Kantelpunten in het klimaat

dr. Robbin Bastiaansen

Docentennetwerk wiskunde (14 april 2025)

Klimaatconferentie Parijs 2015

Akkoord van Parijs:
→Beperk opwarming tot 2°C

Klimaatconferentie Parijs 2015

Akkoord van Parijs:
→Beperk opwarming tot 1.5°C


PLANNING

Planning

1. Context – klimaatverandering en modellen
2. Analyse van modellen - bifurcatiediagram
3. Voorbeelden klimaatmodellen
4. Kantelpunten
5. (Analyse van ingewikkeldere modellen)
6. Context – kantelpunten in het klimaat
7. Uitdagingen analyse kantelpunten
8. Mogelijke verdiepingen

slides: http://bastiaansen.github.io/talks/2025-04-14_WS-docenten.pdf

dr. Robbin Bastiaansen

Sinds 2022	Universitair Docent @	Mathematisch Intituut (MI) & Instituut voor Instituut voor Marien en Atmosferisch onderzoek Utrecht (IMAU), Universiteit Utrecht (UU)
2020-2022	Postdoc @	Instituut voor Marien en Atmosferisch onderzoek Utrecht (IMAU), Universiteit Utrecht (UU)
2015-2019	PhD @	Mathematisch Instituut (MI), Universiteit Leiden (UL)

Onderzoeksonderwerpen:

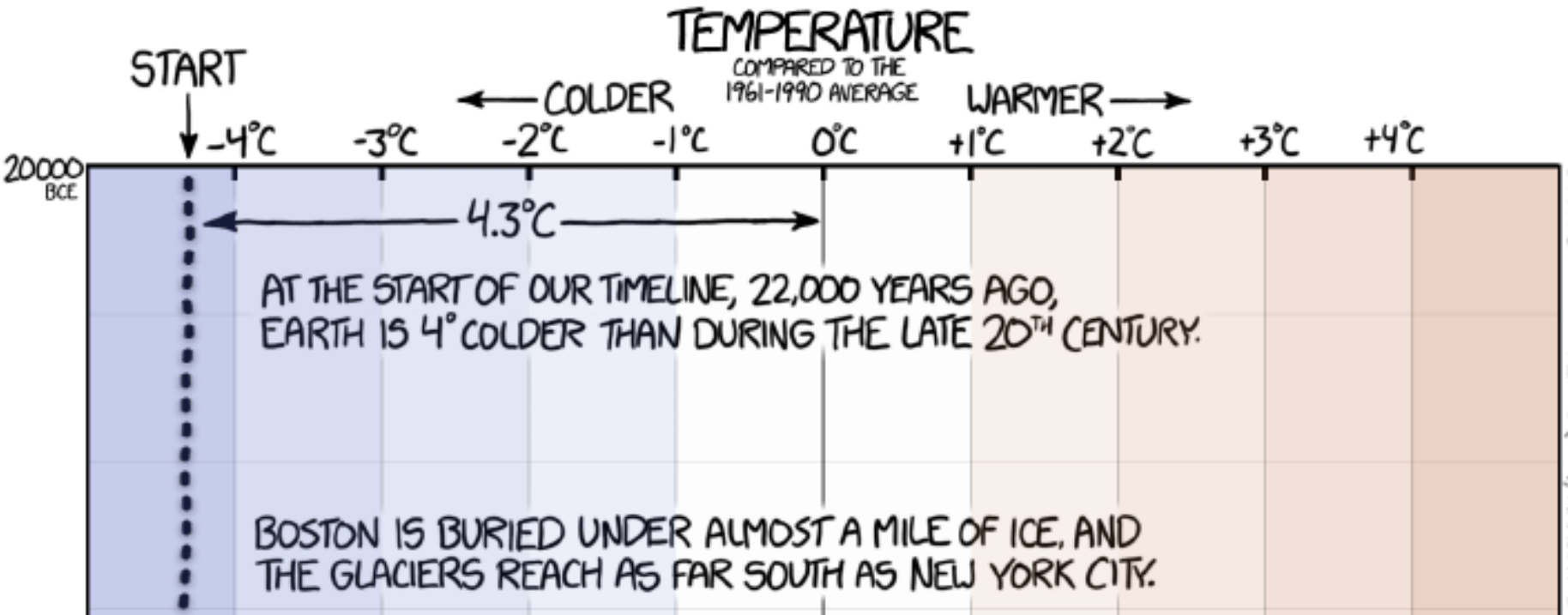
Tipping Points, Pattern Formation, Asymptotic Analysis, Response Theory, Climate Sensitivity, Climate Dynamics, Dynamical Systems, Mathematical Modelling, Bifurcation Theory, Differential Equations, Climate Change, Nonlinear Dynamics, Singular Perturbation Theory, Matched Asymptotics, Spatial Patterns, Complex Systems, Ecosystem Dynamics, Resilience, Fast-Slow dynamics


KLIMAATVERANDERING

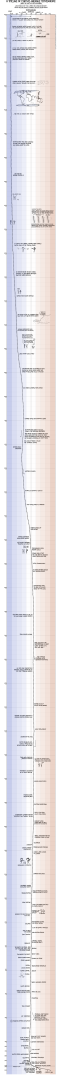
A TIMELINE OF EARTH'S AVERAGE TEMPERATURE

SINCE THE LAST ICE AGE GLACIATION

WHEN PEOPLE SAY "THE CLIMATE HAS CHANGED BEFORE,"
THESE ARE THE KINDS OF CHANGES THEY'RE TALKING ABOUT.



SOURCES: SHAKUN ET AL. (2012), MARCOTT ET



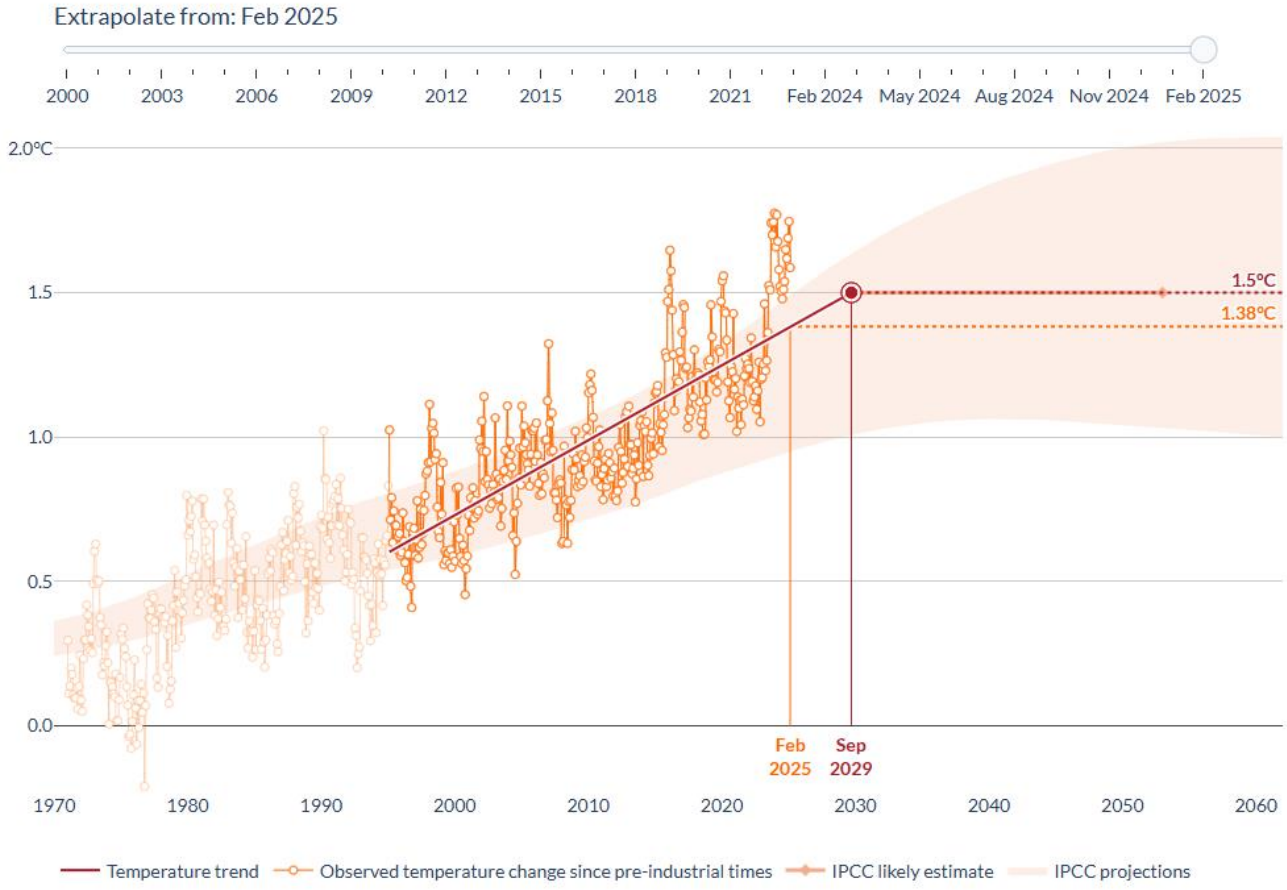
4 graden opwarming in meer dan 20 000 jaar

1 graad opwarming in 150 jaar

https://apps.climate.copernicus.eu/global-temperature-trend-monitor/

Global warming reached an estimated **1.38°C** in **February 2025**.

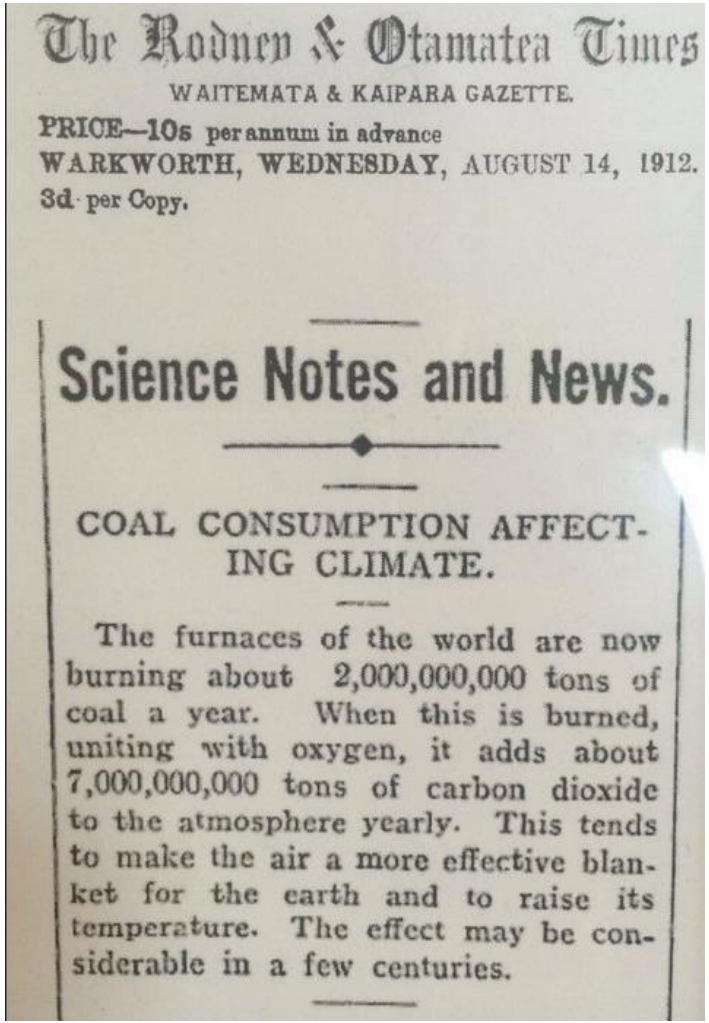
If the 30-year warming trend leading up to then continued, global warming would reach **1.5°C** by **September 2029**.




GEVOLGEN VAN KLIMAATVERANDERING

Voorspellingen

- Meer zware neerslag
- Langere perioden van droogte
- Meer en intensere bosbranden
- Zeespiegelstijging
- Verzilting van water
- Meer stormen en orkanen
- Hogere landbouwopbrengsten



The Rodney & Otamatea Times
WAITEMATA & KAIPARA GAZETTE.
PRICE—10s per annum in advance
WARKWORTH, WEDNESDAY, AUGUST 14, 1912.
3d per Copy.

Science Notes and News.

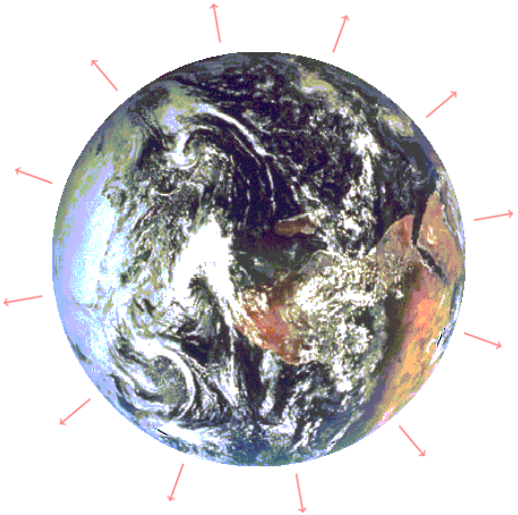
COAL CONSUMPTION AFFECTING CLIMATE.

The furnaces of the world are now burning about 2,000,000,000 tons of coal a year. When this is burned, uniting with oxygen, it adds about 7,000,000,000 tons of carbon dioxide to the atmosphere yearly. This tends to make the air a more effective blanket for the earth and to raise its temperature. The effect may be considerable in a few centuries.

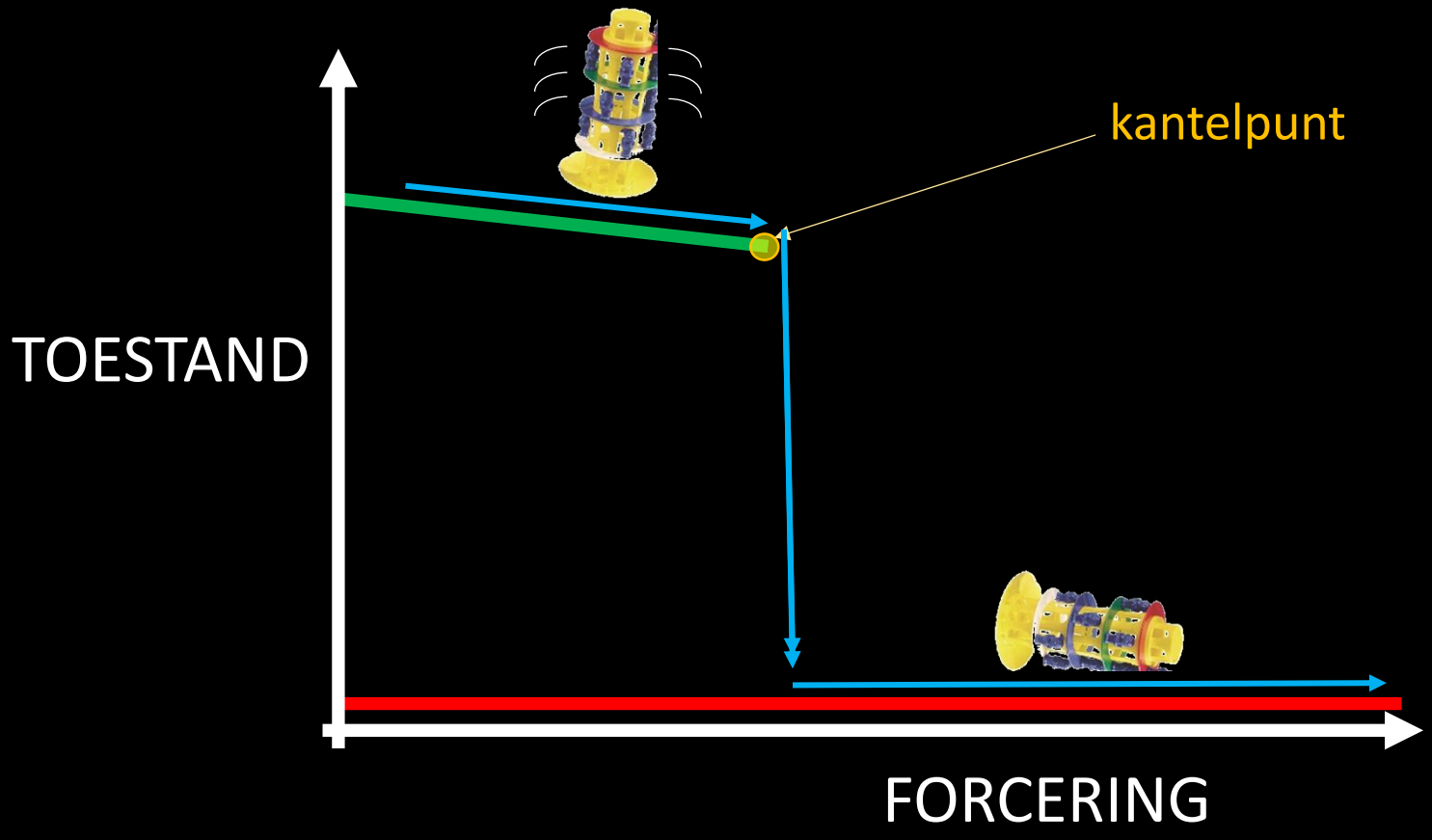
KERN VAN DE MOEILIJKHEID

Niet-lineaire terugkoppeling

- veel ijs/sneeuw
- veel weerkaatsing zonlicht

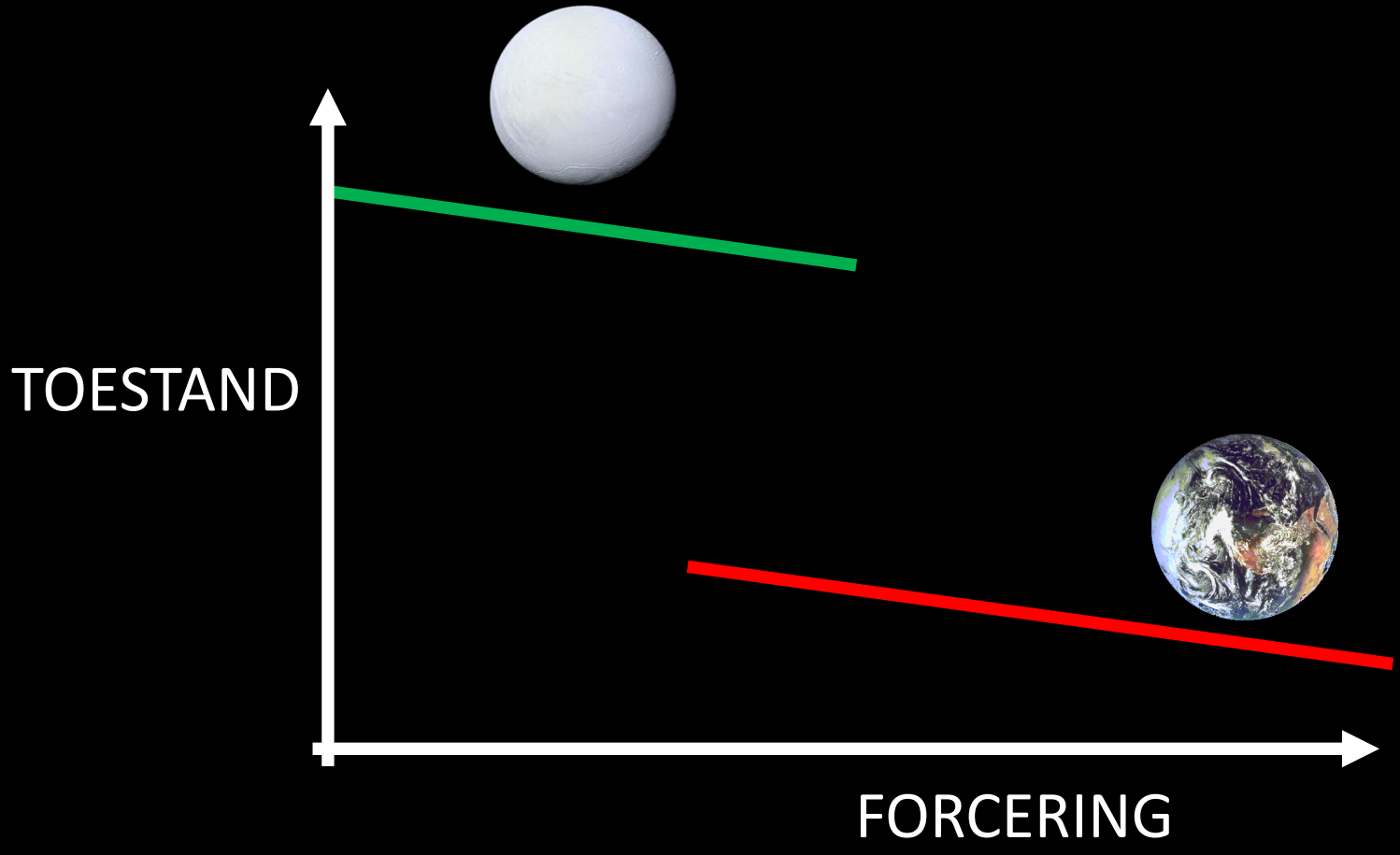


- weinig ijs/sneeuw
- weinig weerkaatsing zonlicht


kantelpunt
TOESTAND
FORCERING

TOESTAND

FORCERING


TOESTAND
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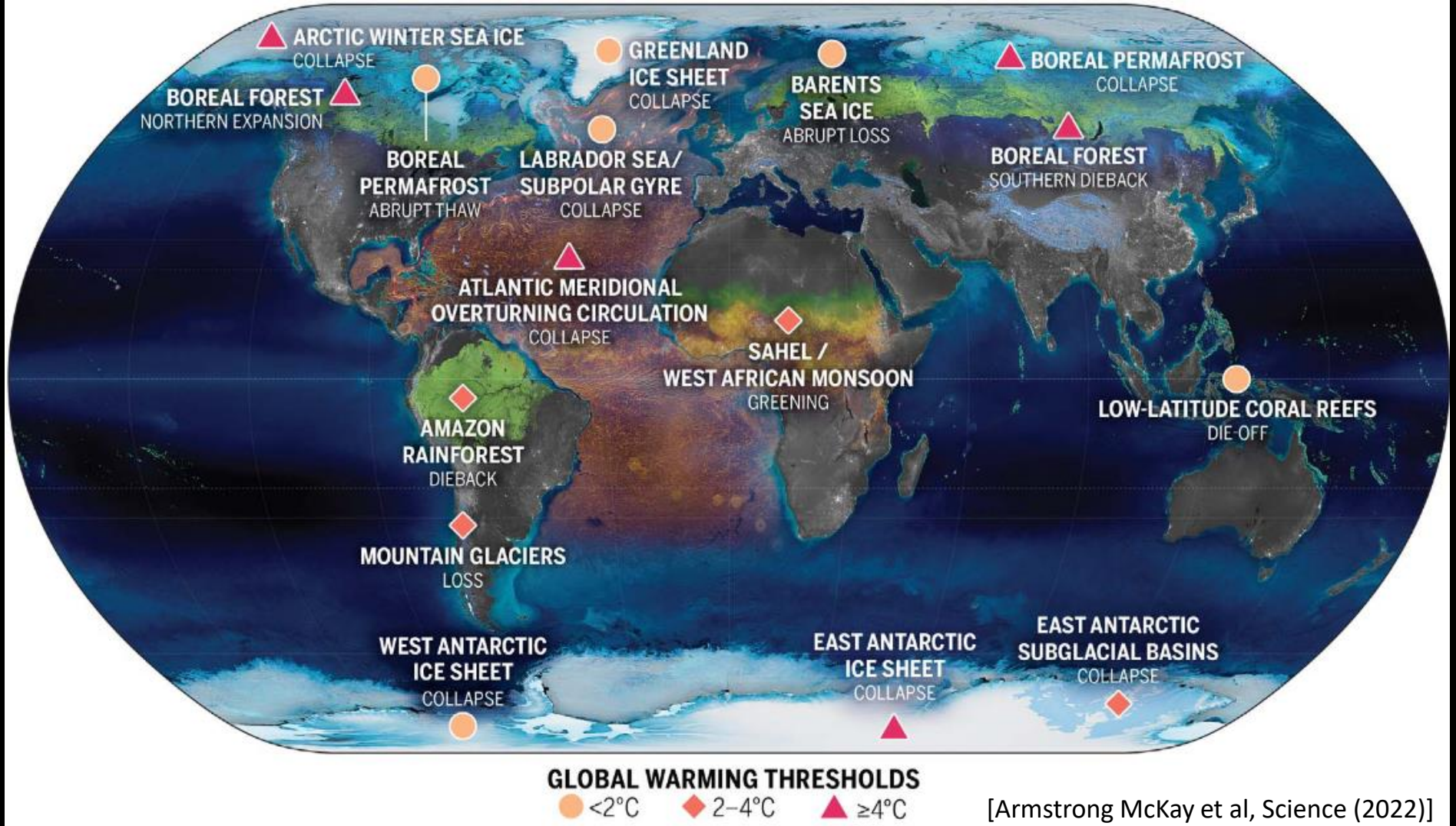
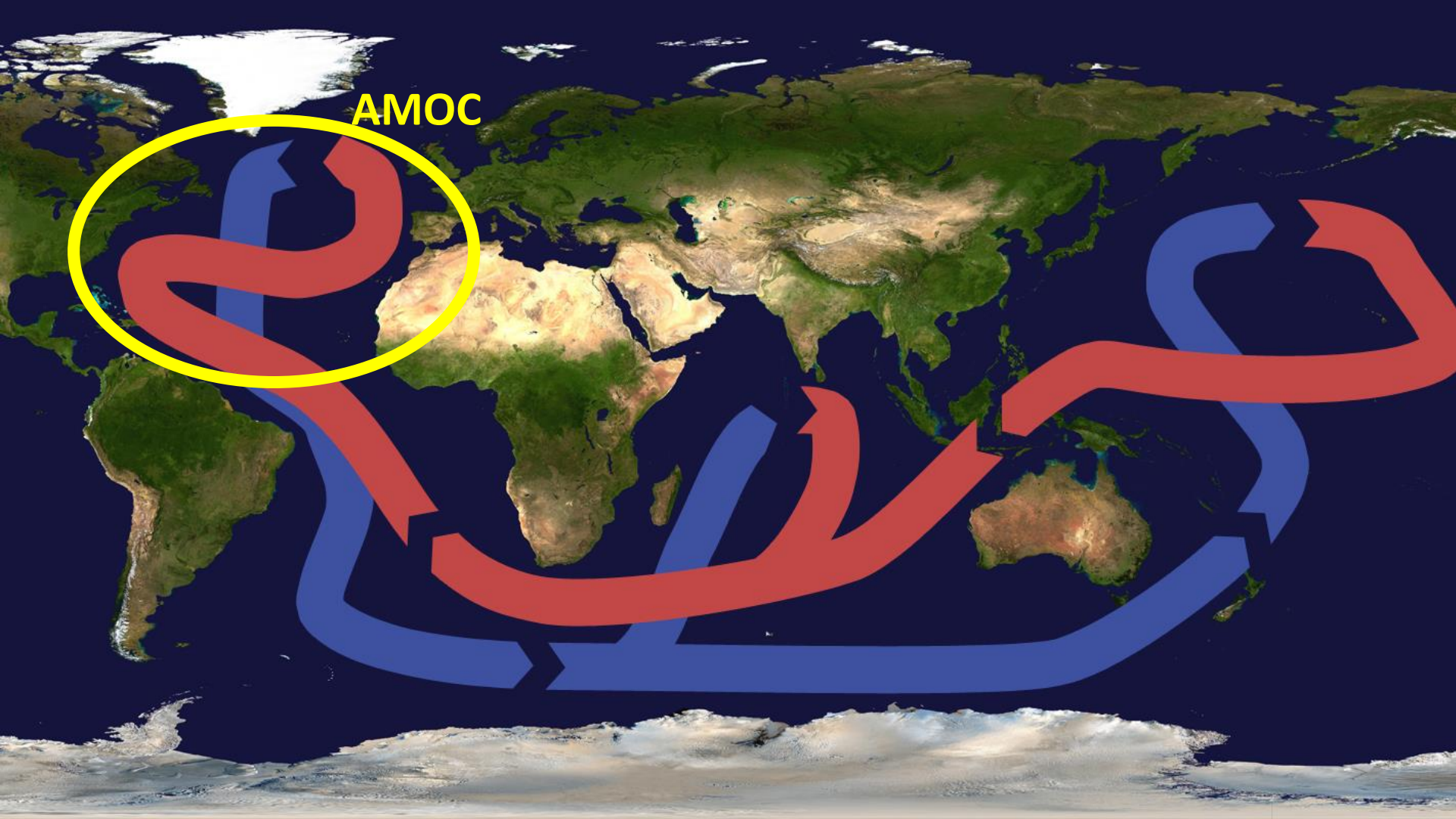
regenwoud ↔ savanne

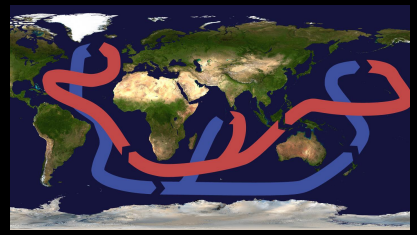
uitsterven diersoorten

verwoestijning



[Armstrong McKay et al, Science (2022)]


AMOC



AMOC on

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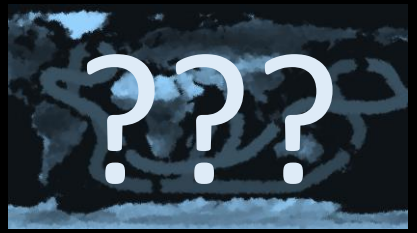

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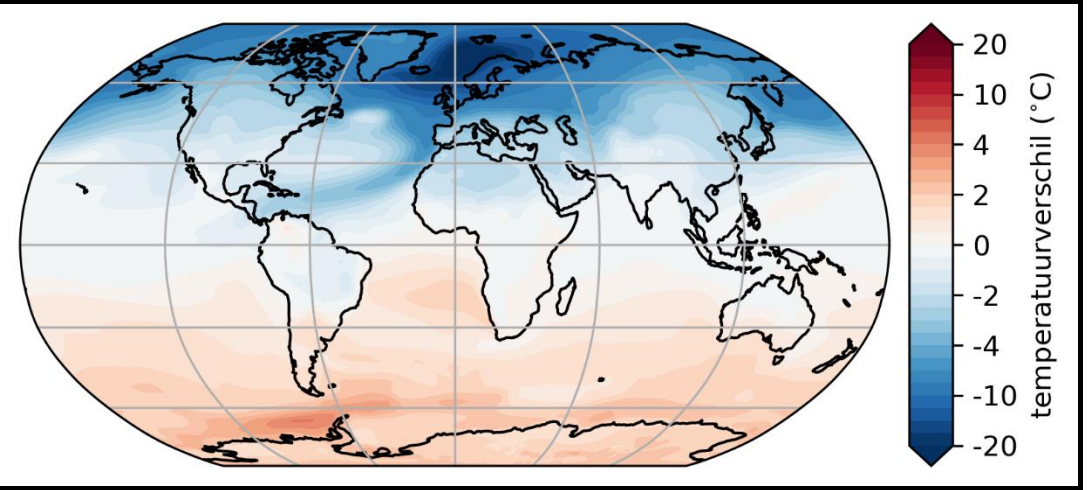
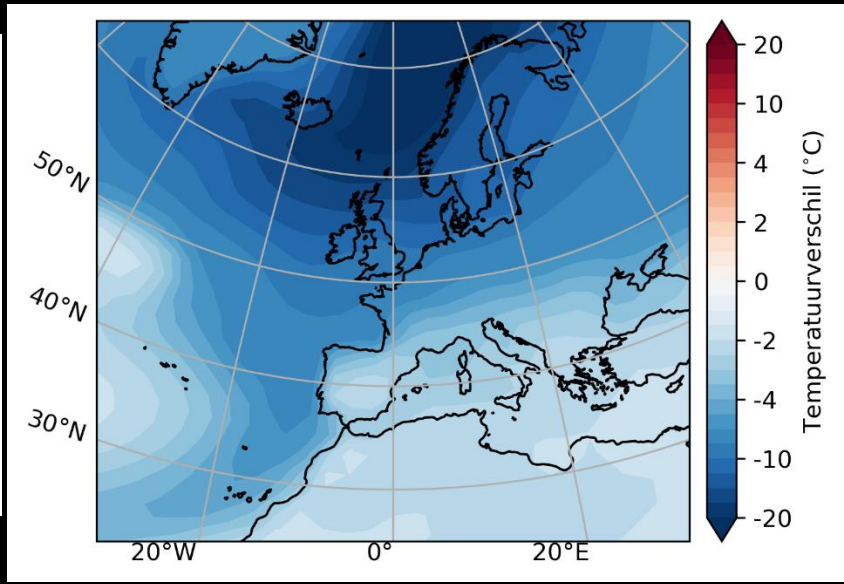
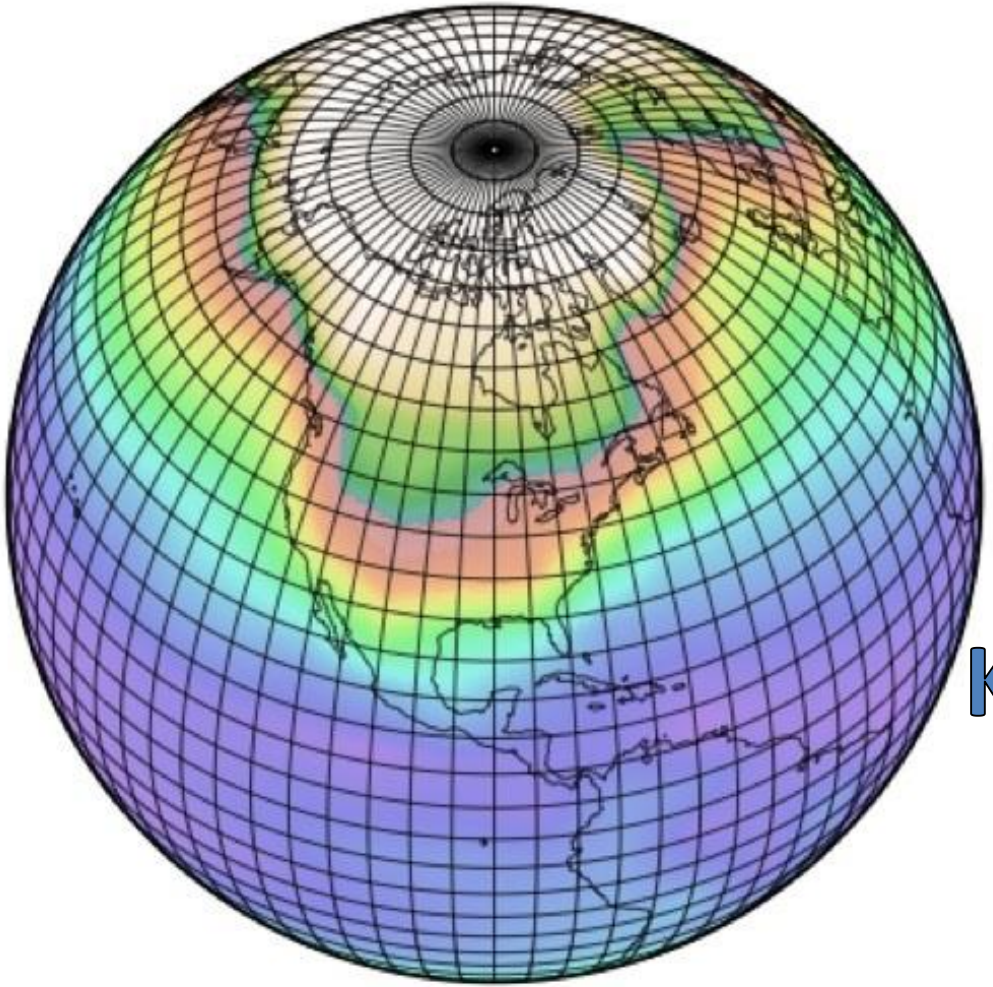
THE DAY AFTER TOMORROW





[Van Westen et al, Science Advances. (2024)]



KLIMAATMODELLEN

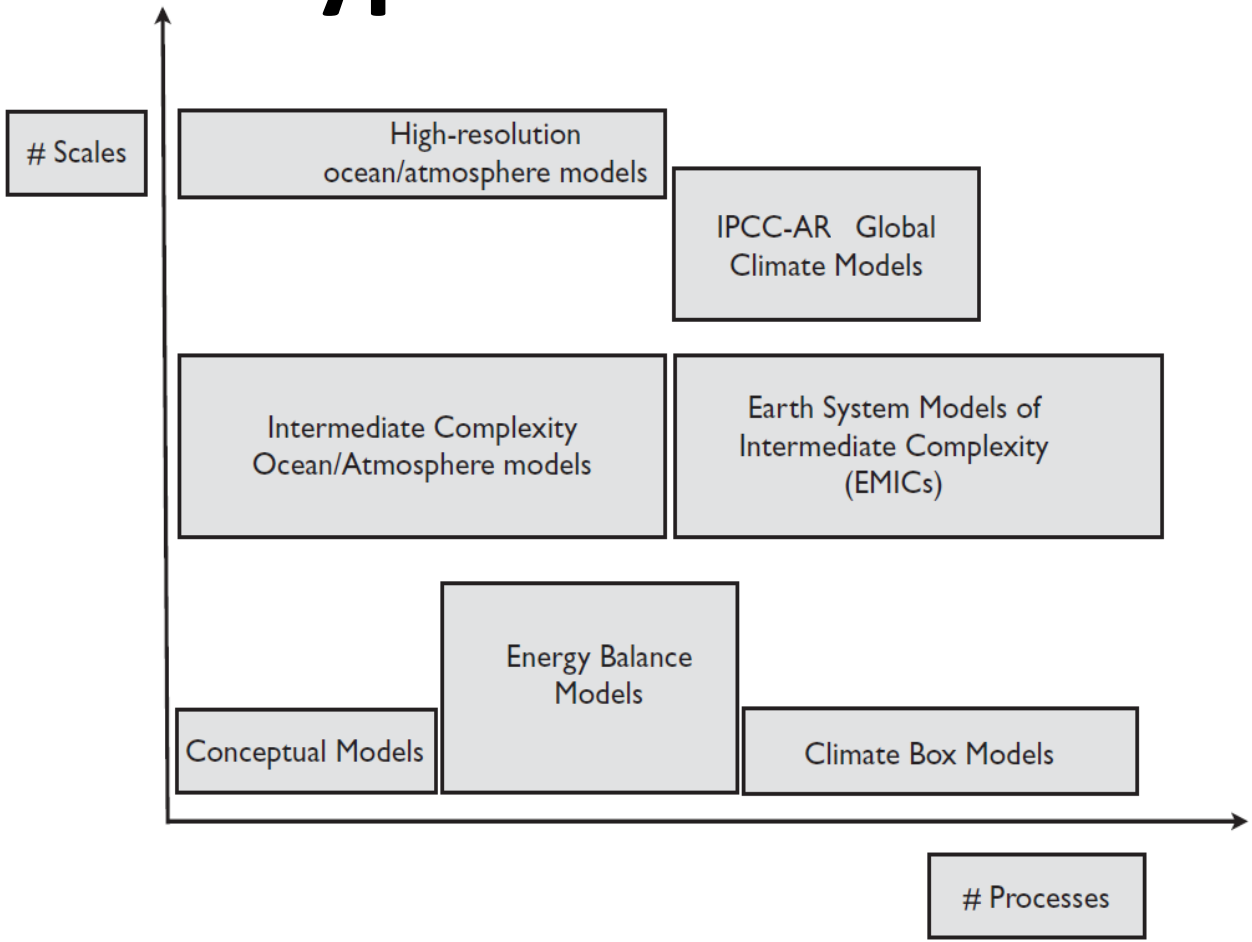
Wiskundige modellen

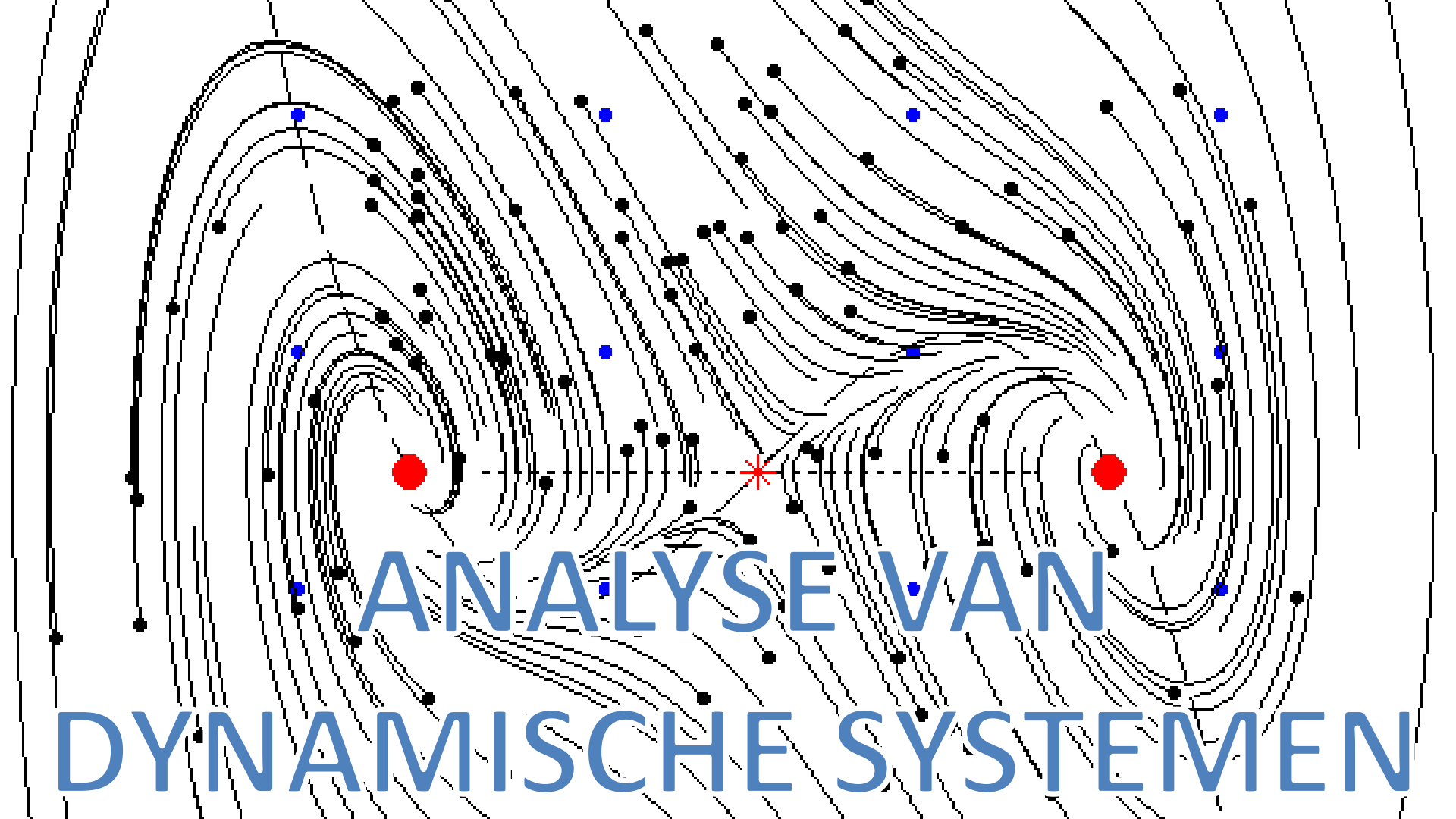
$\boldsymbol{x}$: toestand van het systeem

<u>Dynamisch systeem:</u>

$$\frac{d\boldsymbol{x}(t)}{dt} = f(\boldsymbol{x}(t), t)$$

Type modellen





ANALYSE VAN DYNAMISCHE SYSTEMEN

Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x, t)$$

Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x,t)$$

f alleen afhankelijk van t

$$\frac{dx}{dt} = f(t)$$

$$\int_{x(0)}^{x(t)} dx' = \int_0^t f(t')\, dt'$$

$$\rightarrow x(t) = x(0) + \int_0^t f(t')\, dt'$$

Dynamische systemen – de basis

$$x \in \mathbb{R}$$

$$\frac{dx}{dt} = f(x,t)$$

f alleen afhankelijk van t	f alleen afhankelijk van x
$\frac{dx}{dt} = f(t)$	$\frac{dx}{dt} = f(x)$
$\int_{x(0)}^{x(t)} dx' = \int_0^t f(t')\,dt'$	$\int_{x(0)}^{x(t)} \frac{dx'}{f(x')} = \int_0^t dt'$
$\rightarrow x(t) = x(0) + \int_0^t f(t')\,dt'$	$\rightarrow x(t) =$ … ?

$$\frac{dx}{dt} = f(x) \coloneqq \mu - x^2$$

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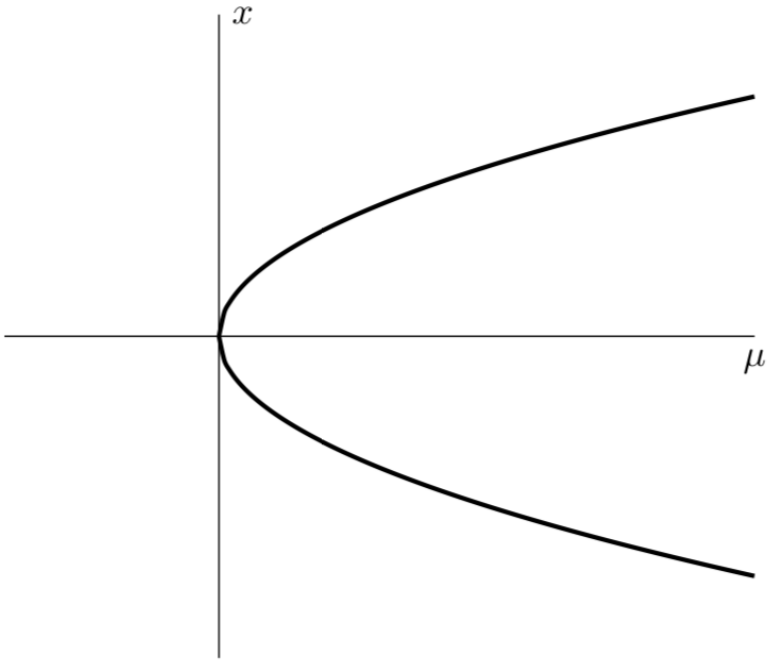
1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

$$\rightarrow x_* = \pm\sqrt{\mu}$$



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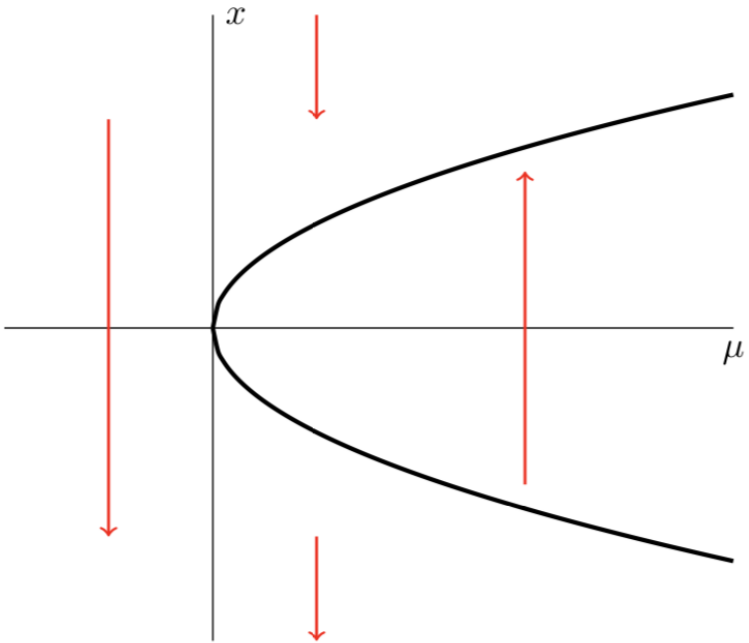
2. Stabiliteit



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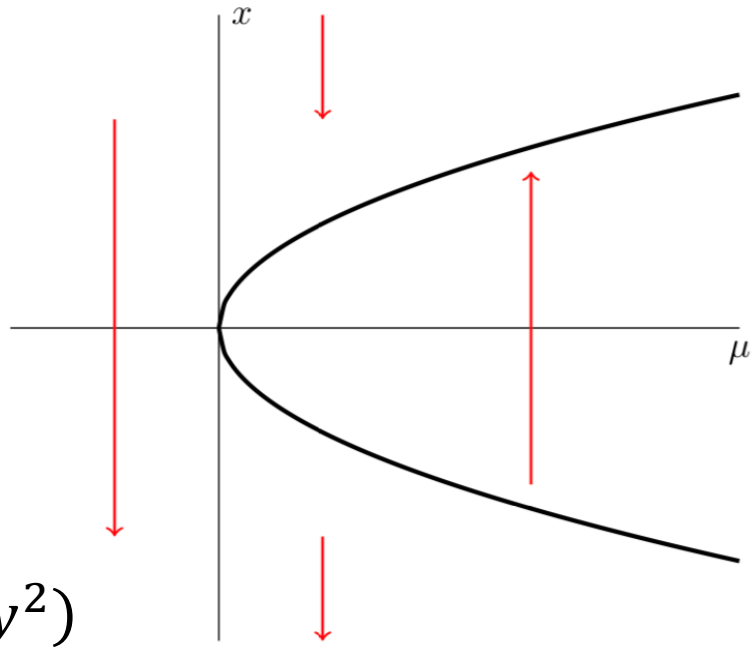


2. Stabiliteit

Kleine verstoring $x = x_* + y$

$$\rightarrow \frac{d(x_* + y)}{dt} = f(x_* + y)$$

$$\rightarrow \frac{dy}{dt} = f(x_*) + f'(x_*)y + \mathcal{O}(y^2)$$

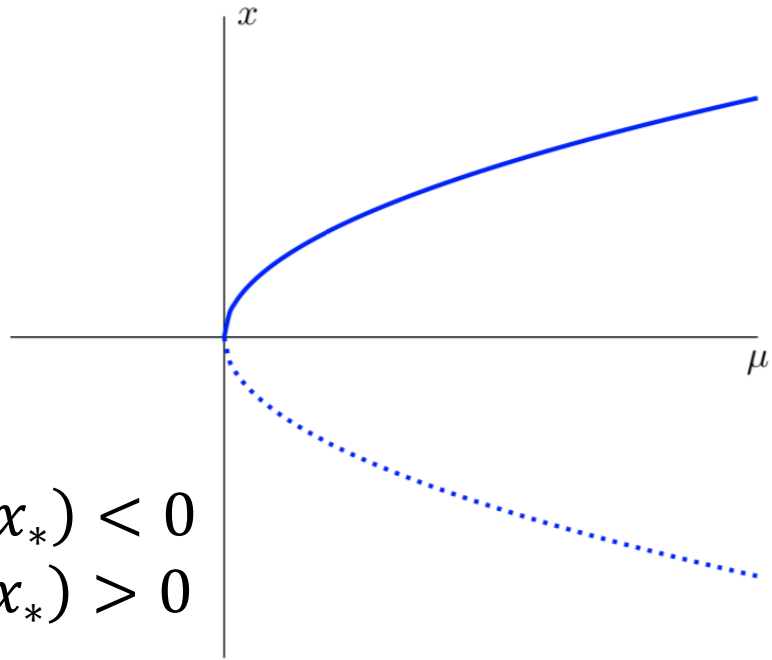
$$\rightarrow \frac{dy}{dt} = f'(x_*)y$$

$$\frac{dx}{dt} = f(x) \coloneqq \mu - x^2$$

1. Vaste Punten

$$\frac{dx_*}{dt} = 0 = f(x_*) = \mu - x_*^2$$

$$\rightarrow x_* = \pm\sqrt{\mu}$$



2. Stabiliteit

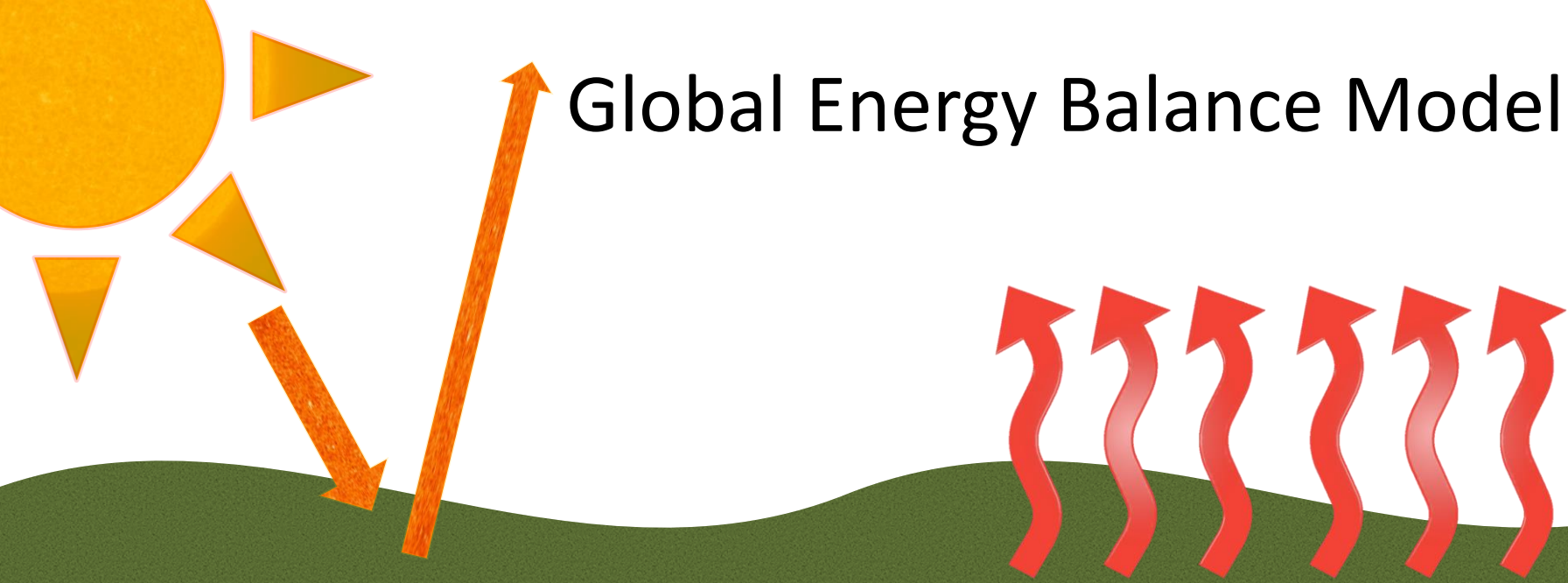
$$f'(x_*) = -2\,x_* = \mp 2\sqrt{\mu}$$

Stabiel (aantrekkend) $\Leftrightarrow \lambda \coloneqq f'(x_*) < 0$

Instabiel (afstotend) $\Leftrightarrow \lambda \coloneqq f'(x_*) > 0$

VOORBEELD:
IJS-ALBEDO FEEDBACK

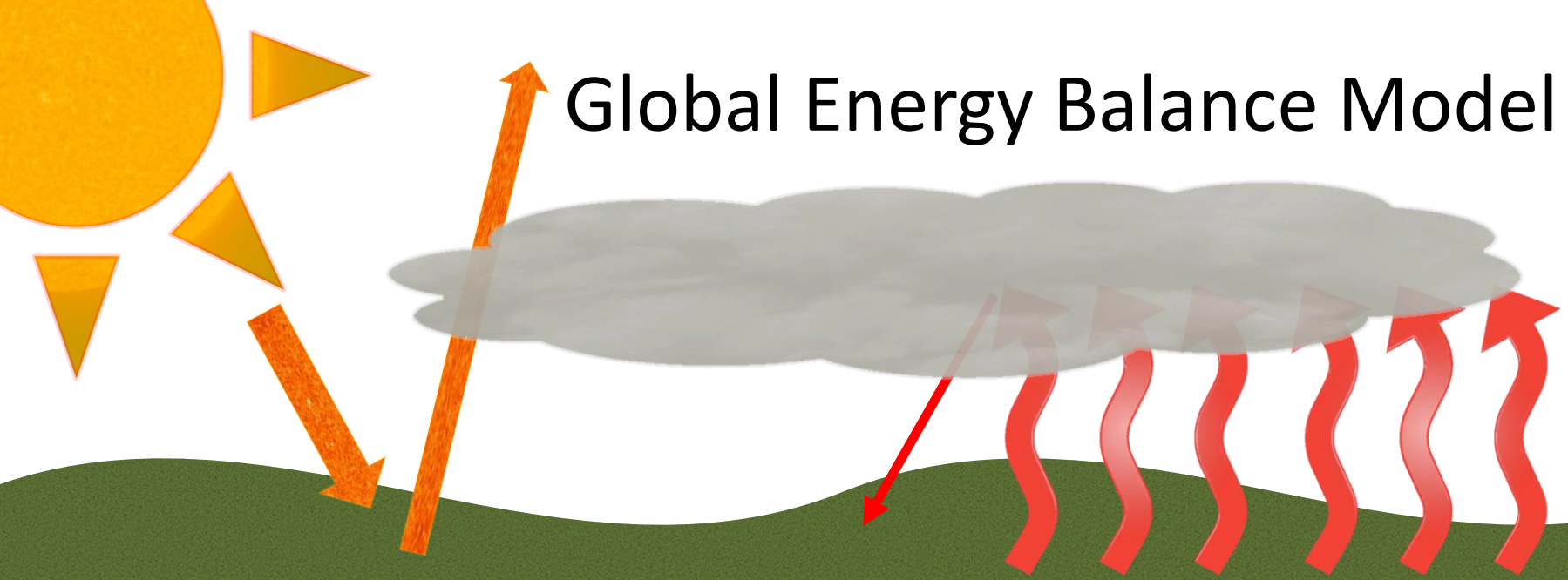
Global Energy Balance Model



$$\text{opwarming} = \text{zonlicht} - \text{weerkaatst zonlicht} - \text{straling}$$

$$\frac{dT}{dt} = 1 - \alpha(T) - T^4$$

Global Energy Balance Model

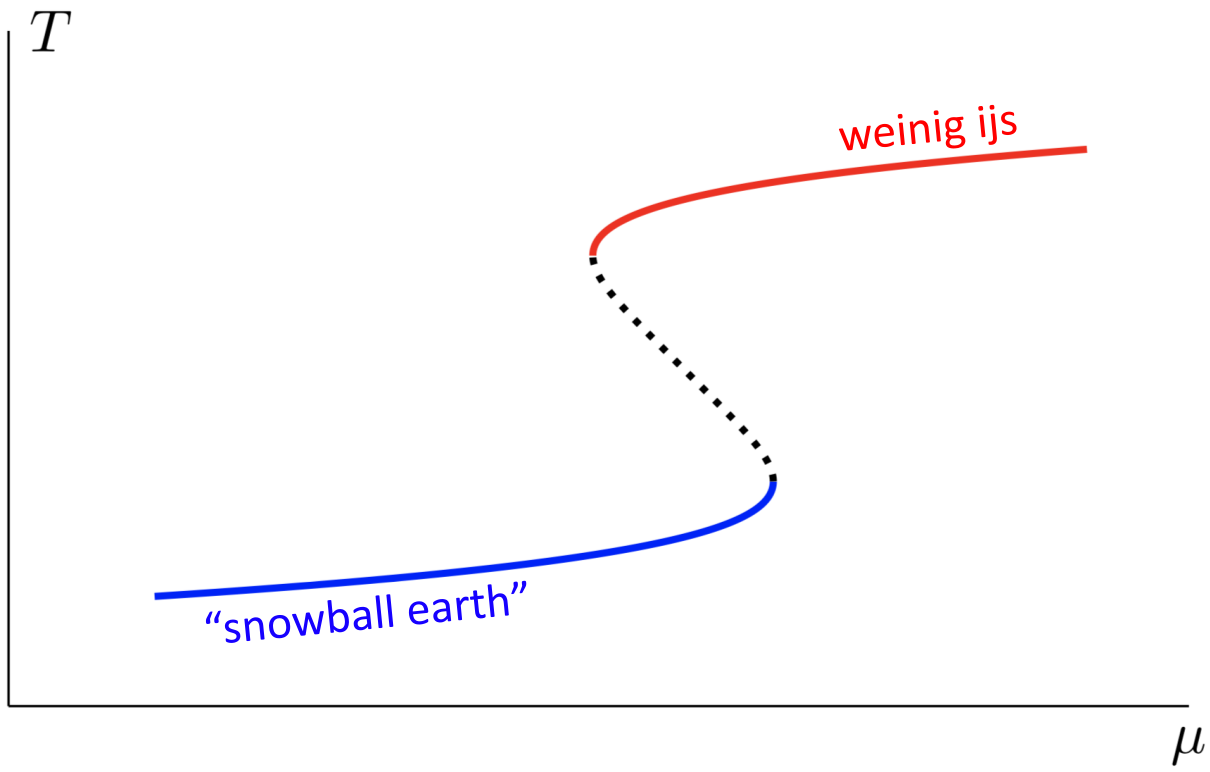


$$\frac{dT}{dt} = 1 - \alpha(T) - T^4 + \mu$$

$$\text{opwarming} = \frac{dT}{dt} = 1 - \alpha(T) - T^4 + \mu$$

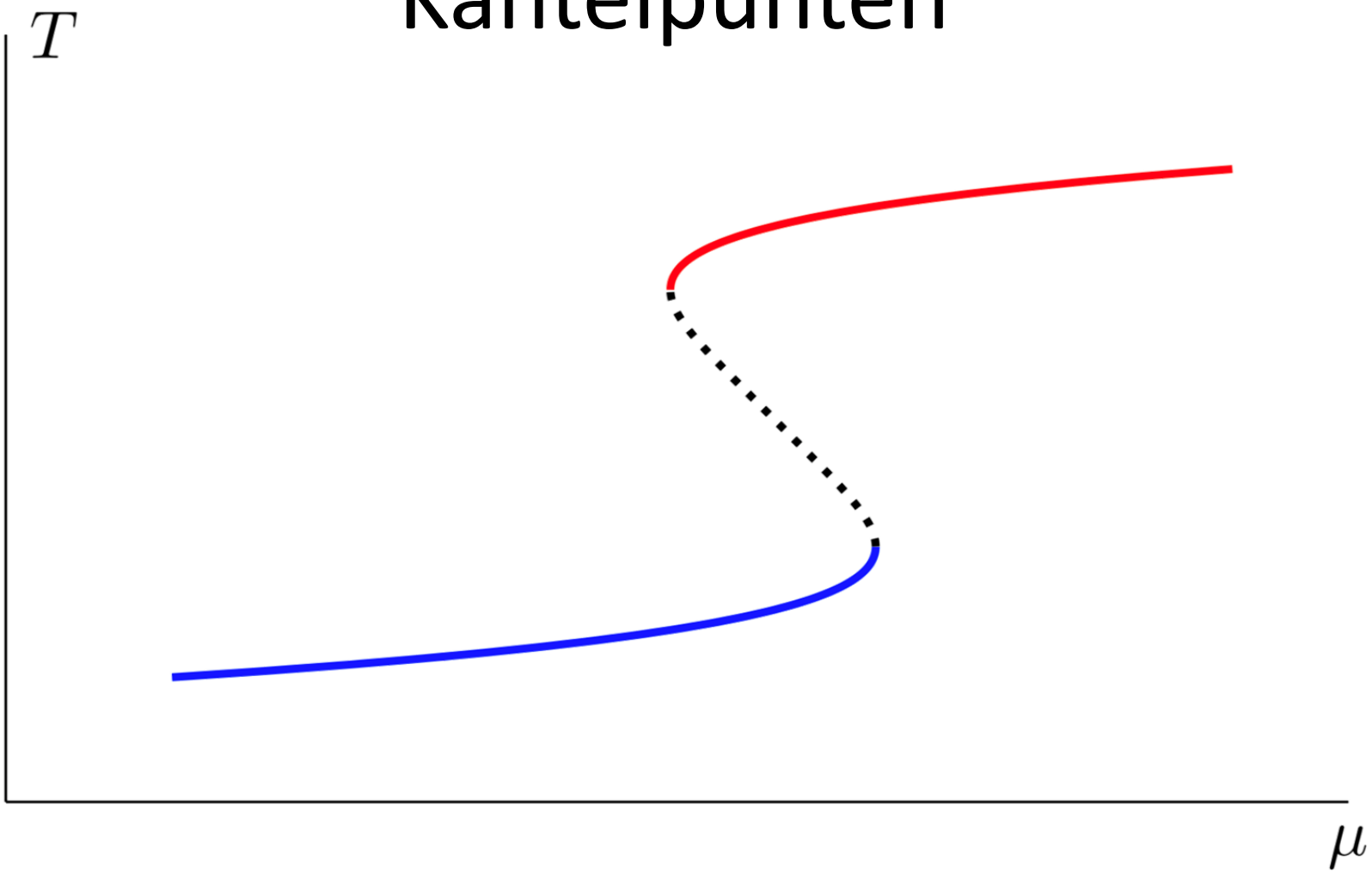
Voor gegeven CO2 (μ)
Hoe warm wordt het?

$$0 = 1 - \alpha(T) - T^4 + \mu$$

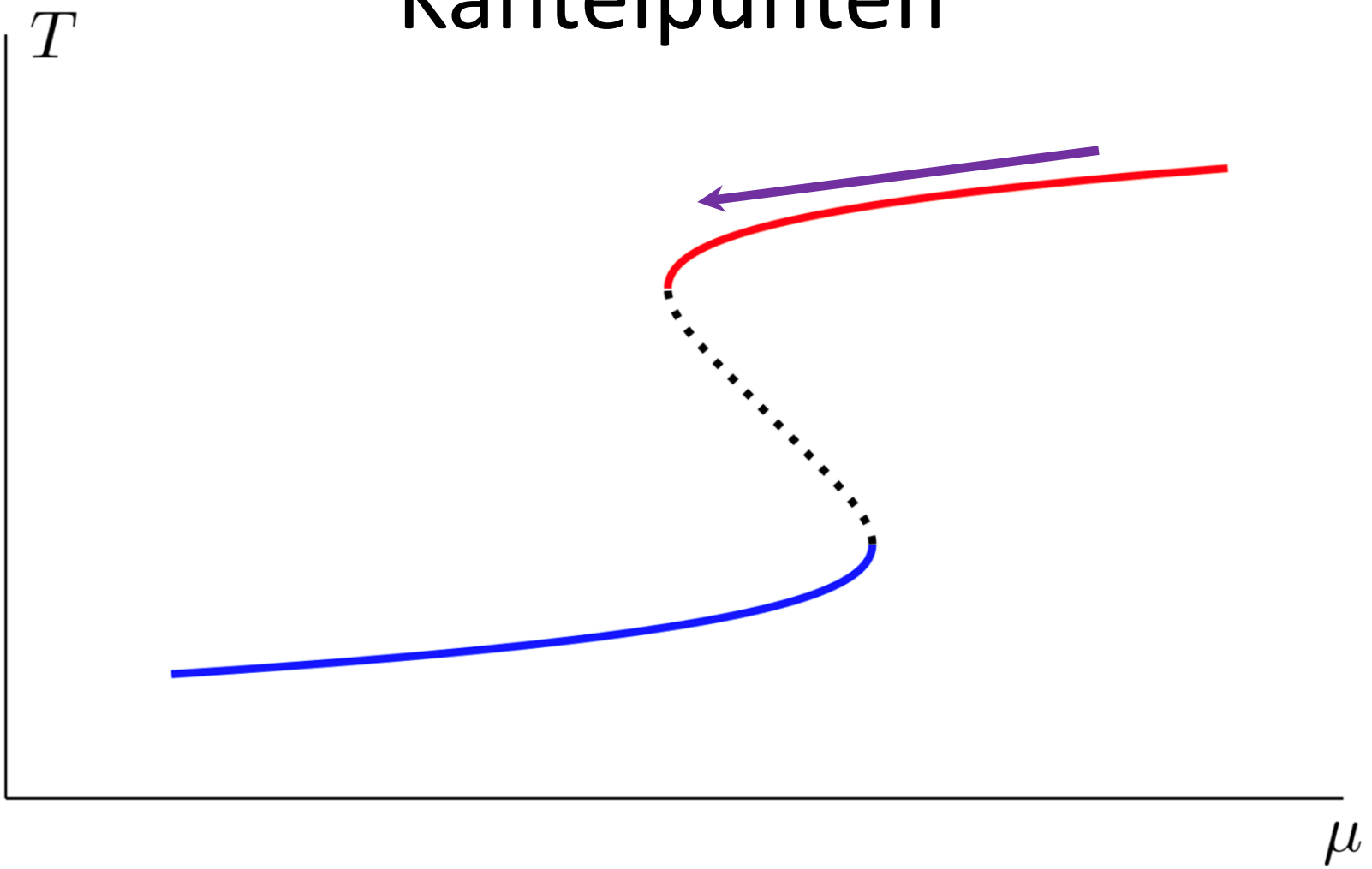



KANTELPUNTEN

Kantelpunten



Kantelpunten



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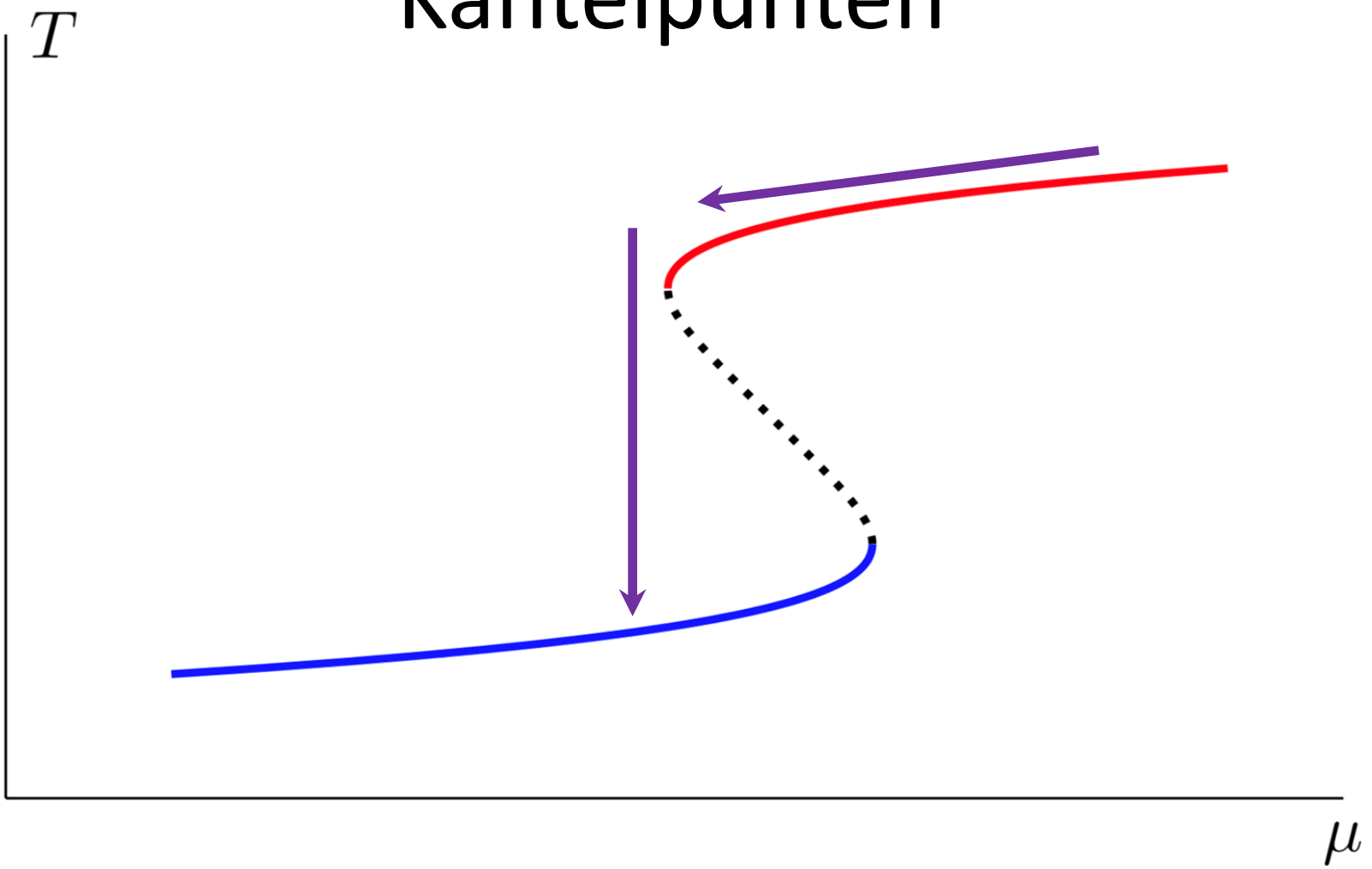
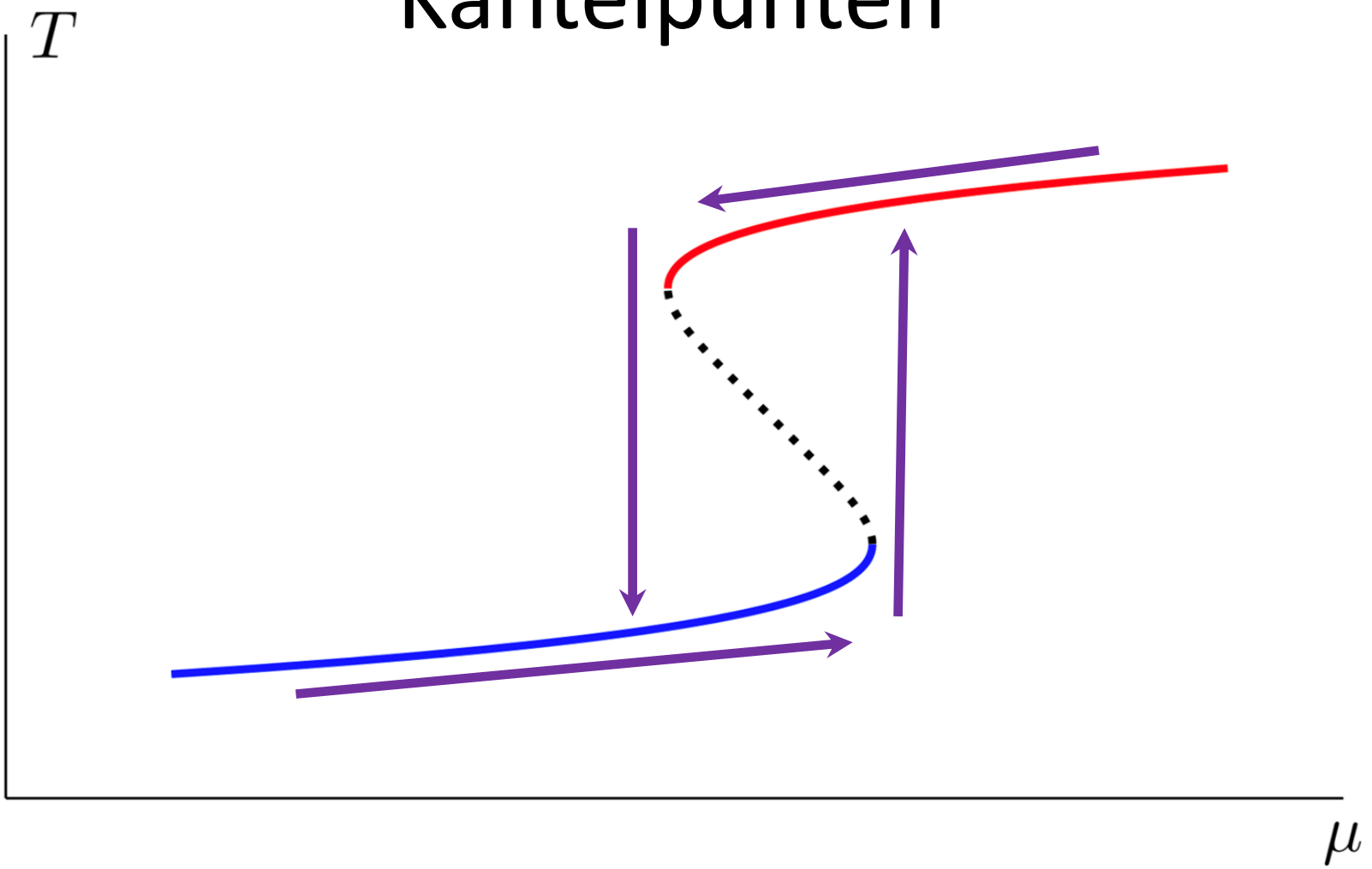
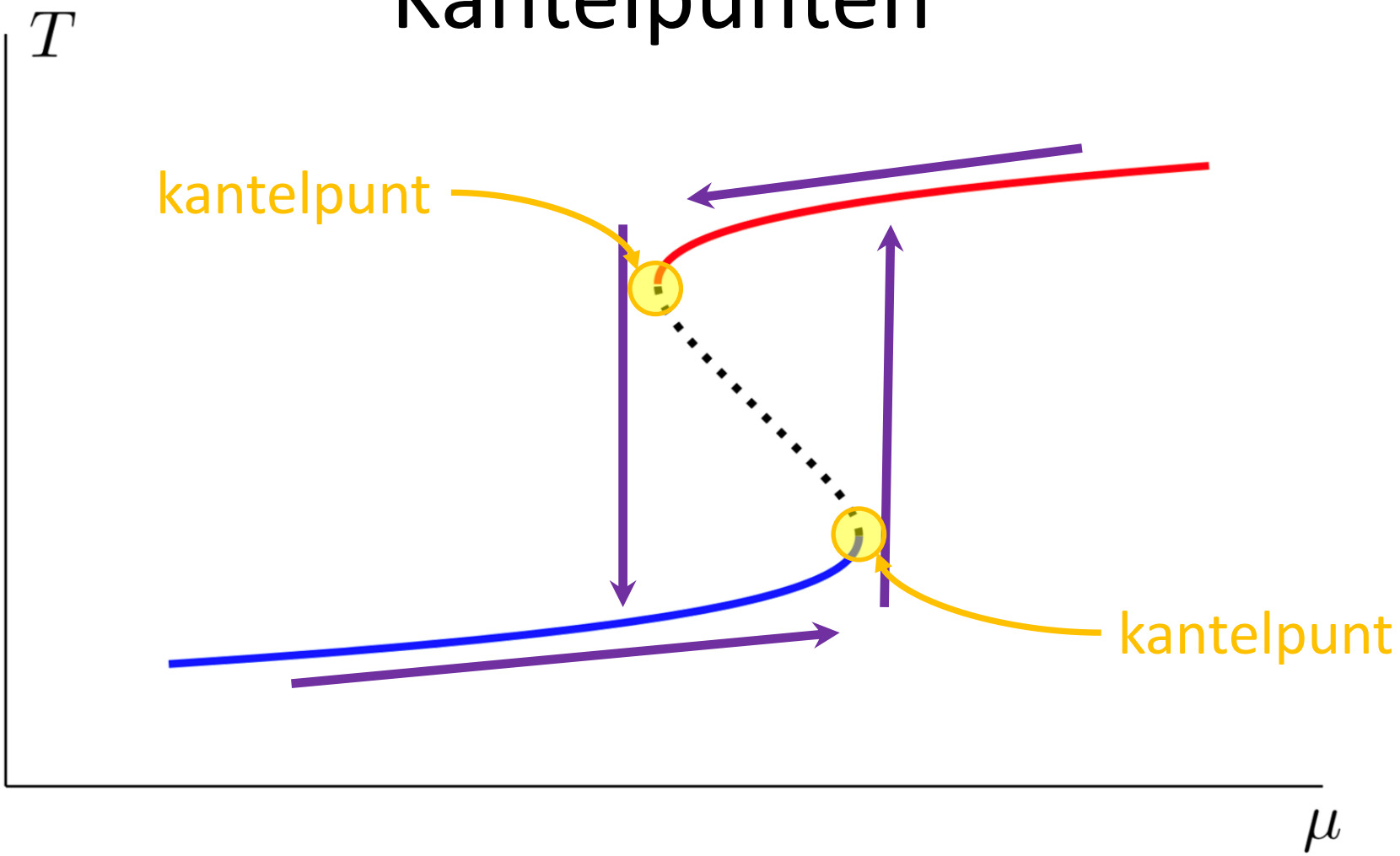


Kantelpunten



Kantelpunten



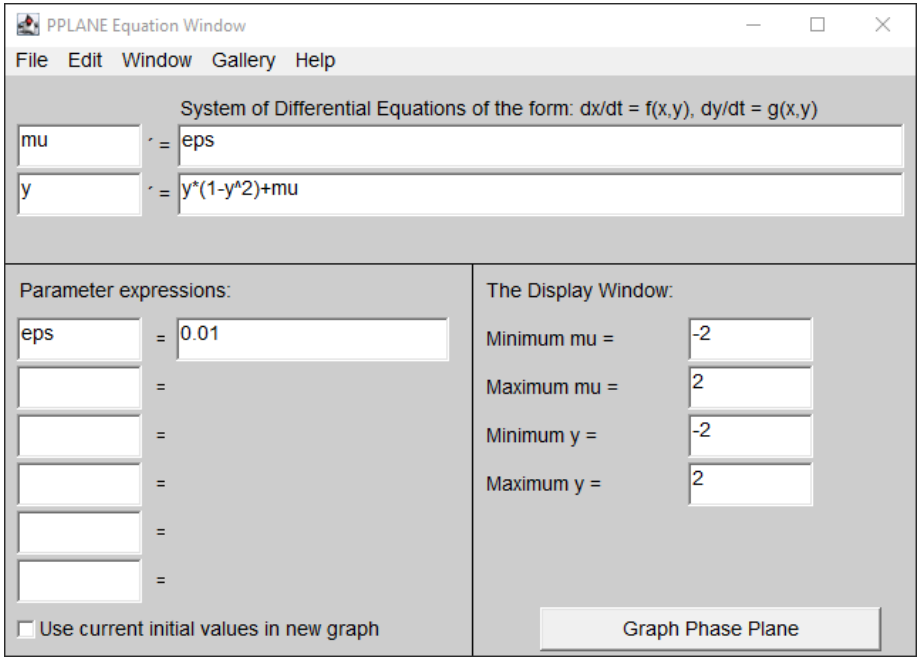
AAN DE SLAG MET VOORBEELDSYSTEMEN

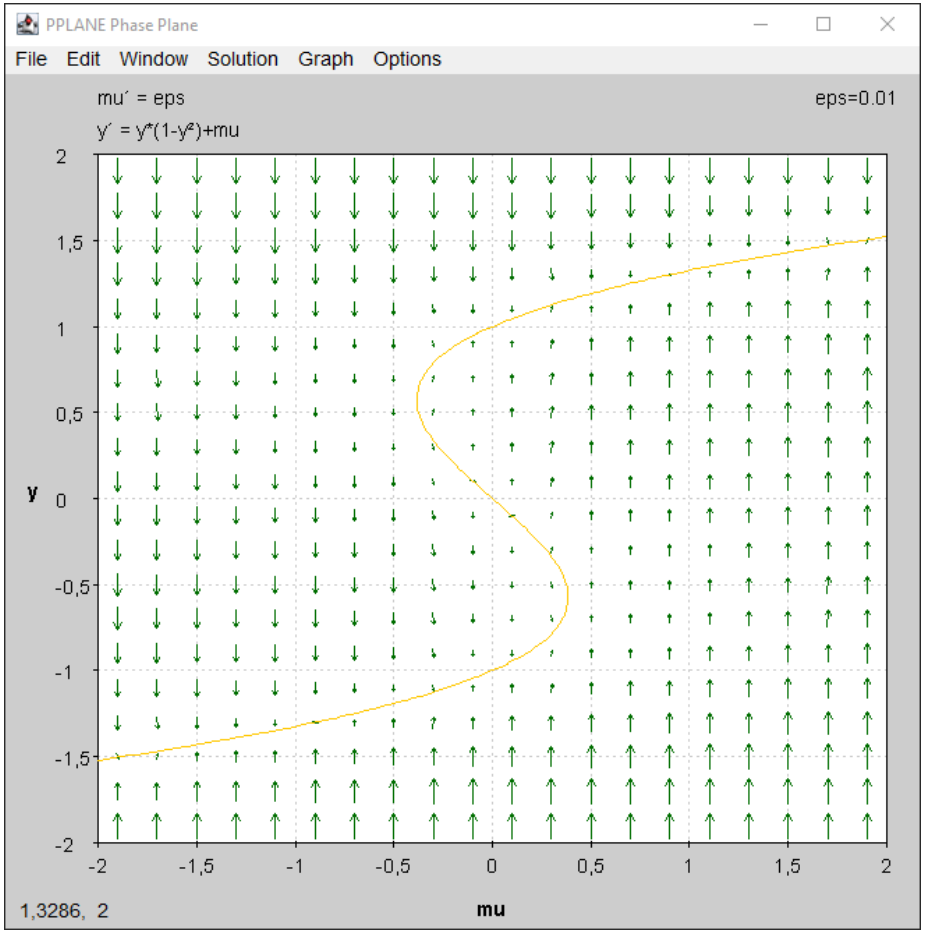
Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf

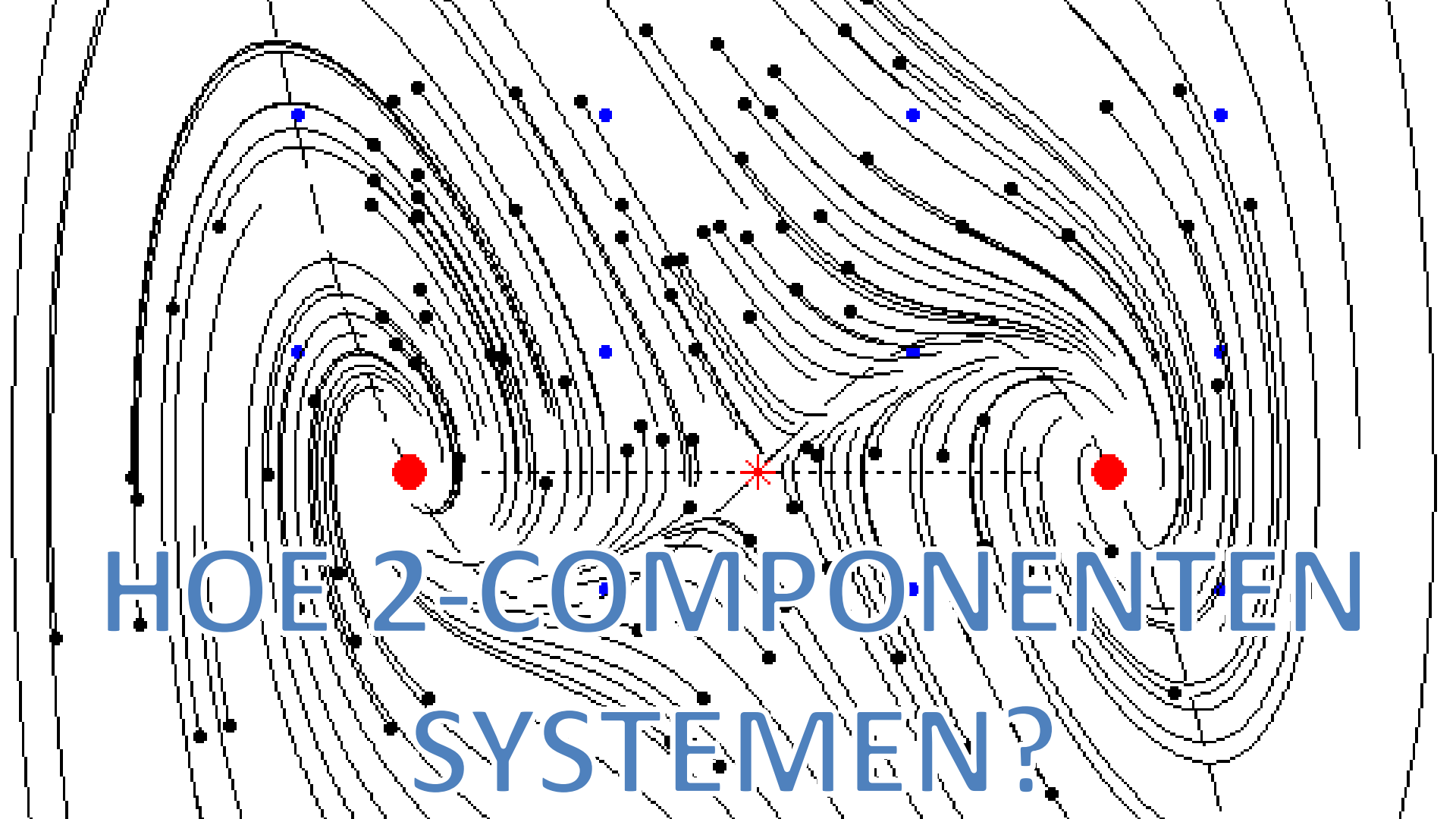
Faseplaatjes

JAVA applicatie:

www.cs.unm.edu/~joel/dfield







HOE 2-COMPONENTEN SYSTEMEN?

1-componenten systemen	N-componenten systemen
$\frac{dx}{dt} = f(x;\mu)$	$\frac{d\vec{x}}{dt} = \vec{f}(\vec{x};\mu)$
Evenwichten: $0 = f(x_*;\mu)$	Evenwichten: $0 = \vec{f}(\vec{x}_*;\mu)$ Nu stelsel van vergelijkingen
Stabiliteit: $\lambda := f'(x;\mu)$ Als $\lambda < 0$: stabiel Als $\lambda > 0$: instabiel Anders: onduidelijk	Stabiliteit: Vind de eigenwaardes λ van $D\vec{f}(\vec{x}_*;\mu)$ Als $Re(\lambda) < 0$ voor allemaal: stabiel Als $Re(\lambda) > 0$ voor minstens één: instabiel Anders: onduidelijk

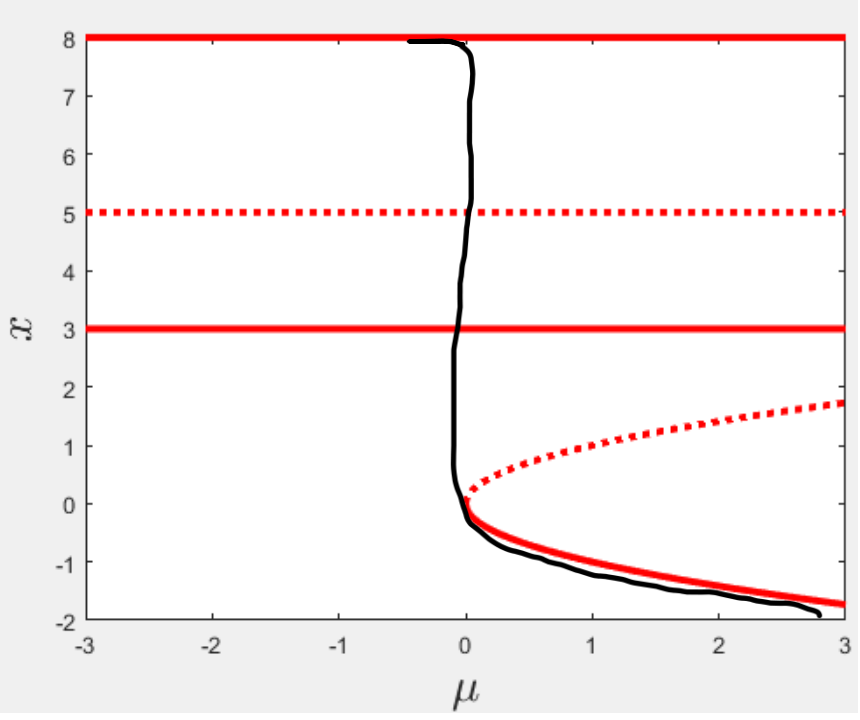
AAN DE SLAG MET VOORBEELDSYSTEMEN

Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf

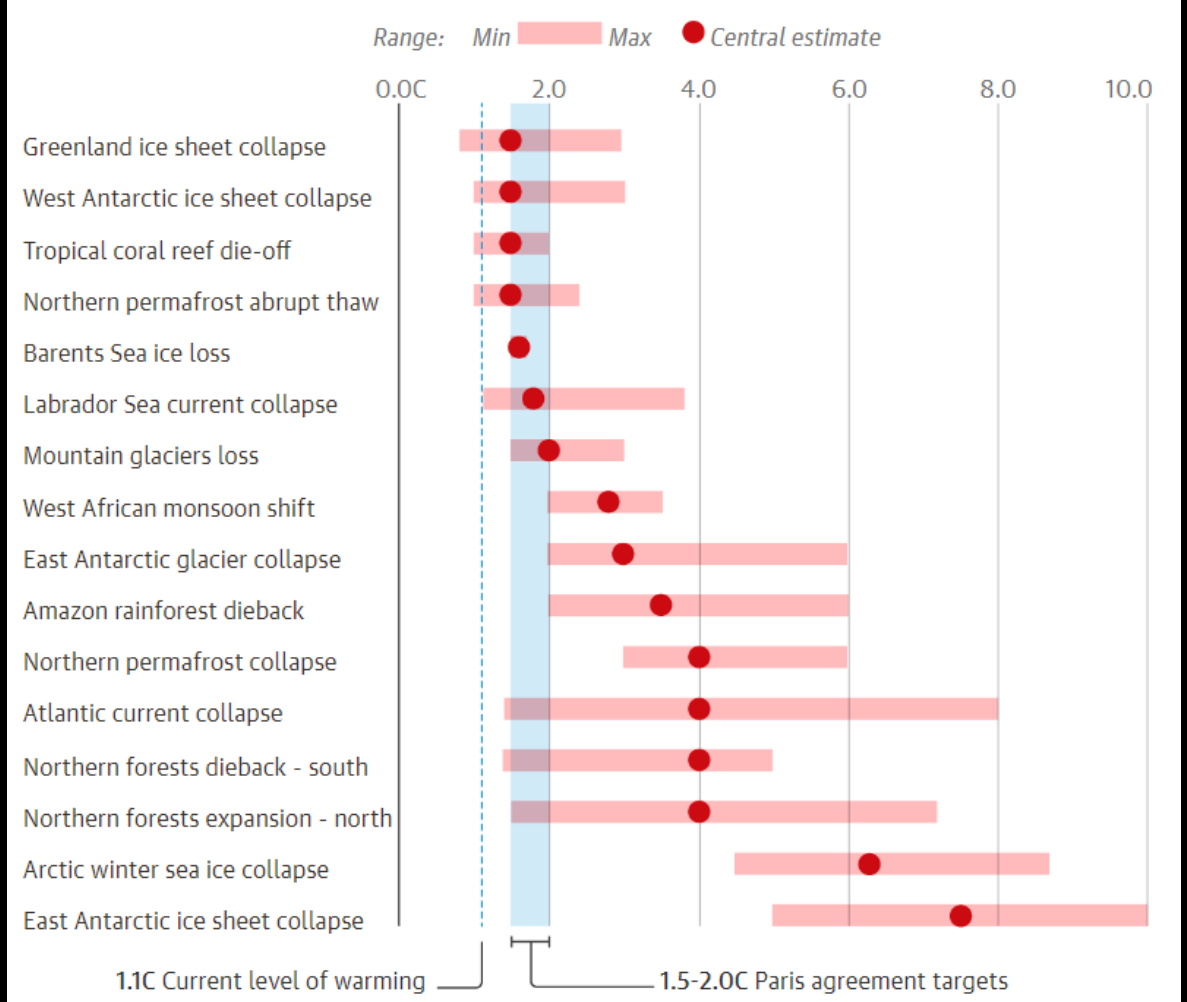
2-COMPONENTEN: WAT NA BIFURCATIE?

Let op: wat ná een bifurcatie?

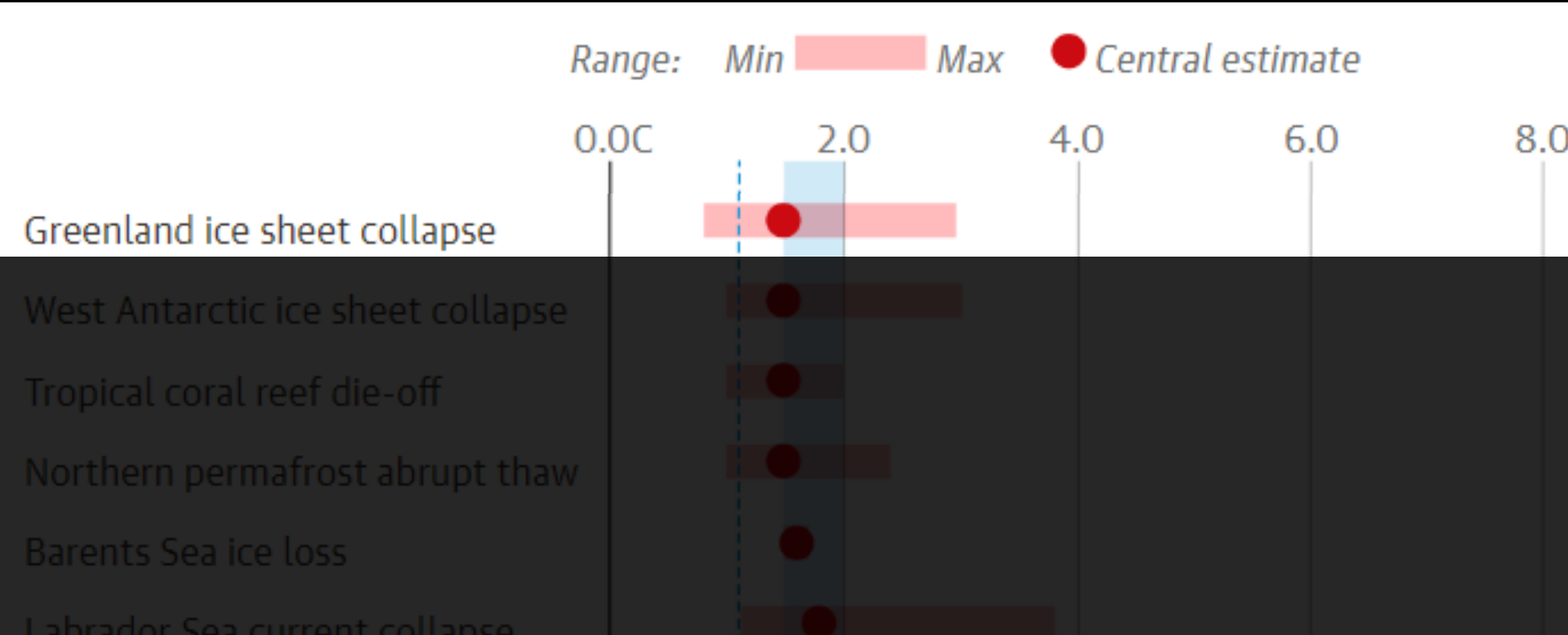
$$\begin{cases} x' = -(x^2 + \mu)\,(\,(x-4)^2 + y + 1)\,(x-8) \\ y' = -y\,(x-3)(x\,-5) \end{cases}$$

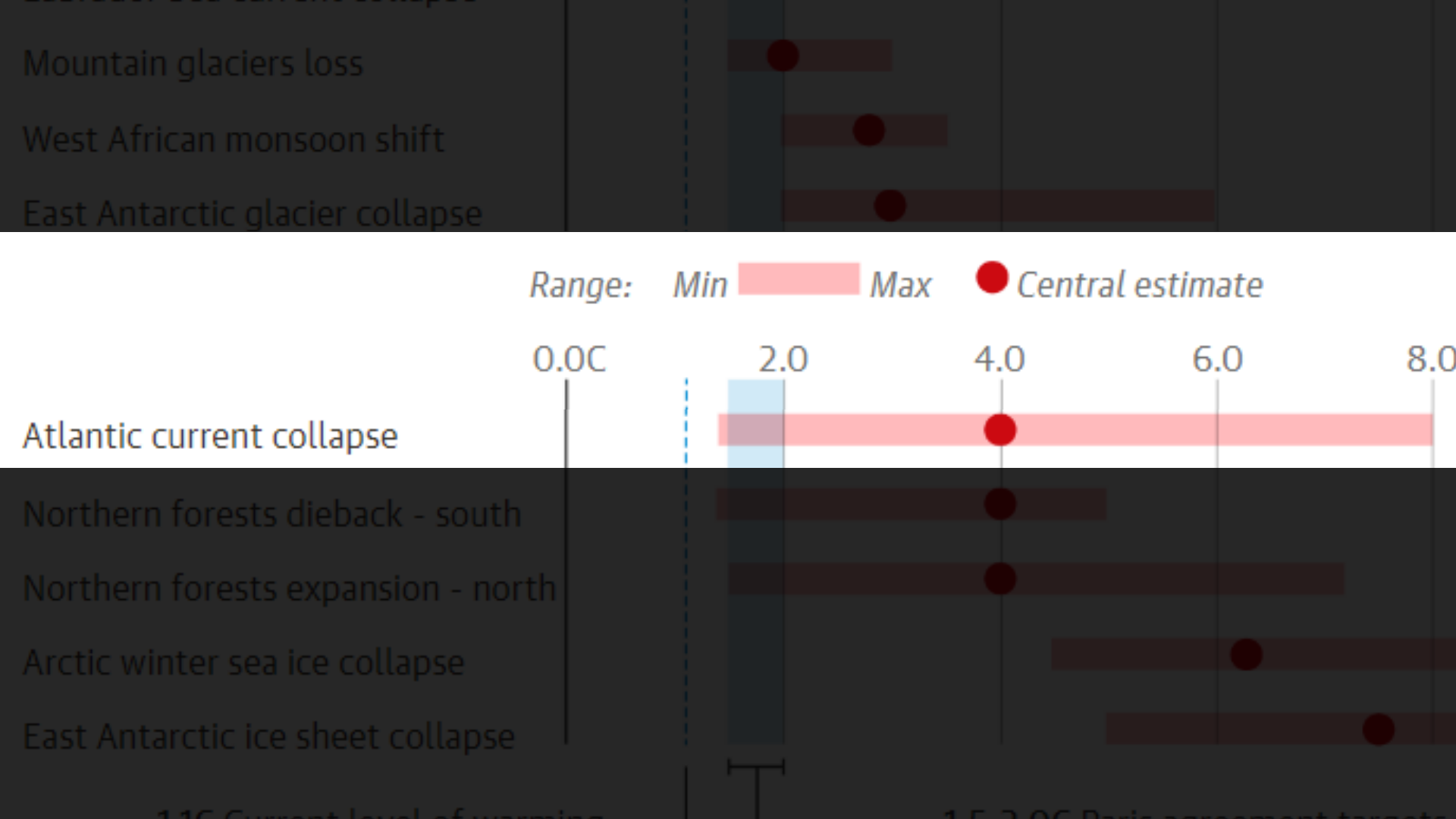



KANTELPUNTEN VOORSPELLEN



Guardian graphic. Source: Armstrong McKay et al, Science, 2022. Note: Current global heating temperature rise 1.1C Paris agreement targets 1.5-2.0C


Range:
Min
Max
Central estimate
0.0C
2.0
4.0
6.0
8.0
Greenland ice sheet collapse
West Antarctic ice sheet collapse
Tropical coral reef die-off
Northern permafrost abrupt thaw
Barents Sea ice loss


Mountain glaciers loss
West African monsoon shift
East Antarctic glacier collapse
Range:
Min
Max
Central estimate
0.0C
2.0
4.0
6.0
8.0
Atlantic current collapse
Northern forests dieback - south
Northern forests expansion - north
Arctic winter sea ice collapse
East Antarctic ice sheet collapse

Warme Golfstroom, van grote invloed op ons klimaat, komt mogelijk nog eerder tot stilstand: 'Heel zorgwekkend'

Het Parool
26-07-2023

Gulf Stream could collapse as early as 2025, study suggests

The Guardian
25-07-2023

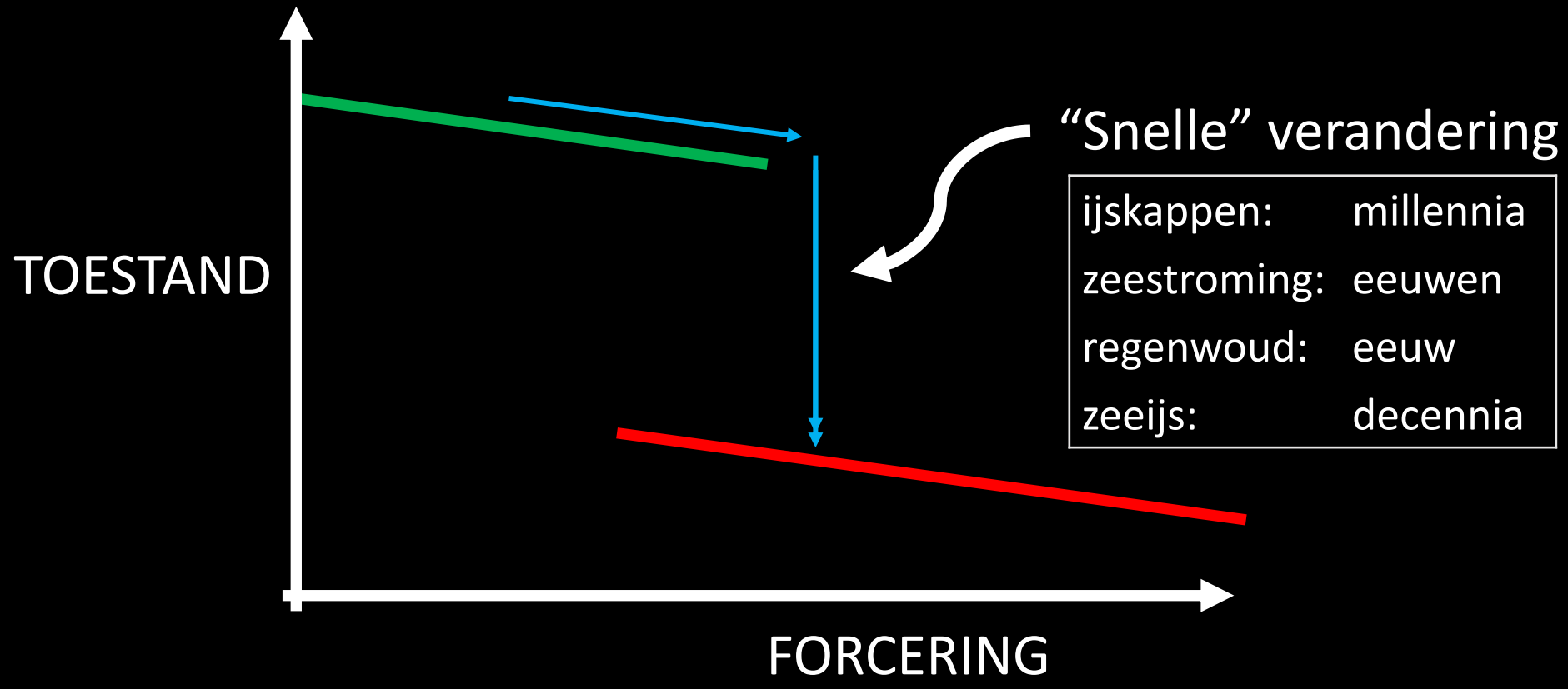
nature communications

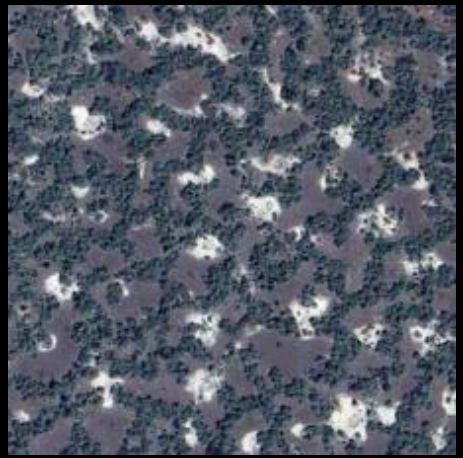
Article

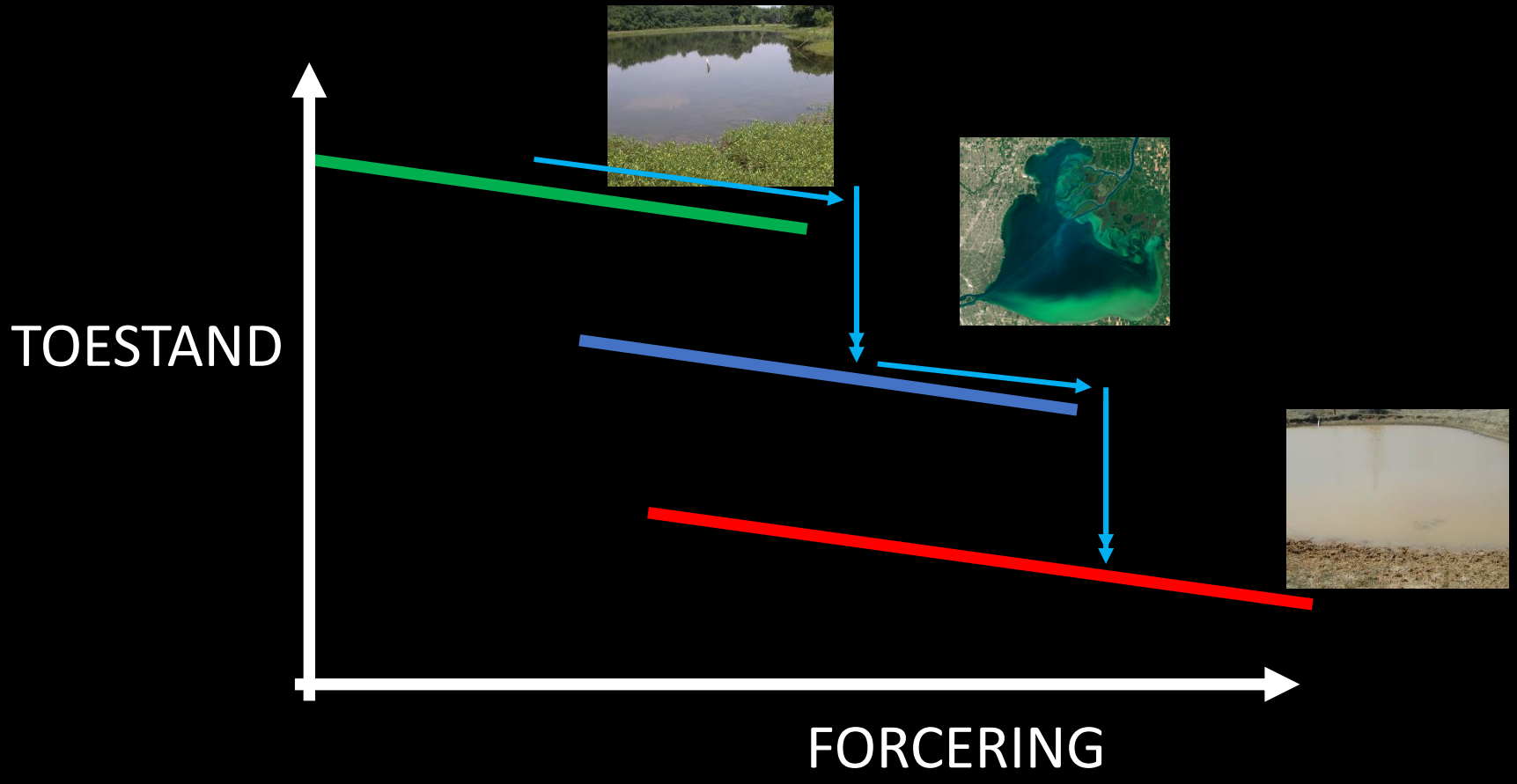
https://doi.org/10.1038/s41467-023-39810-w

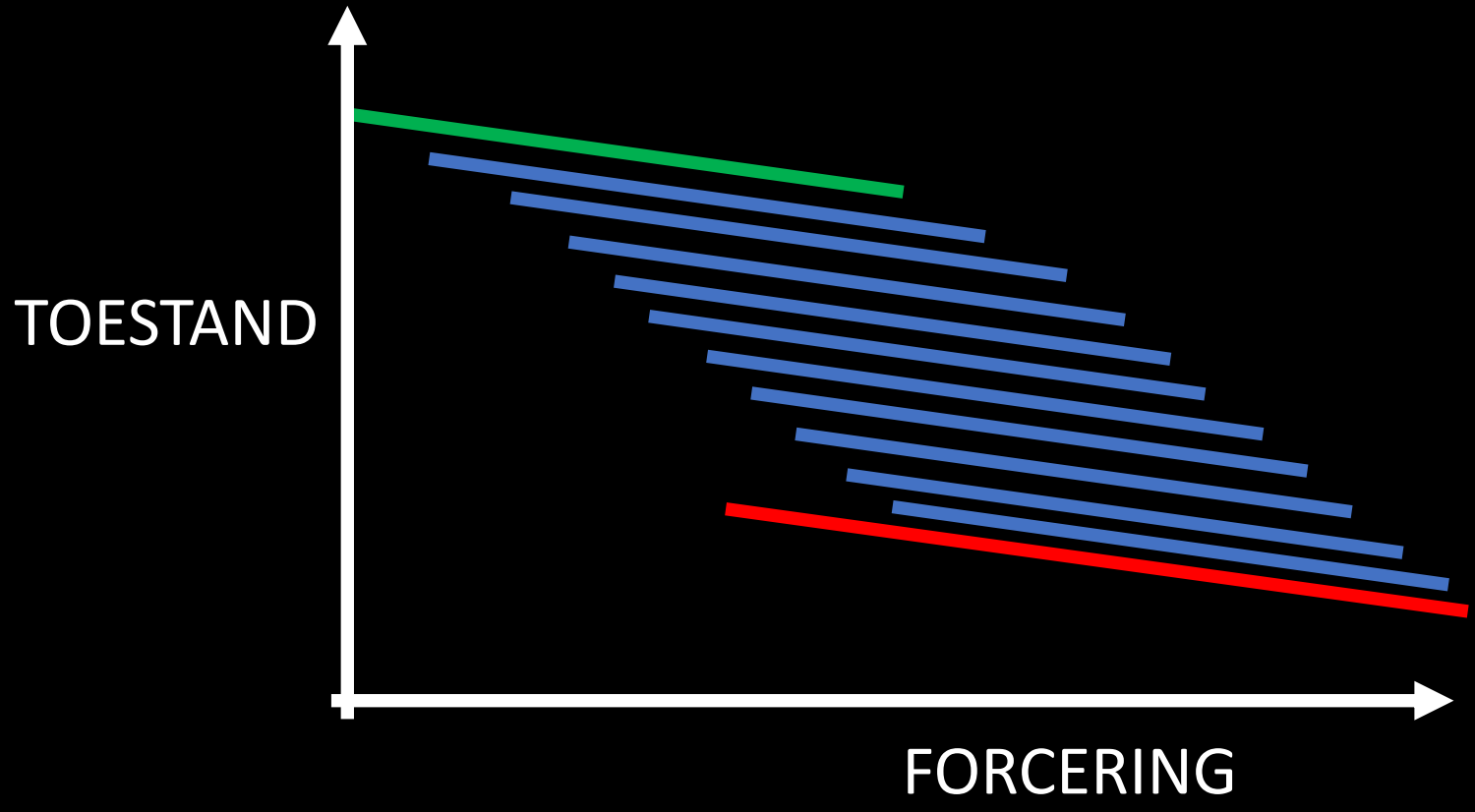
Warning of a forthcoming collapse of the Atlantic meridional overturning circulation

[Ditlevsen and Ditlevsen, Nature Communications (2023)]


TOESTAND
FORCERING
“Snelle” verandering
ijskappen: millennia
zeestroming: eeuwen
regenwoud: eeuw
zeeijs: decennia




TOESTAND
FORCERING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LINK MET WISKUNDE TREFWOORDEN

Verdere wiskundige onderwerpen

- Dynamische systemen, differentiaalvergelijkingen
- Bifurcatietheorie, catastrophy theory
- Topologie
- Numeriek: simulaties (tijdsintegraties), continuaties/nulpunten/algoritmes
- Partiële differentiaalvergelijkingen
- Snelheid van verandering (niet-autonome systemen)

Slides: http://bastiaansen.github.io/talks/2025-04-14_WS-docenten.pdf
Handout: bastiaansen.github.io/talks/2025-04-14_WS-docenten_handout.pdf