTAND

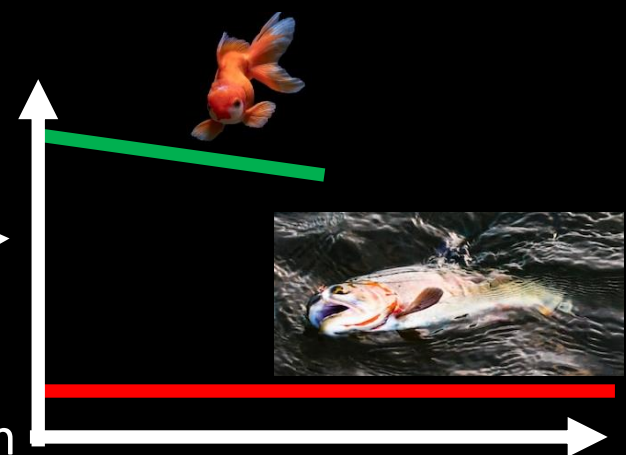


FORCERING

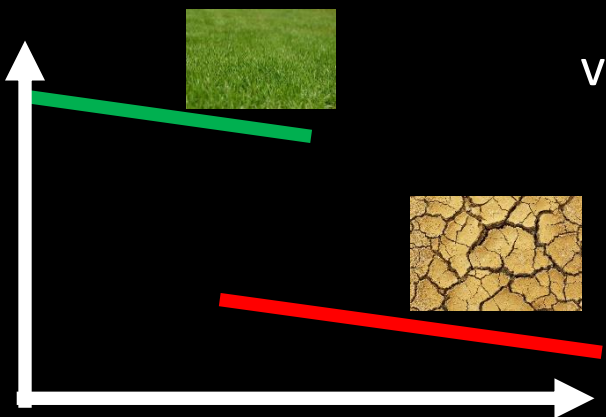




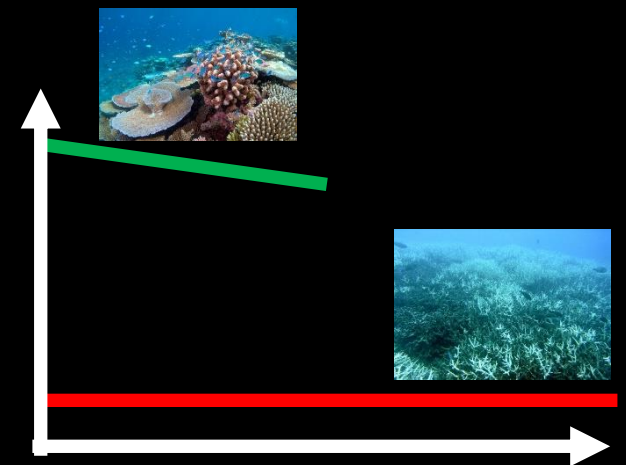
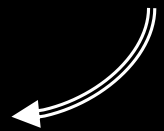
regenwoud ↔ savanne

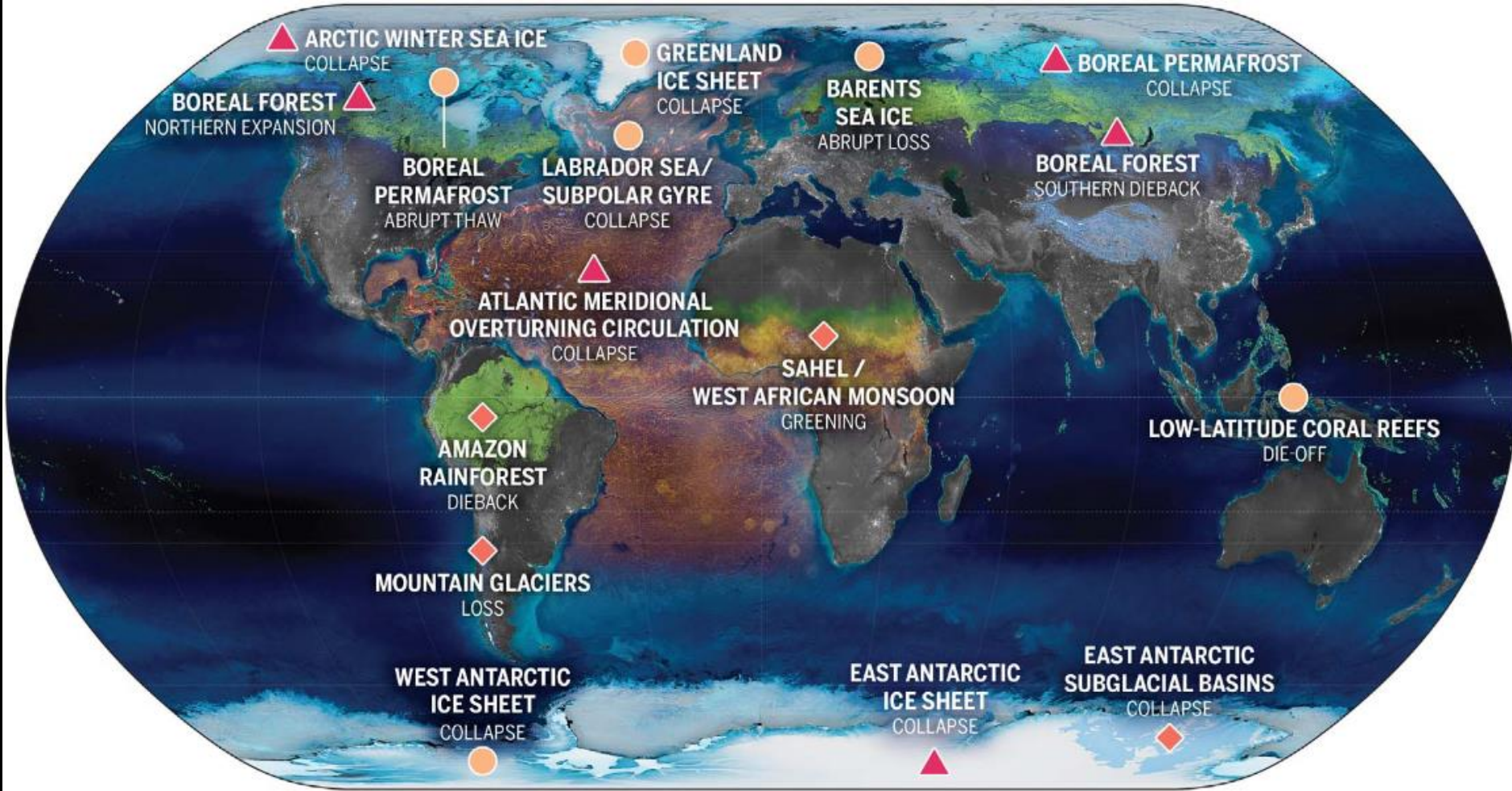


uitsterven diersoorten

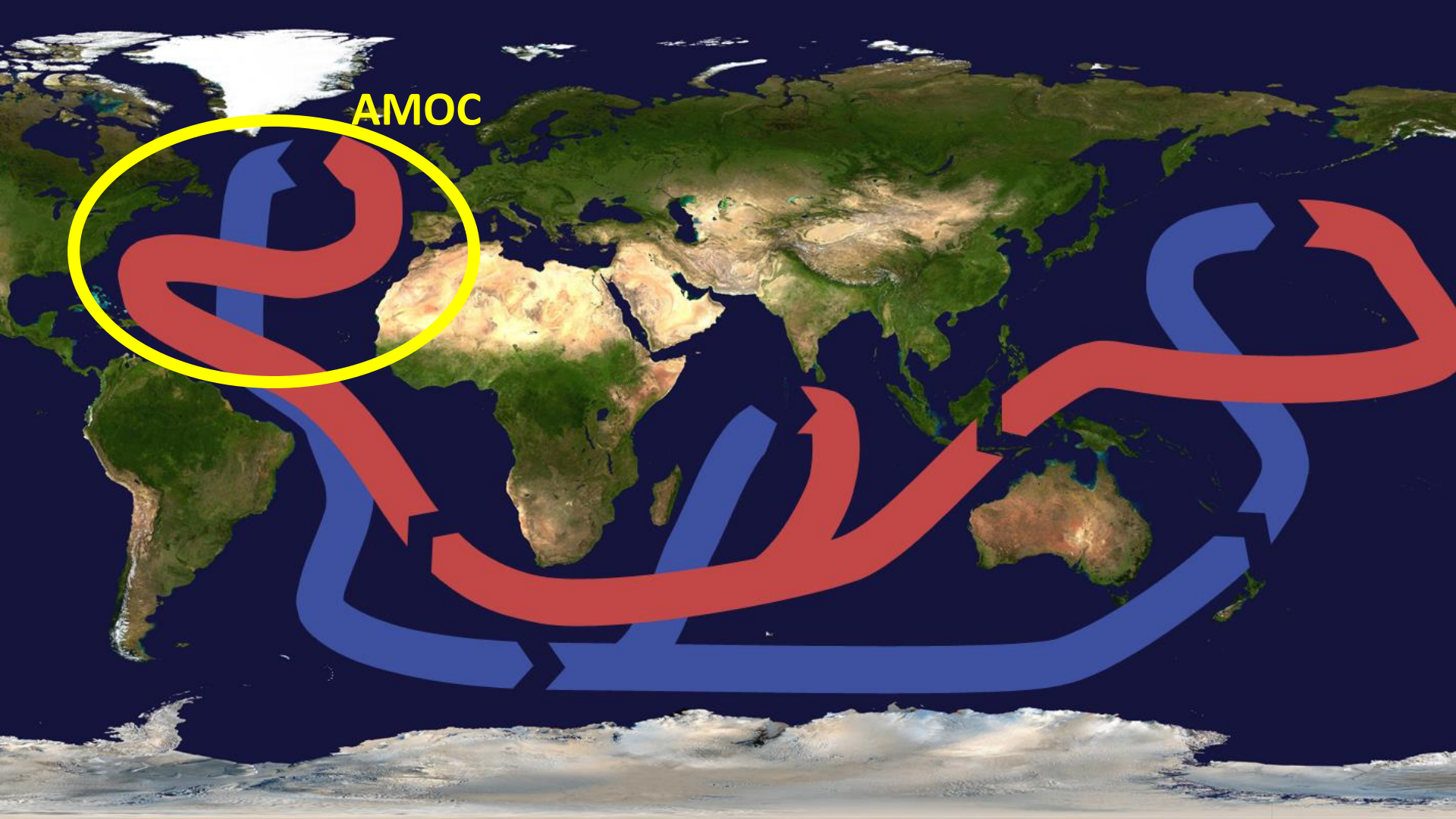


verwoestijning





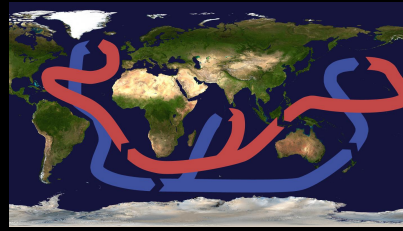
[Armstrong McKay et al, Science (2022)]



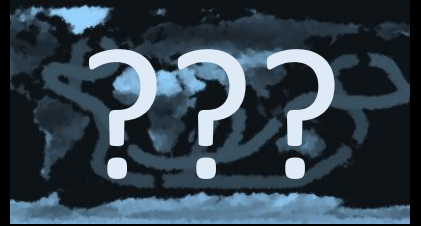
AMOC

TOESTAND

AMOC on



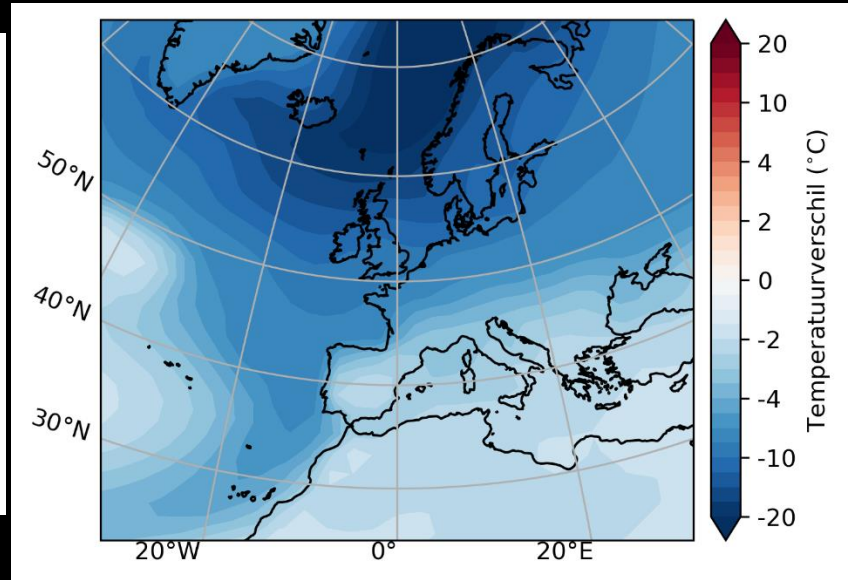
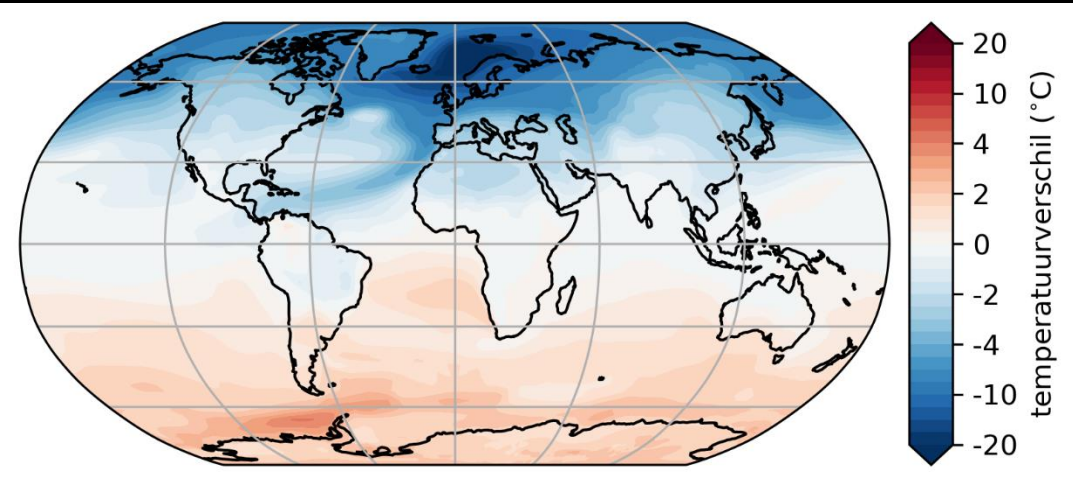
AMOC off

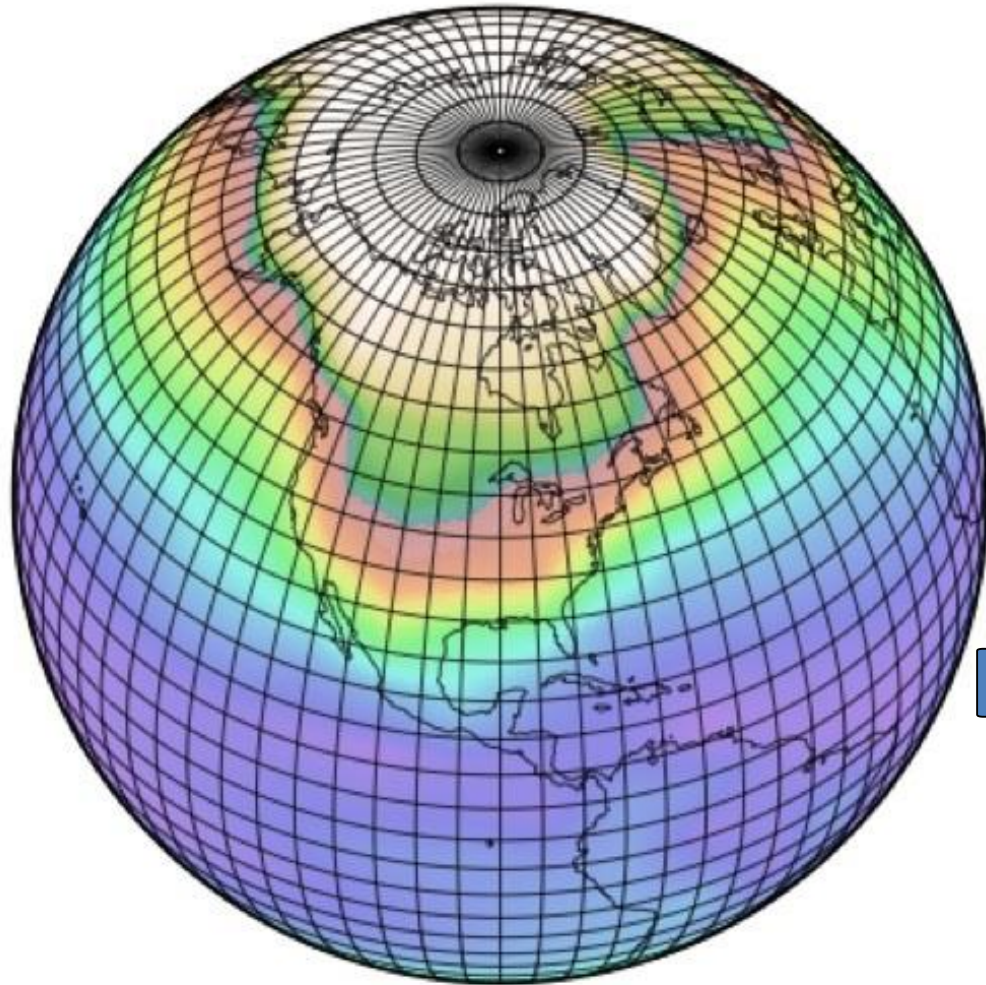


FORCERING

THE DAY AFTER TOMORROW







KLIMAATMODELLEN

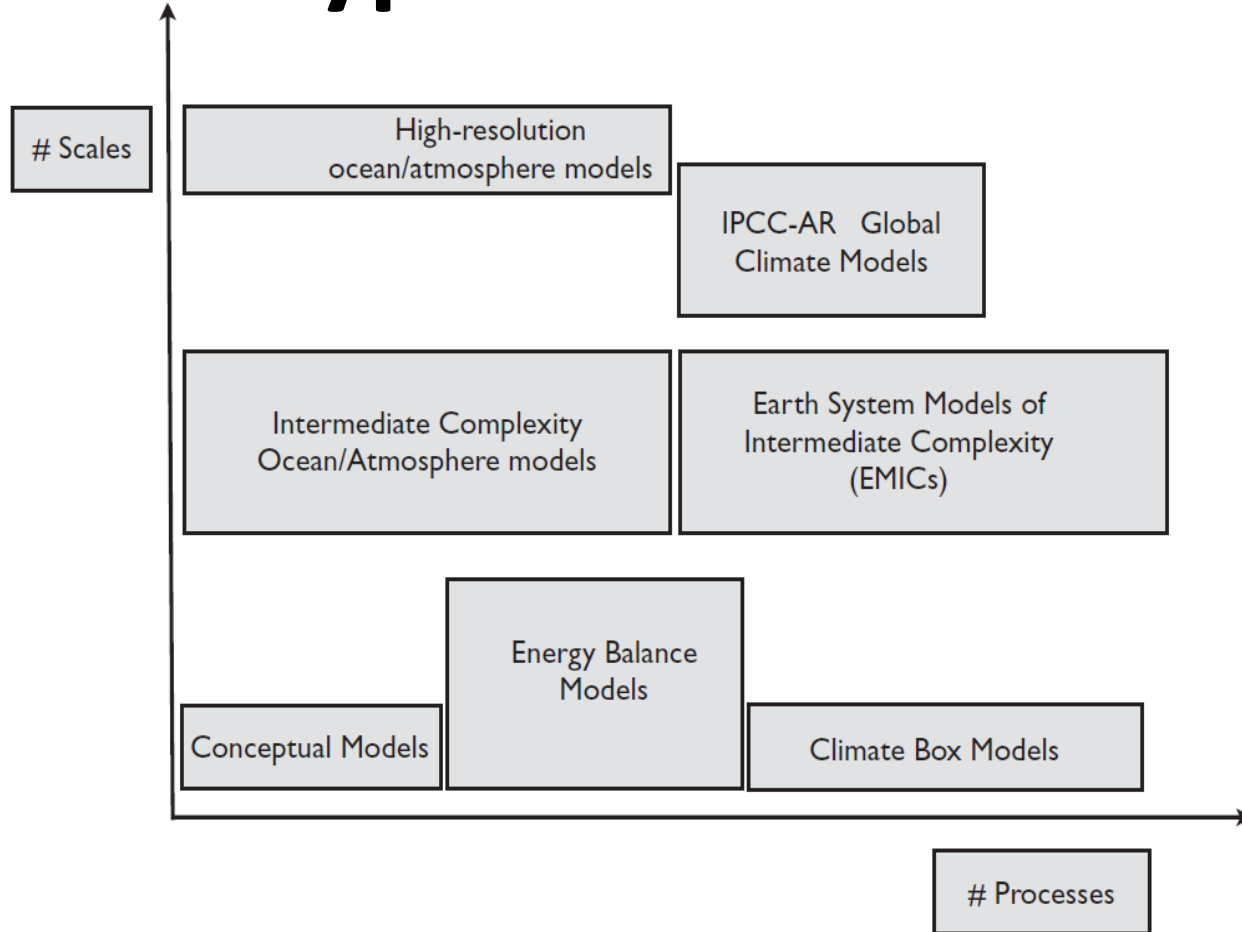
Wiskundige modellen

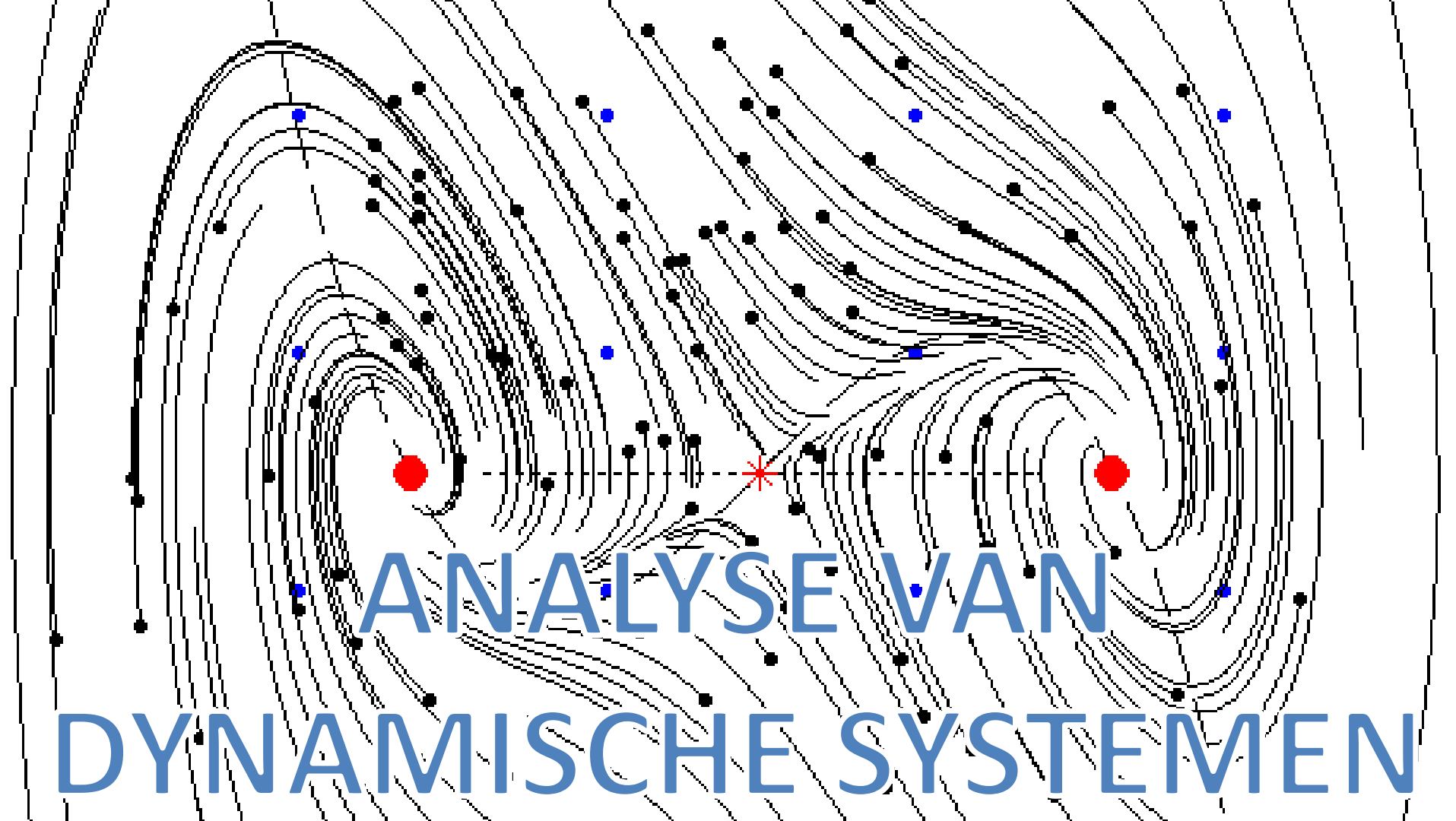
$\boldsymbol{x}$: toestand van het systeem

Dynamisch systeem:

$$\frac{d\boldsymbol{x}(t)}{dt} = f(\boldsymbol{x}(t), t)$$

Type modellen





Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x, t)$$

Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x, t)$$

f alleen afhankelijk van *t*

$$\frac{dx}{dt} = f(t)$$

$$\int_{x(0)}^{x(t)} dx' = \int_0^t f(t') dt'$$

$$\rightarrow x(t) = x(0) + \int_0^t f(t') dt'$$

Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x, t)$$

f alleen afhankelijk van t

f alleen afhankelijk van x

$$\frac{dx}{dt} = f(t)$$

$$\int_{x(0)}^{x(t)} dx' = \int_0^t f(t') dt'$$

$$\rightarrow x(t) = x(0) + \int_0^t f(t') dt'$$

$$\frac{dx}{dt} = f(x)$$

$$\int_{x(0)}^{x(t)} \frac{dx'}{f(x')} = \int_0^t dt'$$

$$\rightarrow x(t) = \dots ?$$

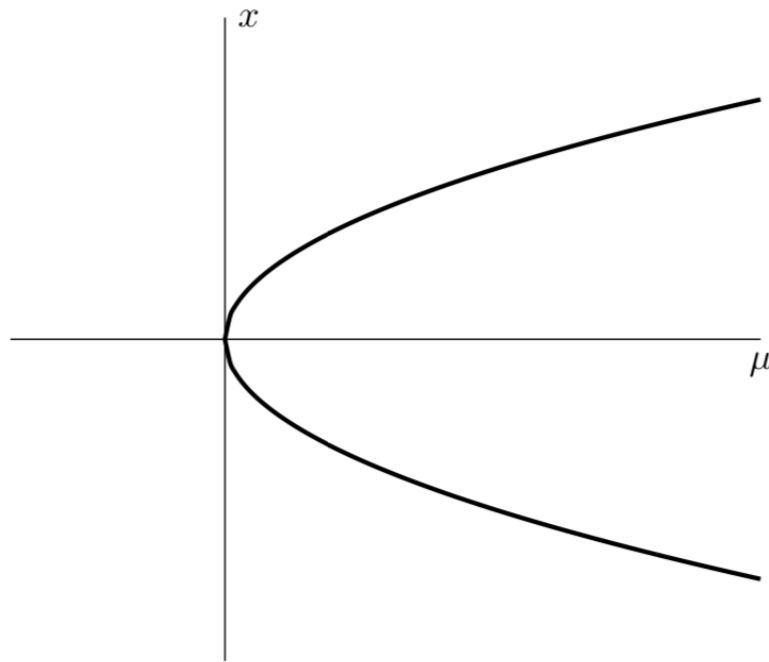
$$\frac{dx}{dt} = f(x) := \mu - x^2$$

$$\frac{dx}{dt} = f(x) := \mu - x^2$$

1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

$$\rightarrow x_* = \pm\sqrt{\mu}$$



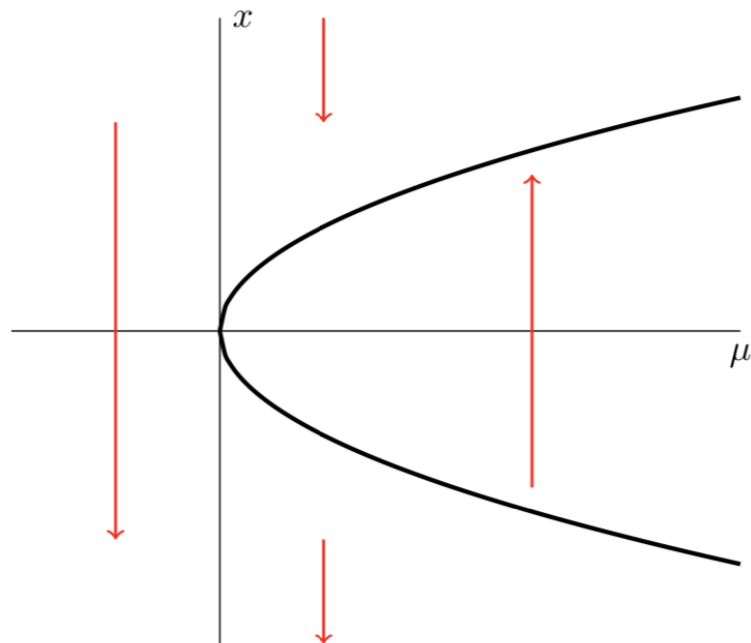
$$\frac{dx}{dt} = f(x) := \mu - x^2$$

1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

$$\rightarrow x_* = \pm\sqrt{\mu}$$

2. Stabiliteit



$$\frac{dx}{dt} = f(x) := \mu - x^2$$

1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

$$\rightarrow x_* = \pm\sqrt{\mu}$$

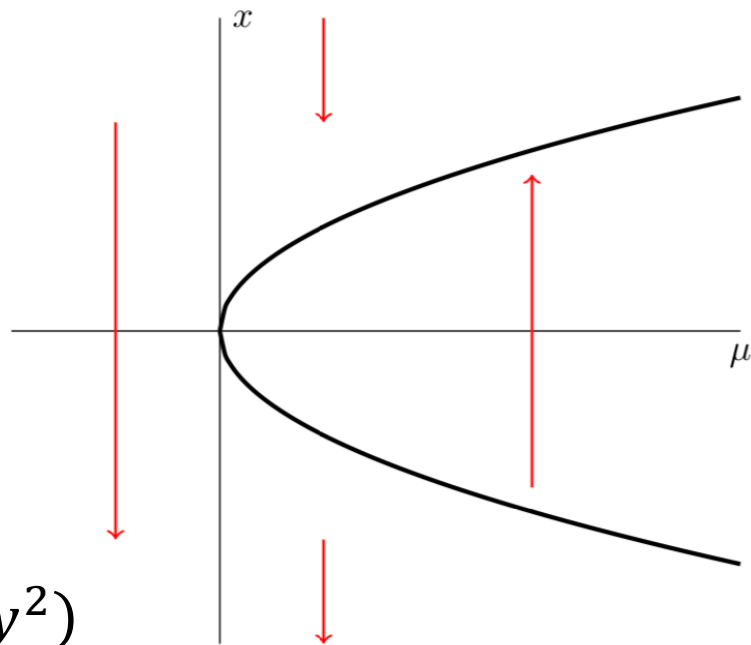
2. Stabiliteit

Kleine verstoring $x = x_* + y$

$$\rightarrow \frac{d(x_* + y)}{dt} = f(x_* + y)$$

$$\rightarrow \frac{dy}{dt} = f(x_*) + f'(x_*)y + \mathcal{O}(y^2)$$

$$\rightarrow \frac{dy}{dt} = f'(x_*)y$$



$$\frac{dx}{dt} = f(x) := \mu - x^2$$

1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

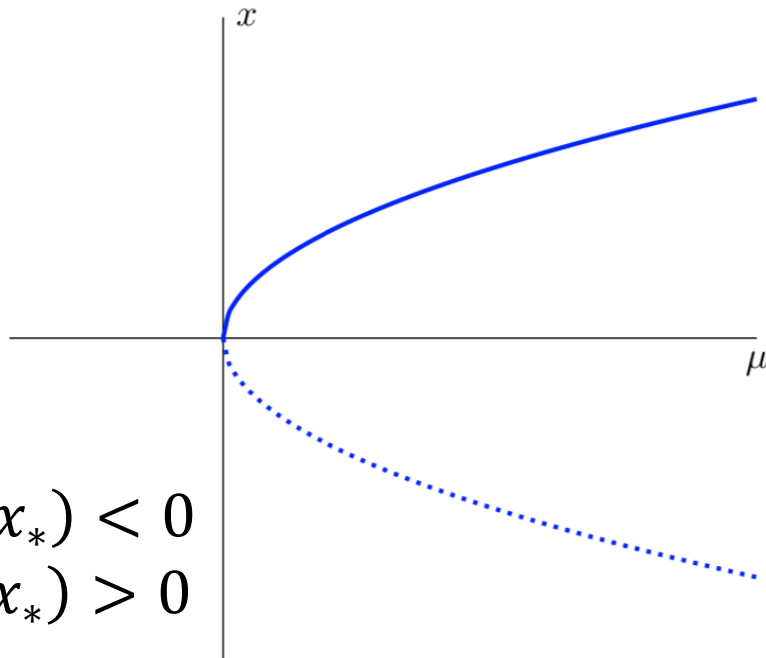
$$\rightarrow x_* = \pm\sqrt{\mu}$$

2. Stabiliteit

$$f'(x_*) = -2x_* = \mp 2\sqrt{\mu}$$

$$\text{Stabiel (aantrekkend)} \Leftrightarrow \lambda := f'(x_*) < 0$$

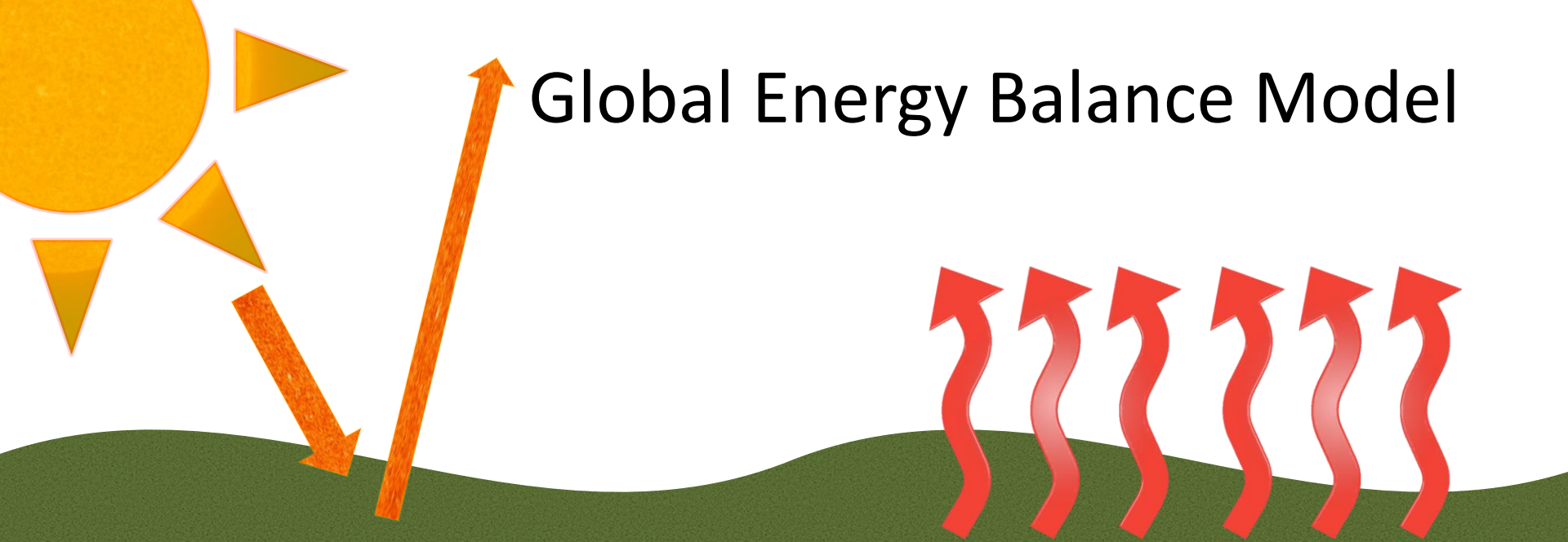
$$\text{Instabiel (afstotend)} \Leftrightarrow \lambda := f'(x_*) > 0$$





**VOORBEELD:
IJS-ALBEDO FEEDBACK**

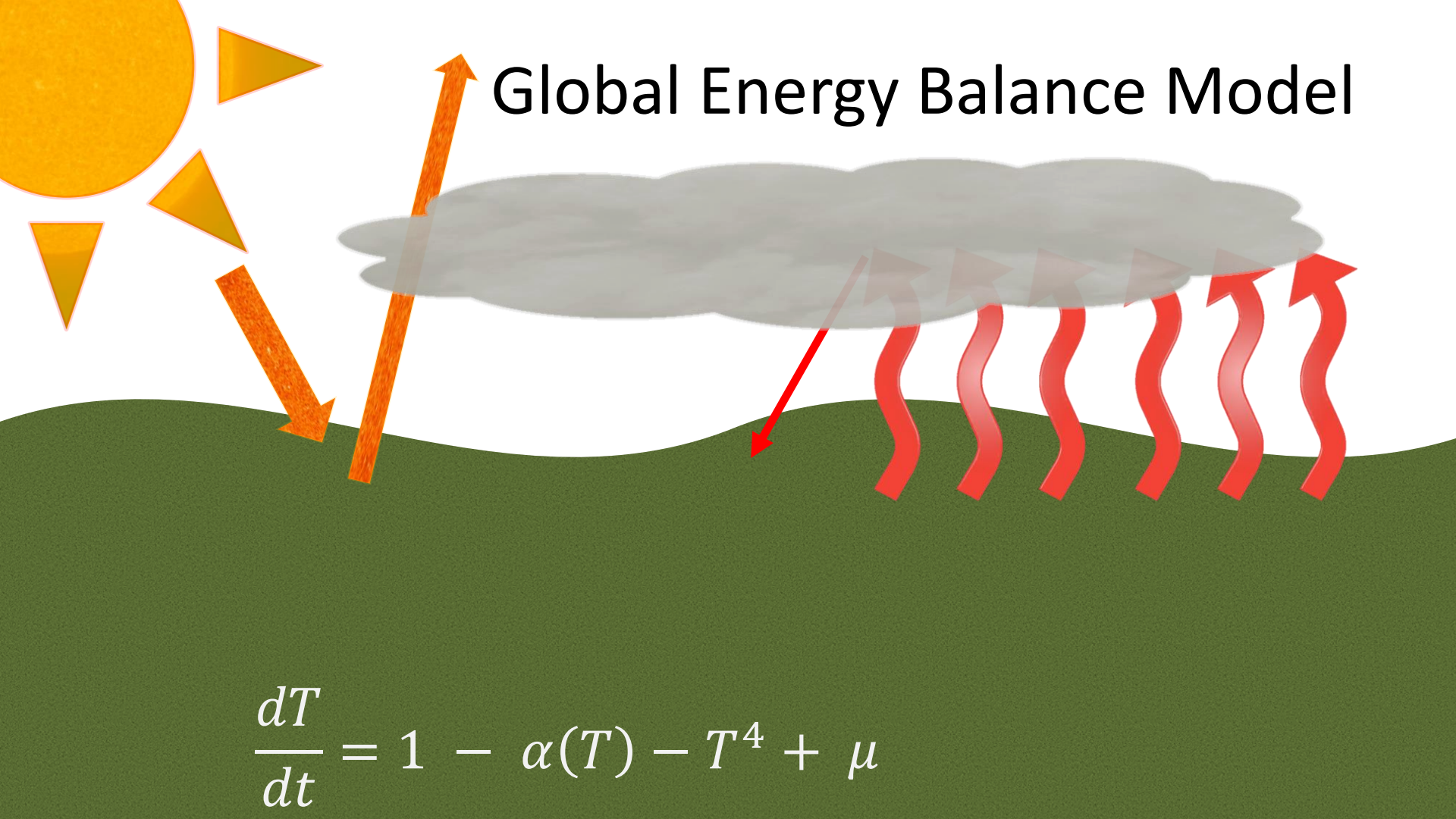
Global Energy Balance Model



opwarming = zonlicht – weerkaatst zonlicht – straling

$$\frac{dT}{dt} = 1 - \alpha(T) - T^4$$

Global Energy Balance Model

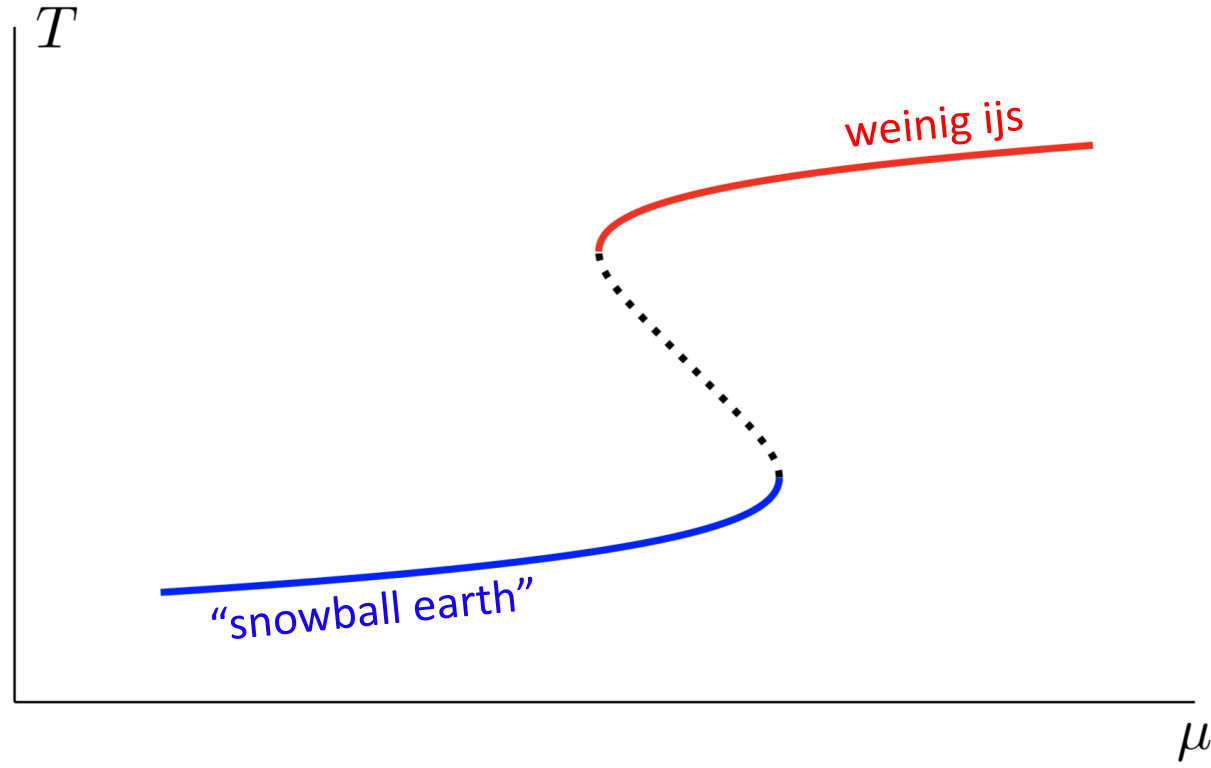


$$\frac{dT}{dt} = 1 - \alpha(T) - T^4 + \mu$$

$$\text{opwarming} = \frac{dT}{dt} = 1 - \alpha(T) - T^4 + \mu$$

Voor gegeven CO₂ (μ)
Hoe warm wordt het?

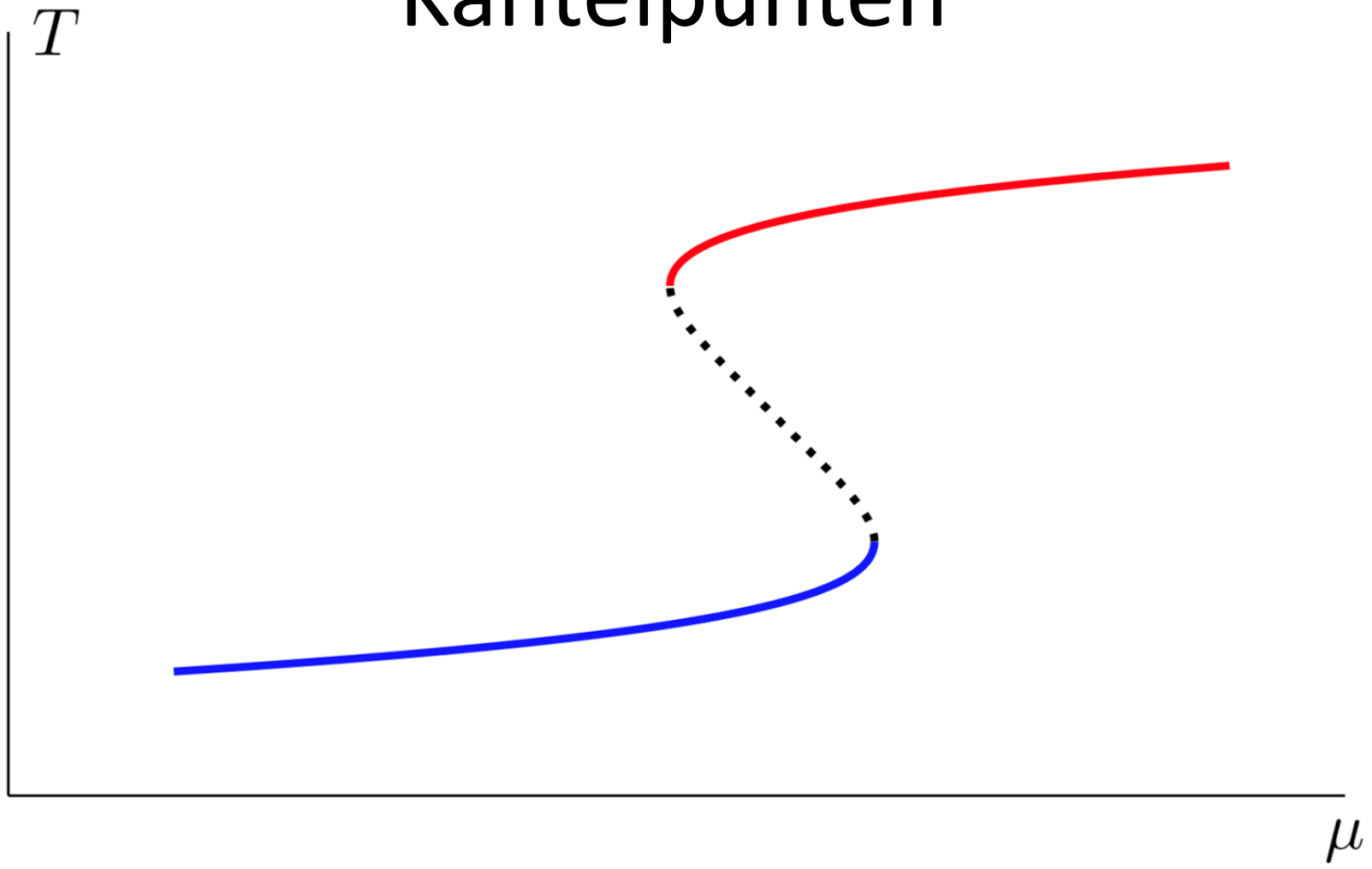
$$0 = 1 - \alpha(T) - T^4 + \mu$$



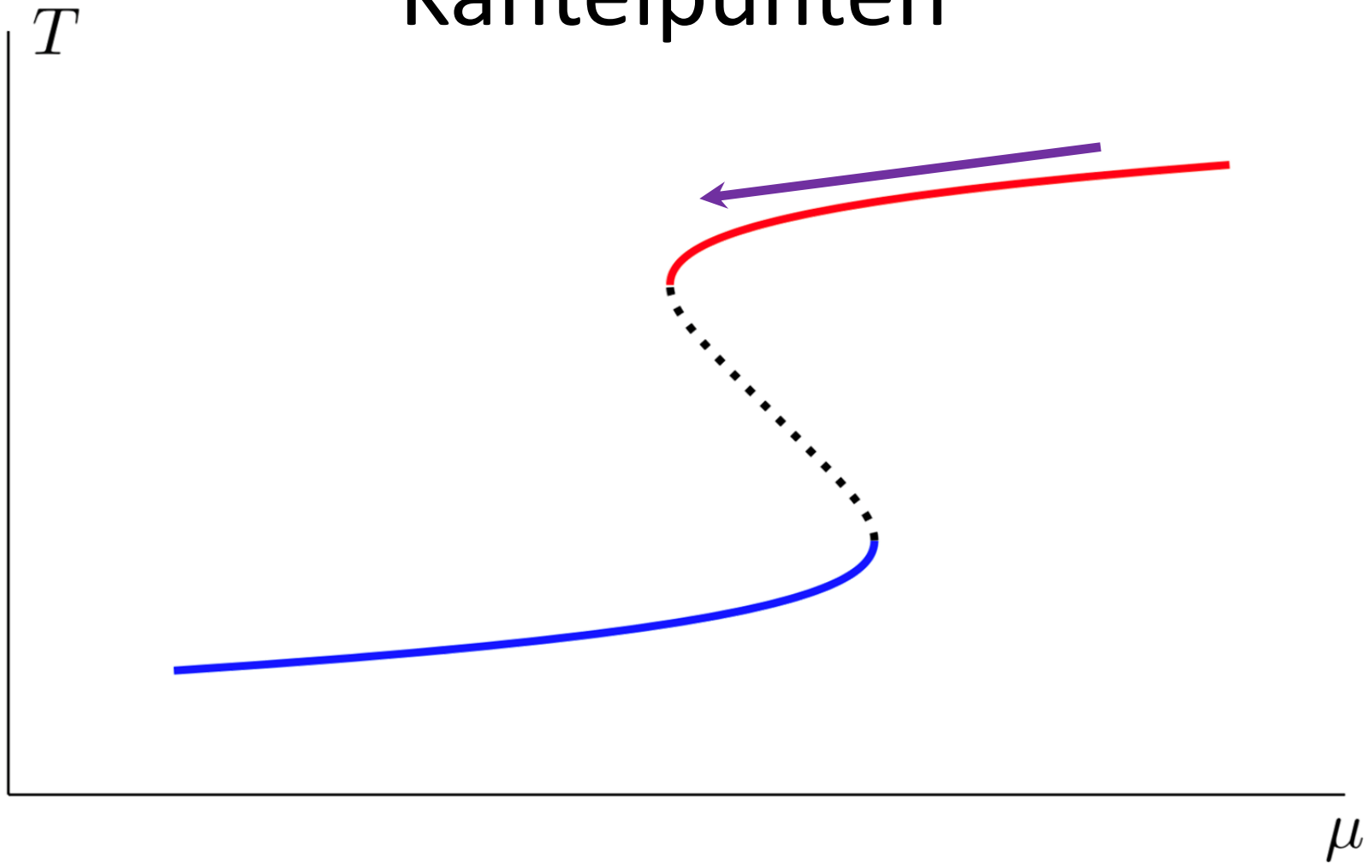
KANTELPUNTEN



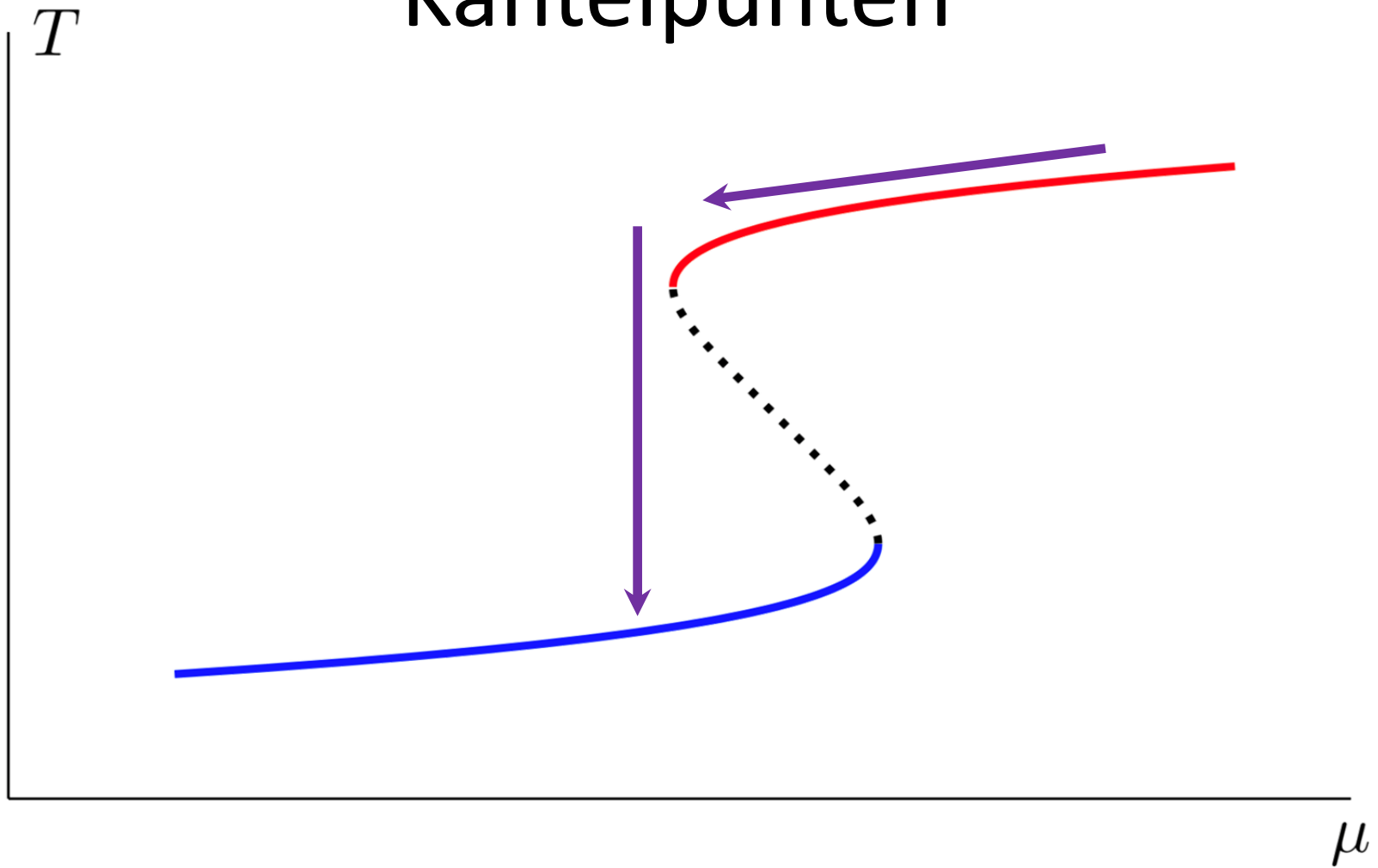
Kantelpunten



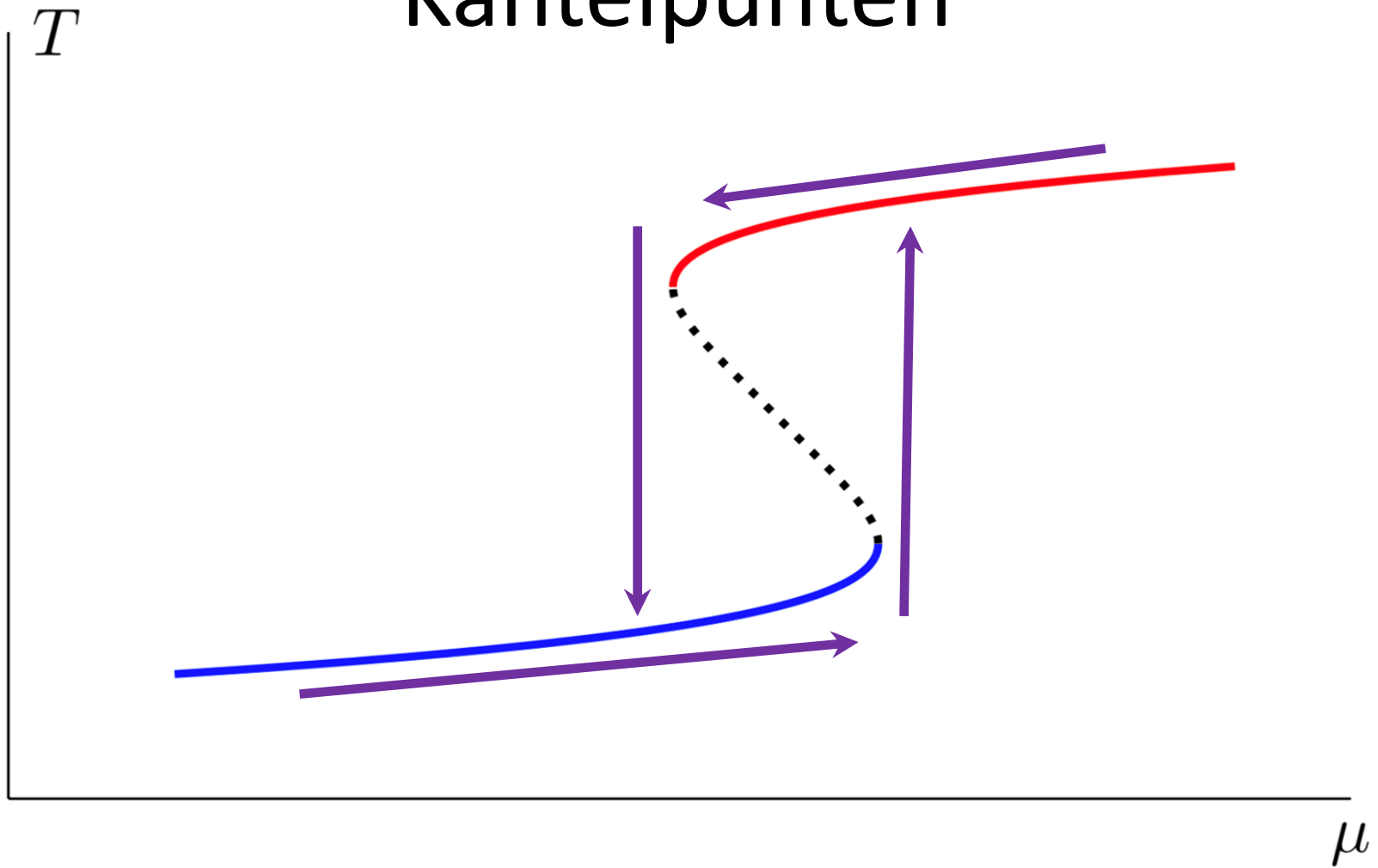
Kantelpunten



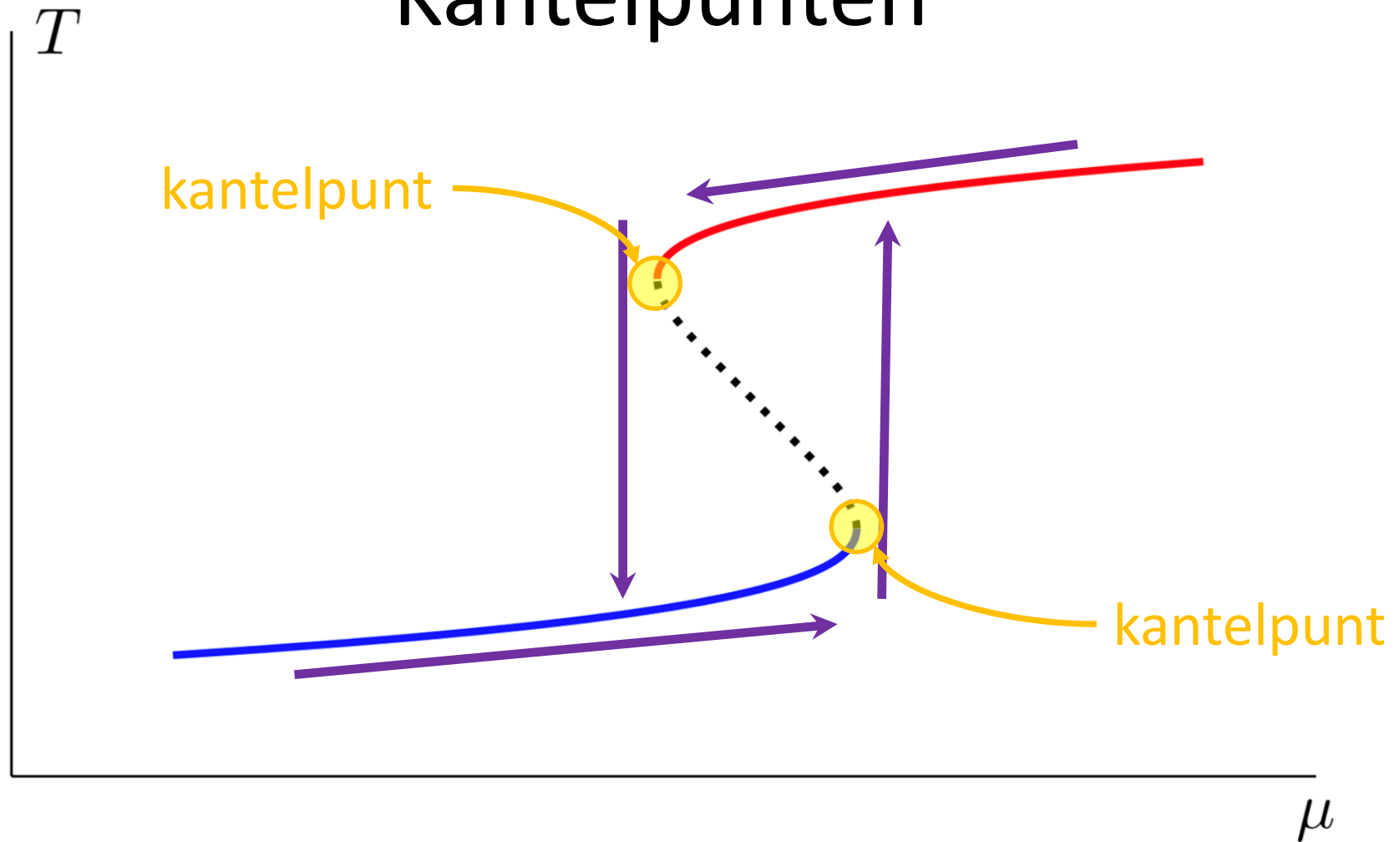
Kantelpunten



Kantelpunten



Kantelpunten





AAN DE SLAG MET VOORBEELDSYSTEMEN

Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf

Faseplaatjes

JAVA applicatie:

www.cs.unm.edu/~joel/dfield

PPLANE Equation Window

File Edit Window Gallery Help

System of Differential Equations of the form: $dx/dt = f(x,y)$, $dy/dt = g(x,y)$

μ =

y =

Parameter expressions:

eps =

The Display Window:

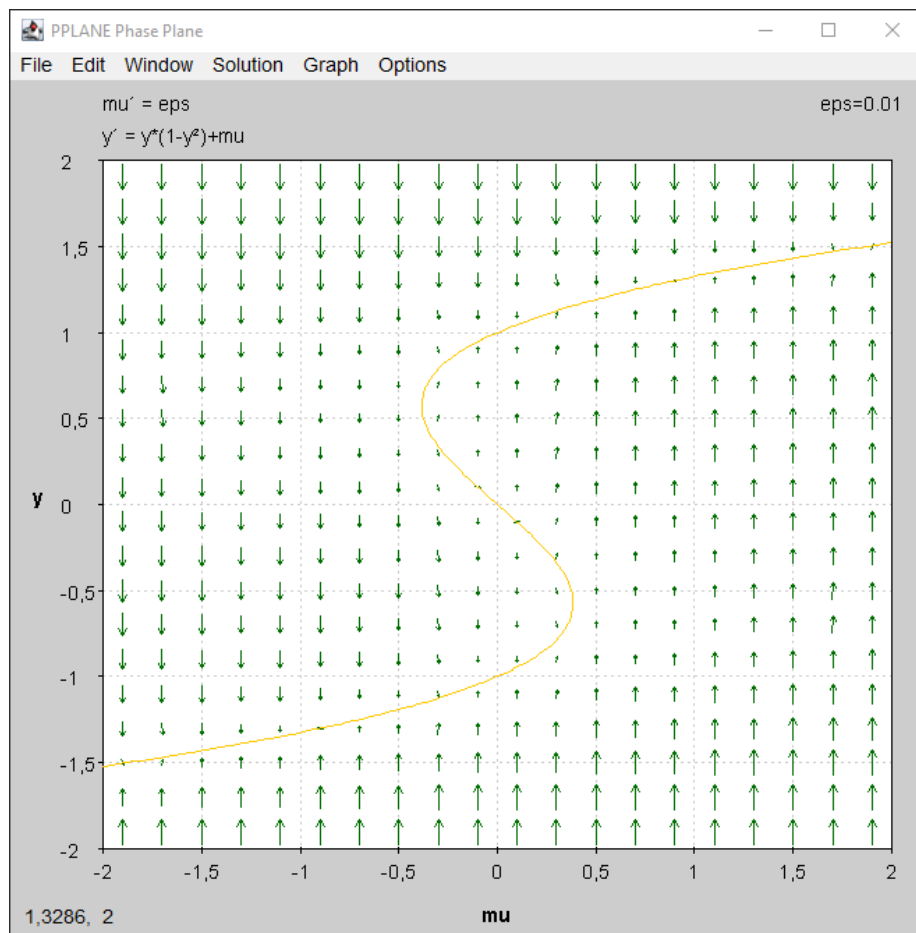
Minimum μ =

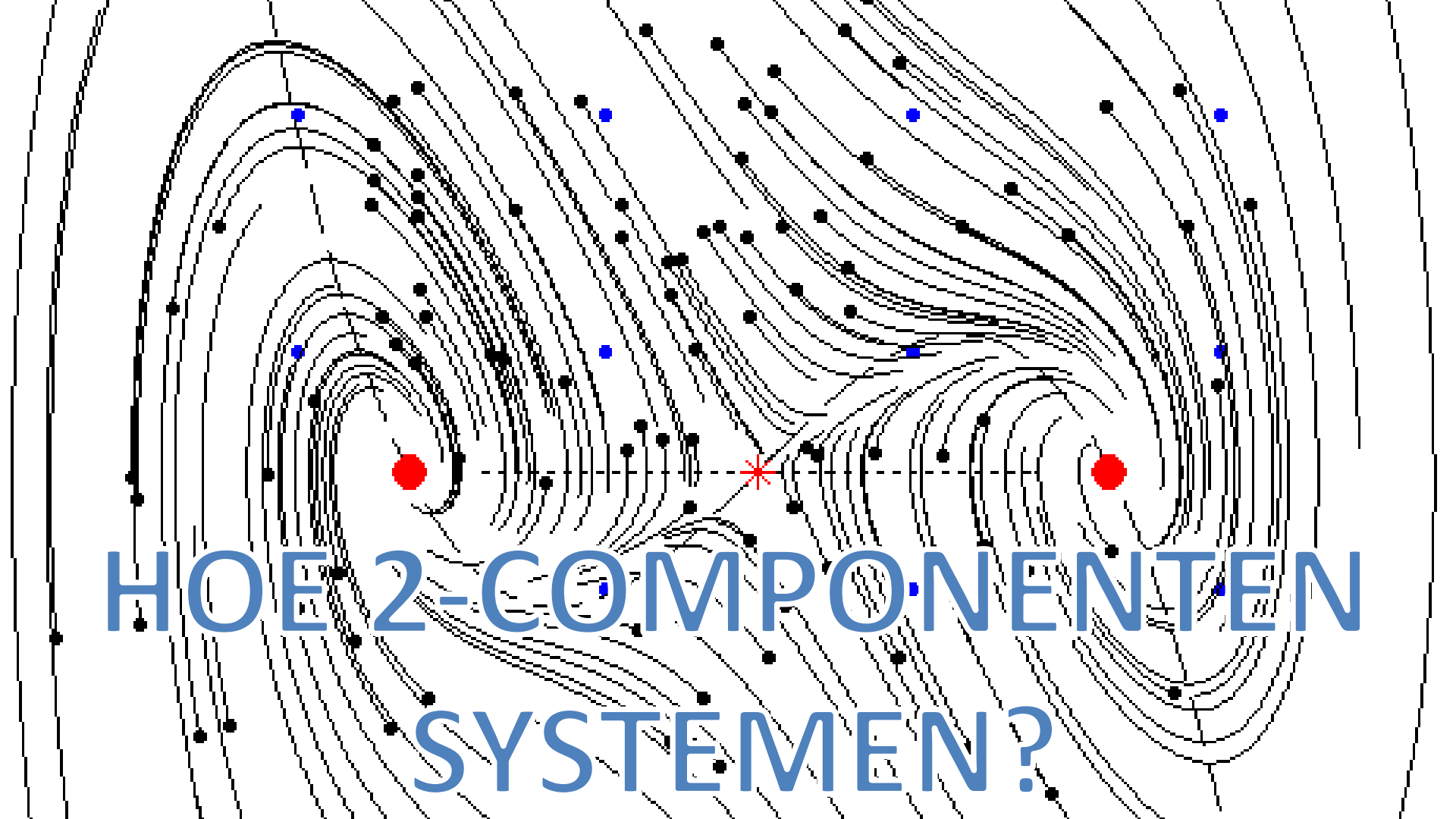
Maximum μ =

Minimum y =

Maximum y =

☐ Use current initial values in new graph





A contour plot illustrating a two-component system. The plot features two distinct peaks, each marked by a red dot, representing the centers of the components. A dashed horizontal line connects these two peaks, with a red asterisk marking the midpoint between them. Numerous black dots are scattered across the plot, following the contours of the peaks. Several blue dots are also present, primarily located in the regions between the two main peaks and along the outer edges of the contours. The text "HOE 2-COMPONENTEN SYSTEMEN?" is overlaid in large blue letters at the bottom of the image.

HOE 2-COMPONENTEN
SYSTEMEN?

1-componenten systemen

$$\frac{dx}{dt} = f(x; \mu)$$

Evenwichten: $0 = f(x_*; \mu)$

Stabiliteit:

$$\lambda := f'(x; \mu)$$

Als $\lambda < 0$: stabiel

Als $\lambda > 0$: instabiel

Anders: onduidelijk

N-componenten systemen

$$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x}; \mu)$$

Evenwichten: $0 = \vec{f}(\vec{x}_*; \mu)$
Nu stelsel van vergelijkingen

Stabiliteit:

Vind de eigenwaardes λ van $D\vec{f}(\vec{x}_*; \mu)$

Als $Re(\lambda) < 0$ voor allemaal: stabiel

Als $Re(\lambda) > 0$ voor minstens één: instabiel

Anders: onduidelijk



AAN DE SLAG MET VOORBEELDSYSTEMEN

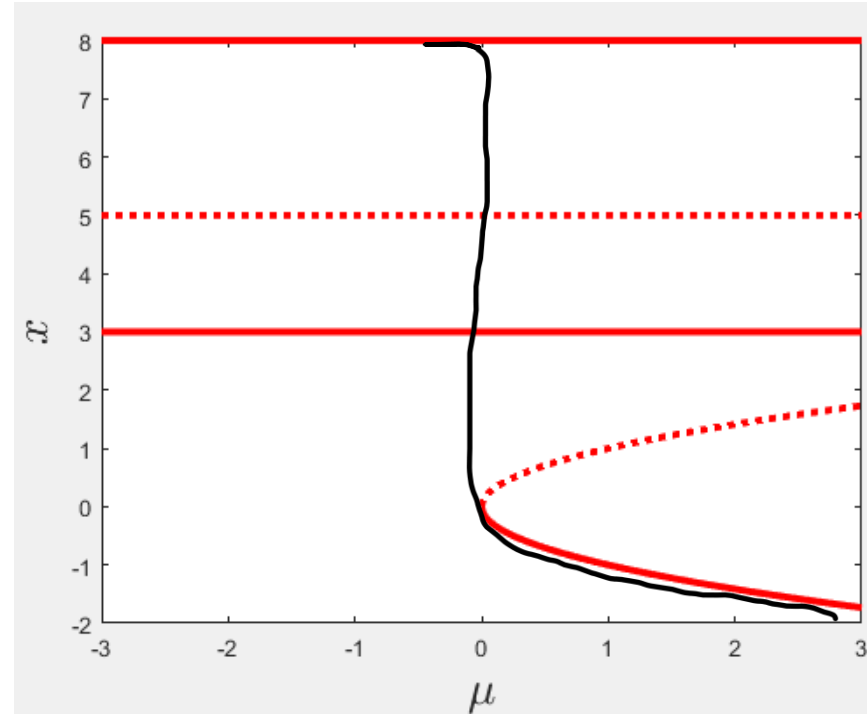
Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf



2-COMPONENTEN: WAT NA BIFURCATIE?

Let op: wat ná een bifurcatie?

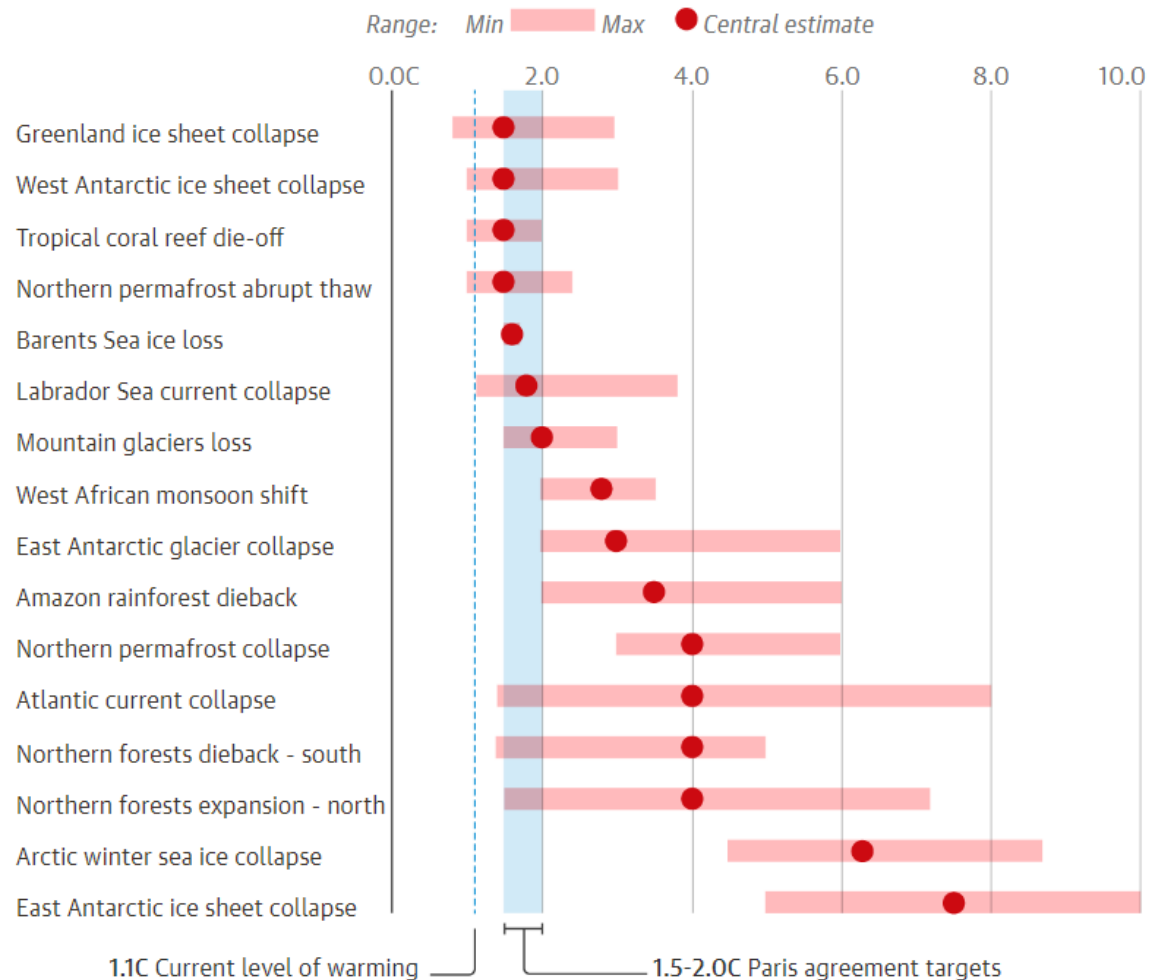
$$\begin{cases} x' = -(x^2 + \mu) ((x - 4)^2 + y + 1) (x - 8) \\ y' = -y (x - 3)(x - 5) \end{cases}$$





KANTELPUNTEN VOORSPELLEN





Guardian graphic. Source: Armstrong McKay et al, Science, 2022. Note: Current global heating temperature rise 1.1C Paris agreement targets 1.5-2.0C

Range: Min Max Central estimate

0.0C

2.0

4.0

6.0

8.0

Greenland ice sheet collapse

West Antarctic ice sheet collapse

Tropical coral reef die-off

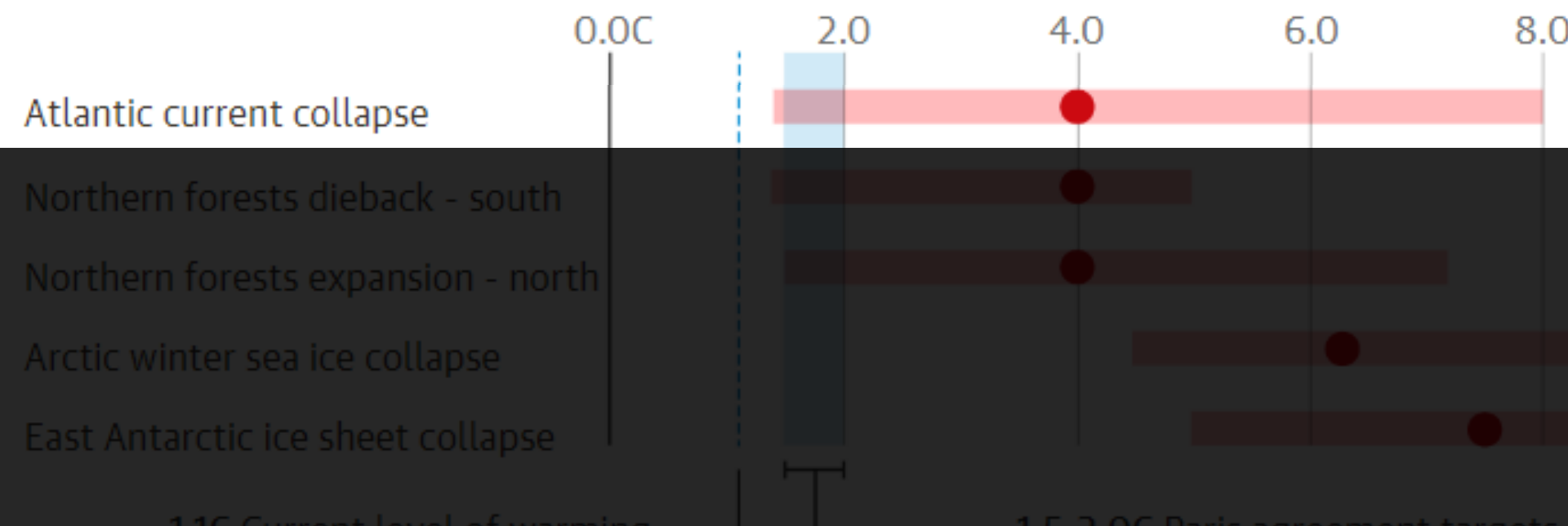
Northern permafrost abrupt thaw

Barents Sea ice loss

Labrador Sea current collapse



Range: Min Max Central estimate



Warme Golfstroom, van grote invloed
op ons klimaat, komt mogelijk nog
eerder tot stilstand: 'Heel
zorgwekkend'

Het Parool
26-07-2023

Gulf Stream could collapse as early as
2025, study suggests

The Guardian
25-07-2023

nature communications

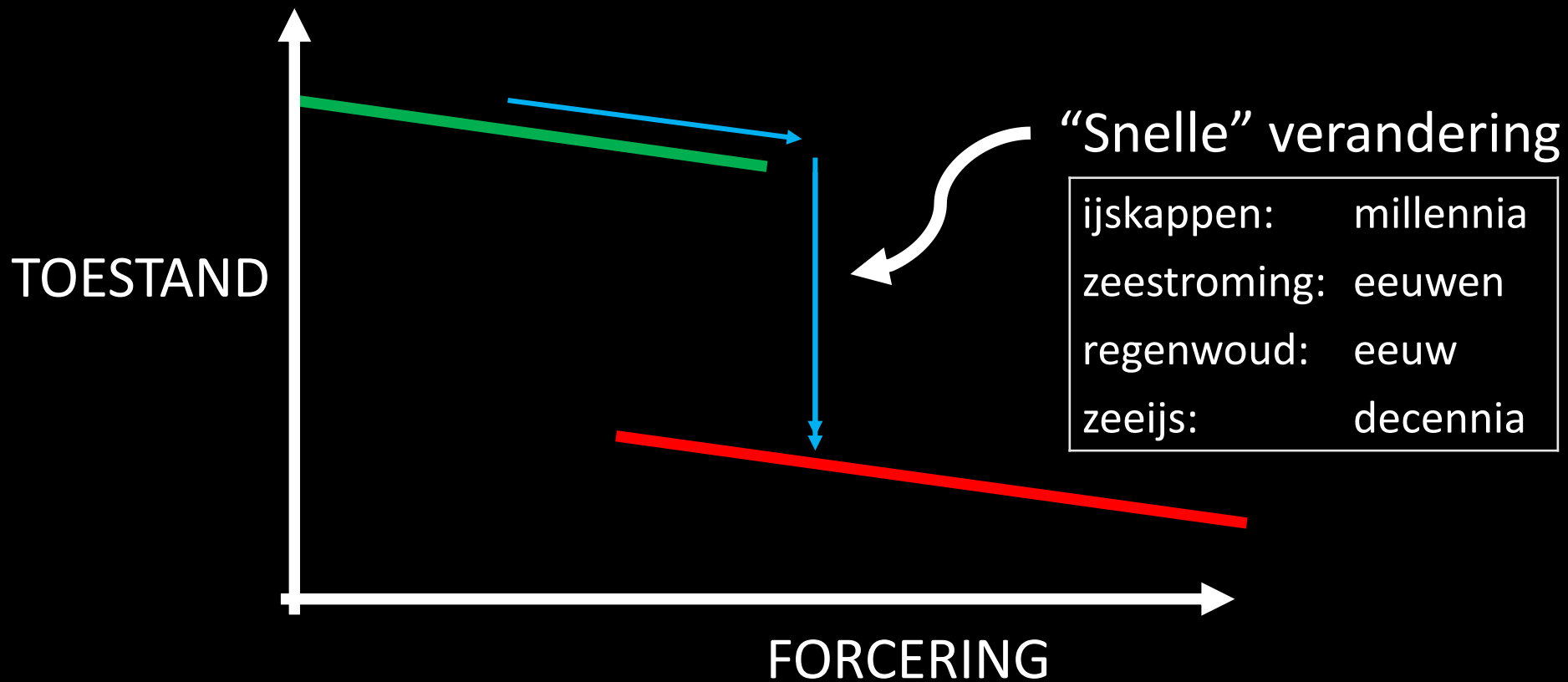


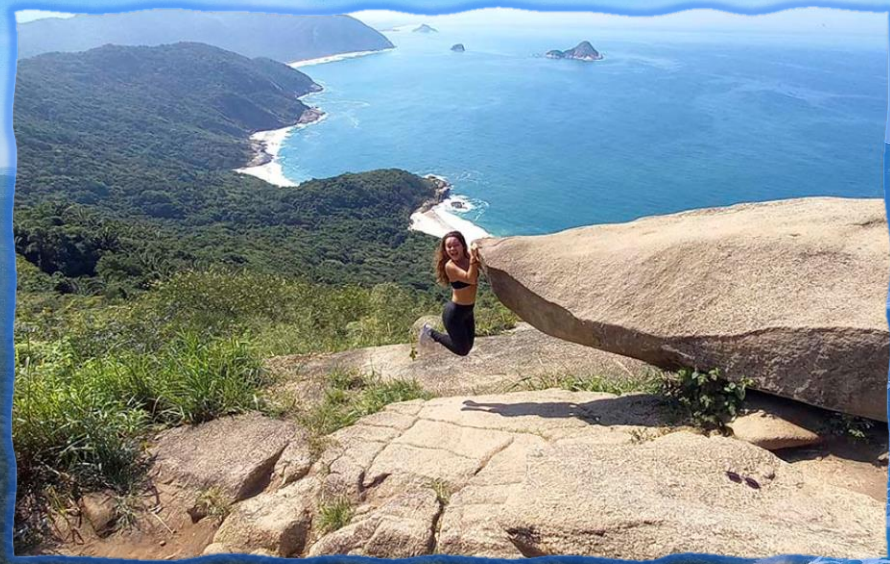
Article

<https://doi.org/10.1038/s41467-023-39810-w>

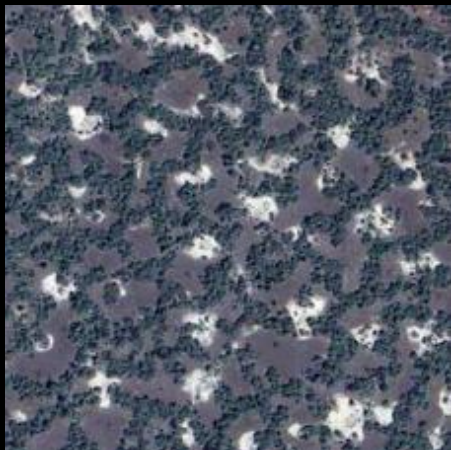
**Warning of a forthcoming collapse of the
Atlantic meridional overturning circulation**

[Ditlevsen and Ditlevsen, Nature Communications (2023)]

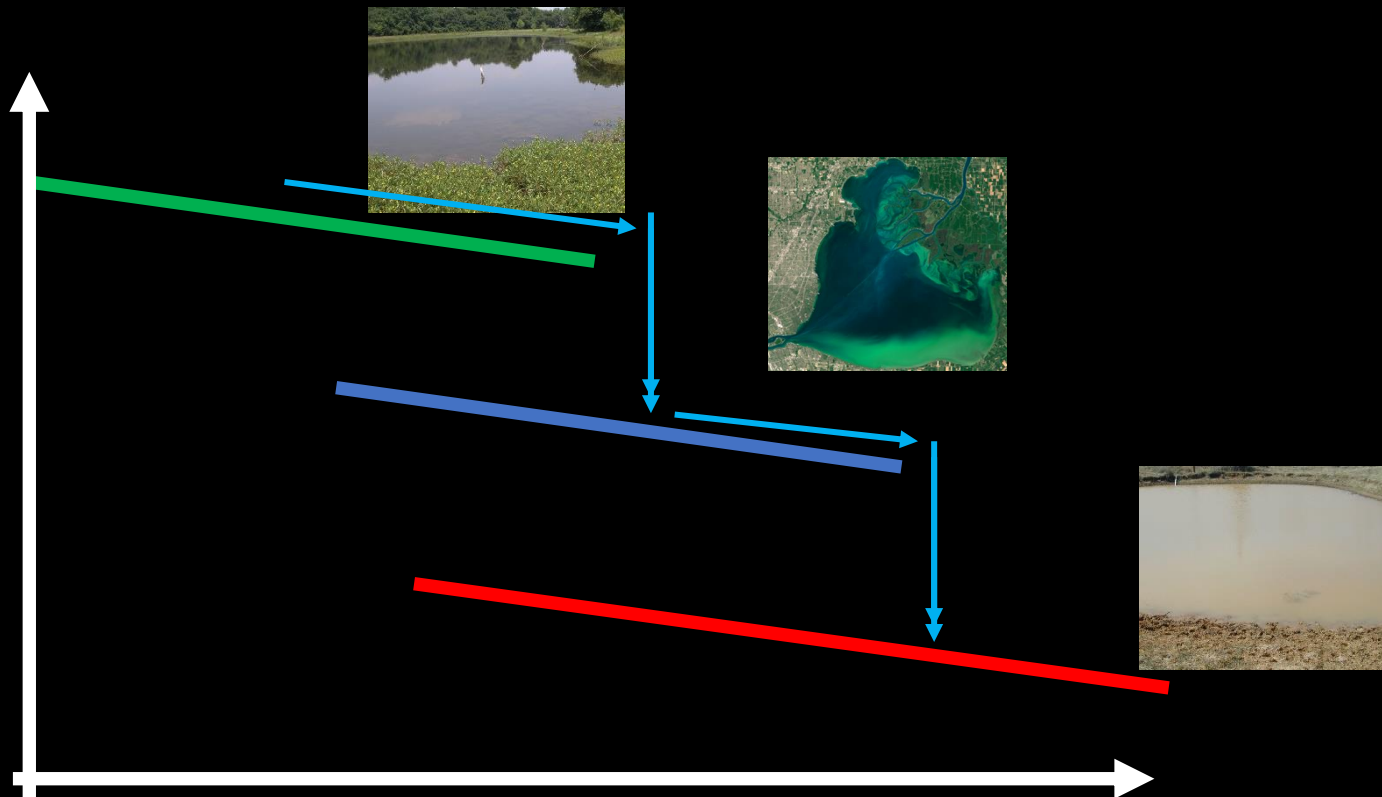




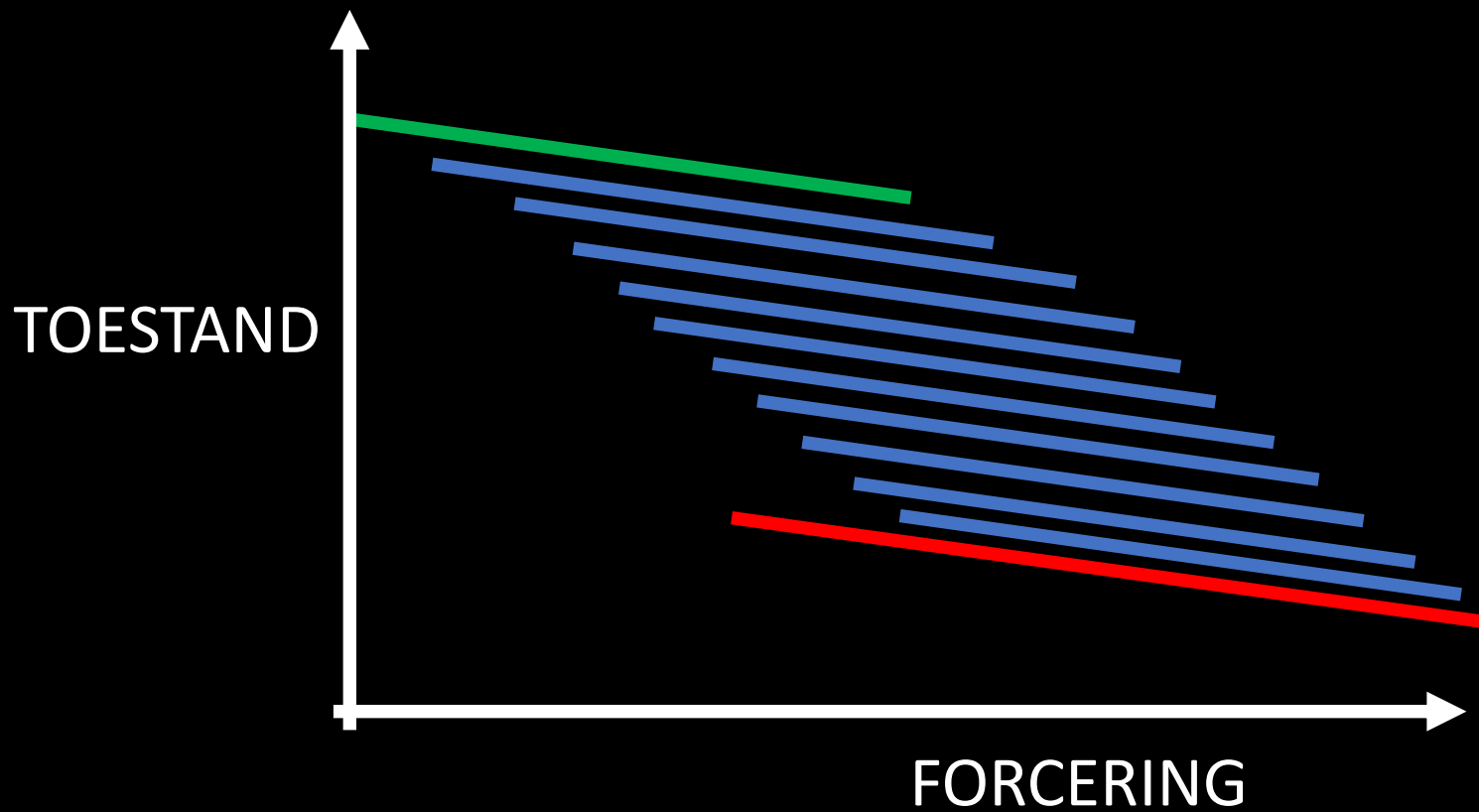




TOESTAND



FORCERING





gemengd



te krap



te snel

LINK MET WISKUNDE

TREFWOORDEN



Verdere wiskundige onderwerpen

- Dynamische systemen, differentiaalvergelijkingen
- Bifurcatietheorie, catastrophe theory
- Topologie
- Numeriek: simulaties (tijdsintegraties), continuaties/nulpunten/algoritmes
- Partiële differentiaalvergelijkingen
- Snelheid van verandering (niet-autonome systemen)

Slides: http://bastiaansen.github.io/talks/2025-04-14_WS-docenten.pdf

Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf

