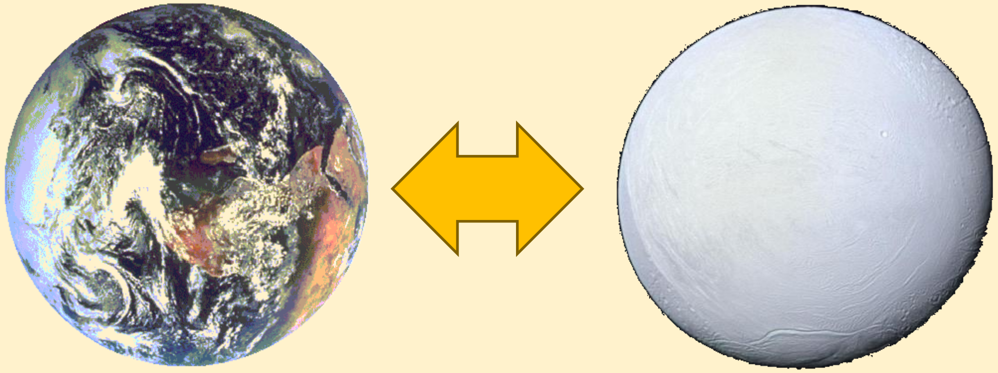
Tipping in Spatially Extended Systems: detection, prediction, control

2025-11-19, ACDC Seminar, VU

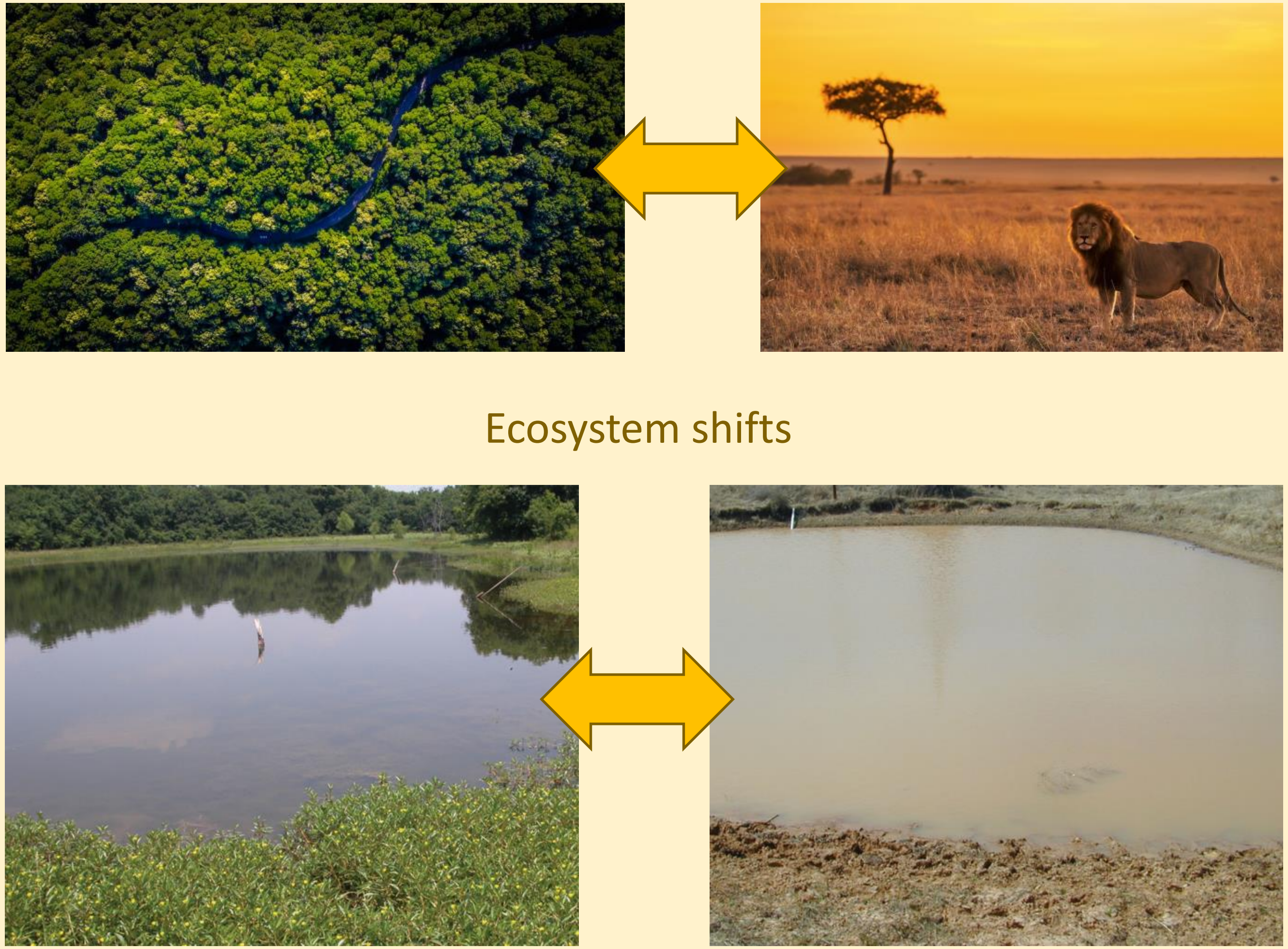
Robbin Bastiaansen (r.bastiaansen@uu.nl)

Tipping Points

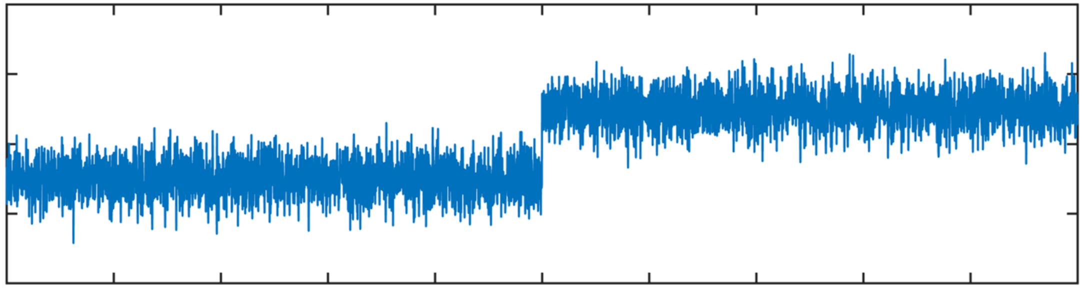
IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”



Planetary transitions



Ecosystem shifts



Tipping Points

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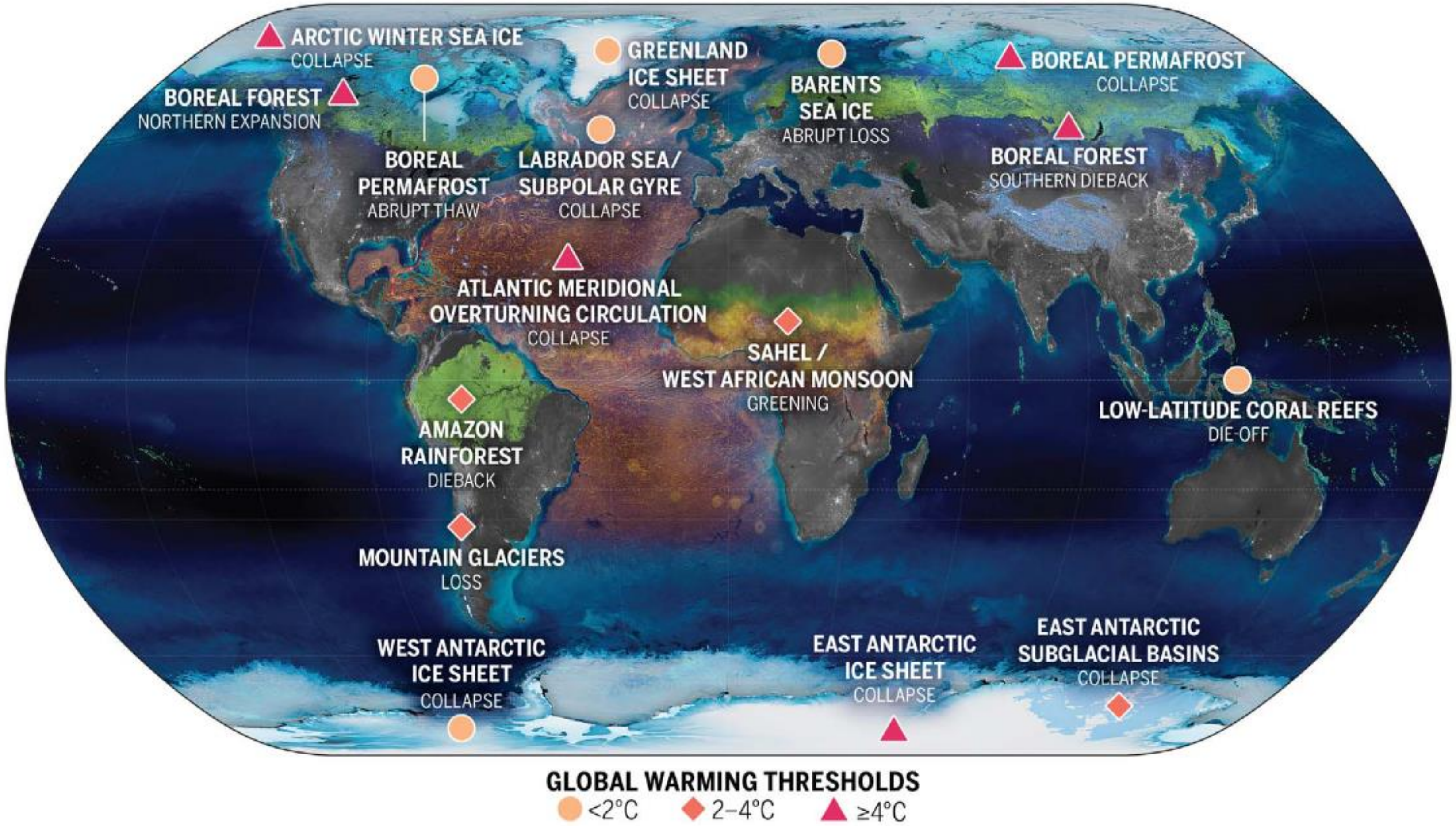
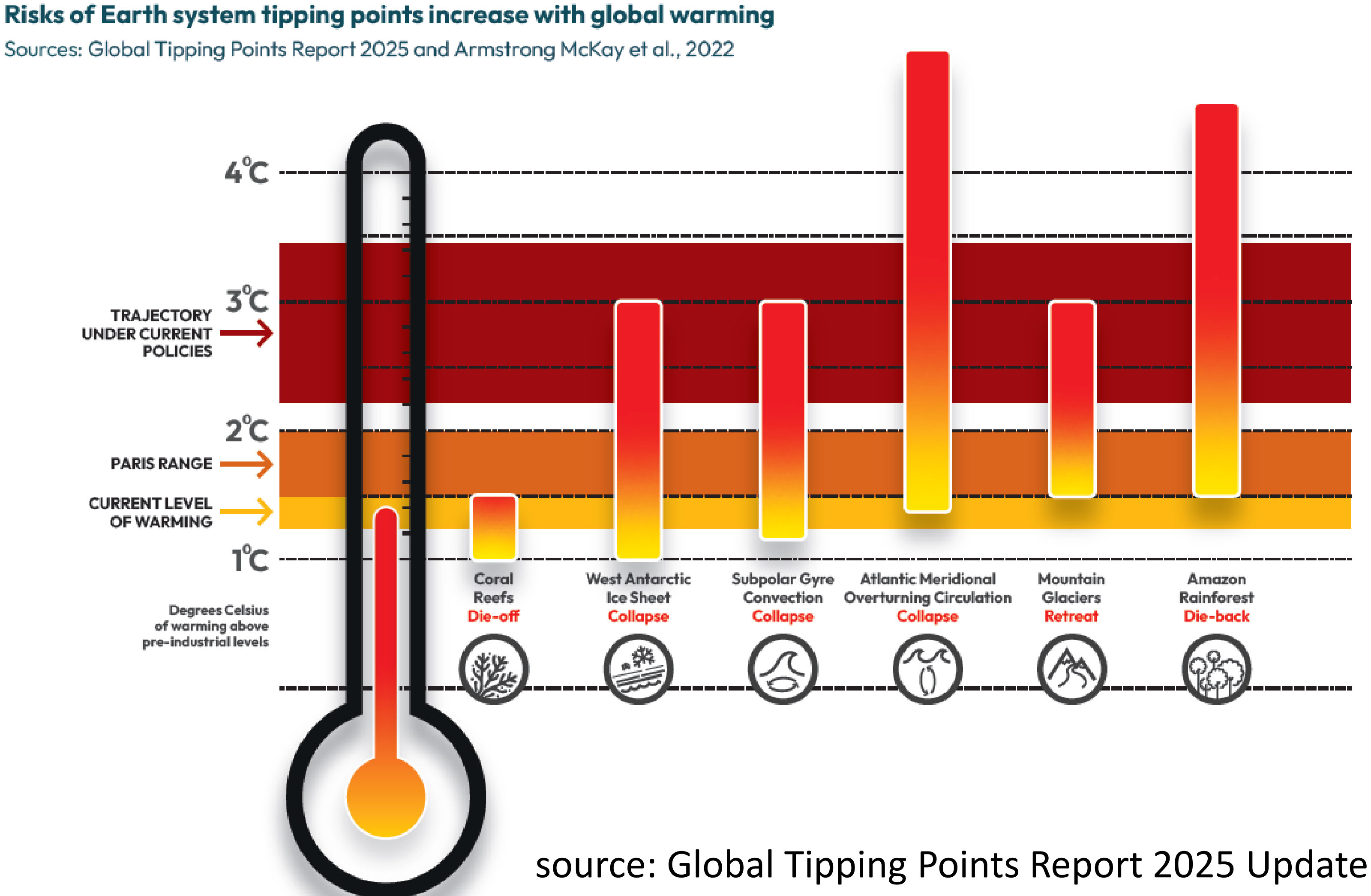


source: McKay et al, 2022

Tipping Points

IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”

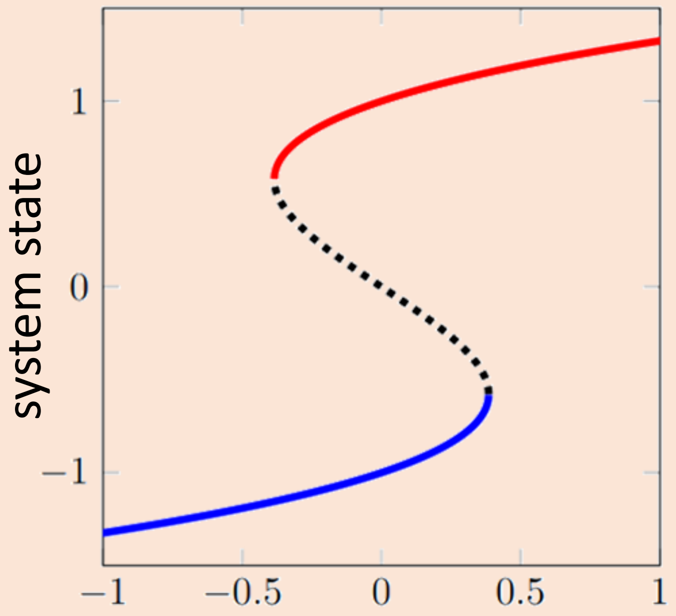


source: Global Tipping Points Report 2025 Update

Mathematics

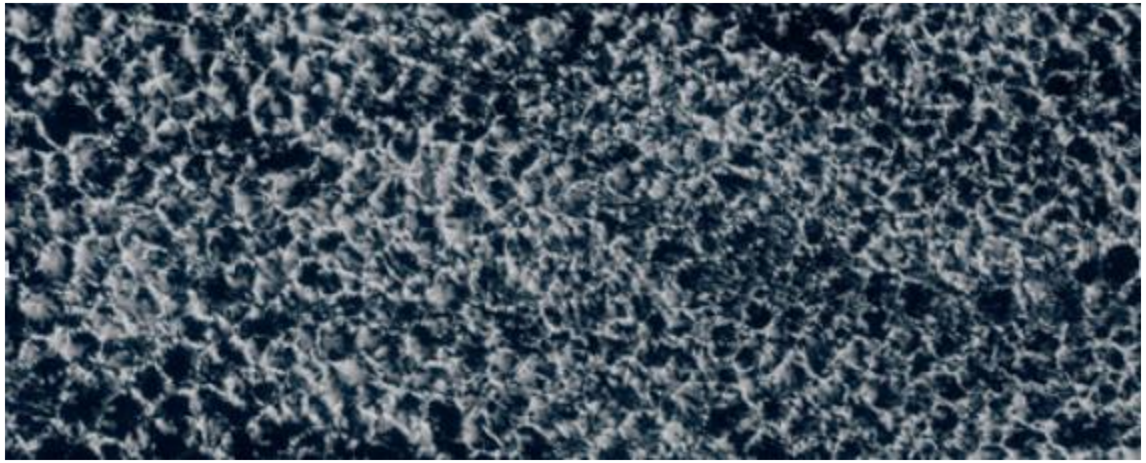
Tipping points ↔ Bifurcations

$$\frac{dy}{dt} = f(y, \mu)$$

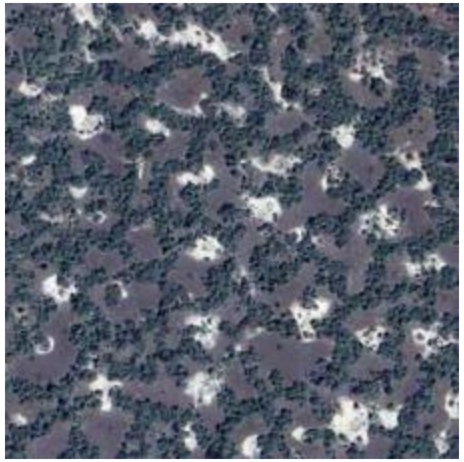


Examples of spatial patterning – regular patterns

mussel beds

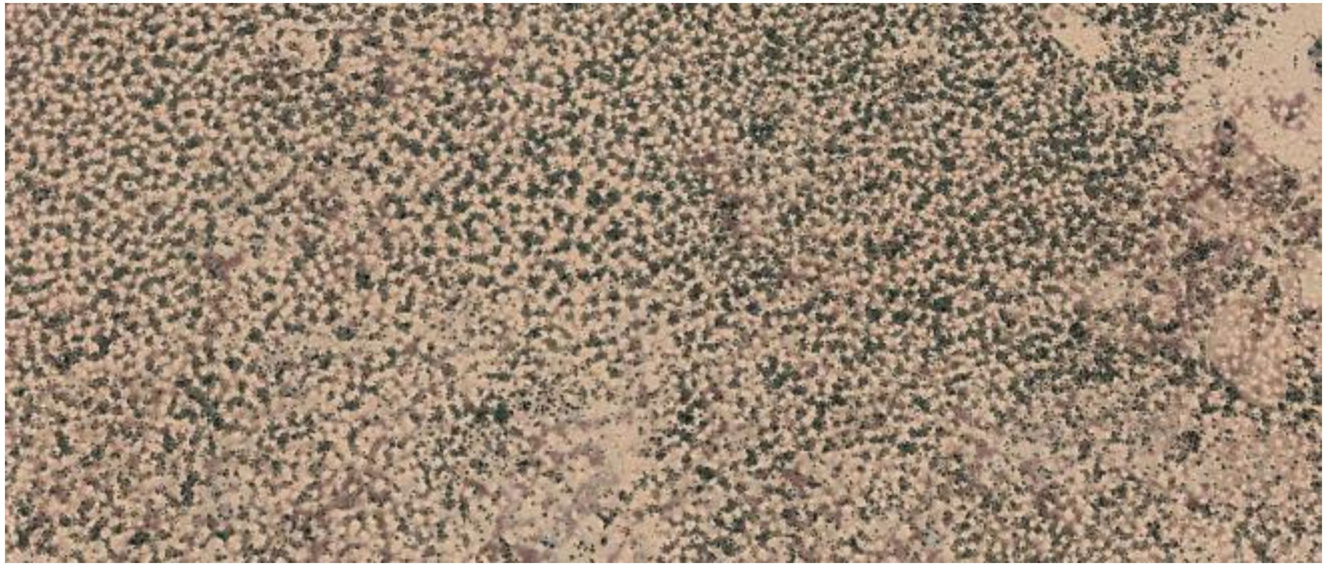


clouds



savannas

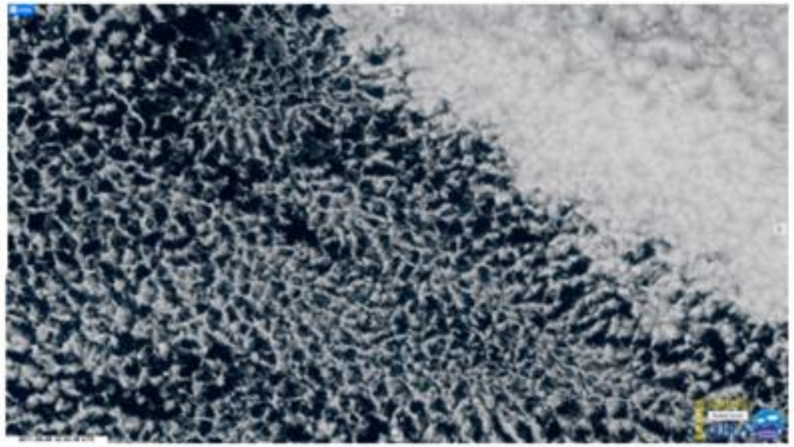
melt ponds



drylands

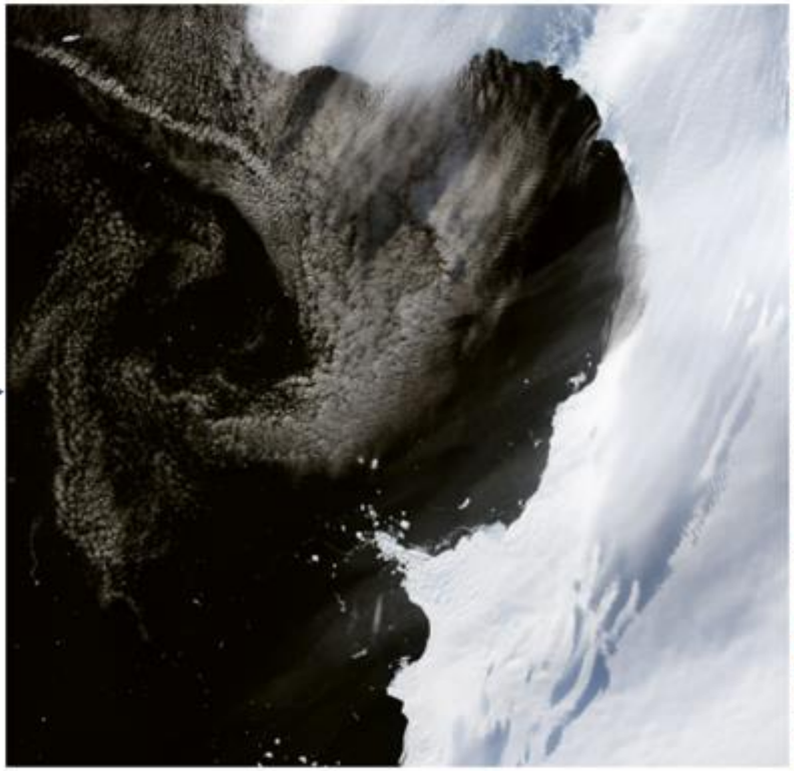
Examples of spatial patterning – spatial interfaces

tropical forest
& savanna
ecosystems
[Google Earth]



types of
stratocumulus
clouds
[RAMMB/CIRA SLIDER]

sea-ice & water
at Eltanin Bay
[NASA's Earth observatory]



algae bloom
in Lake St. Clair
[NASA's Earth observatory]

Tipping
in
Spatially Extended Systems:
detection, prediction, control

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AGU Advances

RESEARCH ARTICLE

10.1029/2025AV001698

Assessment of Abrupt Shifts in CMIP6 Models Using Edge Detection

Sjoerd Terpstra[1,2], Swinda K. J. Falkena[1], Robbin Bastiaansen[1,3], Sebastian Bathiany[4,5], Henk A. Dijkstra[1,2], and Anna S. von der Heydt[1,2]

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Peer Review The peer review history for this article is available as a PDF in the Supporting Information.

Key Points:

- Large-scale abrupt shifts are present in most CMIP6 models
- At higher levels of global warming there is a higher risk of large-scale abrupt shifts in CMIP6 models
- There is a high diversity in the onset and spatial extent of abrupt shifts between the different CMIP6 models

Supporting Information:

Supporting Information may be found in the online version of this article.

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Citation:

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Author Contributions:

Conceptualization: Sjoerd Terpstra, Henk A. Dijkstra, Anna S. von der Heydt
Data curation: Sjoerd Terpstra

Abstract

Past research has shown that multiple climate subsystems might undergo abrupt shifts, such as the Arctic Winter sea ice or the Amazon rainforest, but there are large uncertainties regarding their timing and spatial extent. In this study we investigated when and where abrupt shifts occur in the latest generation of earth system models (CMIP6) under a scenario of 1% annual increase in CO_2. We considered 82 ocean, atmosphere, and land variables across 57 models. We used a Canny edge detection method to identify abrupt shifts occurring on yearly to decadal timescales, and performed a connected component analysis to quantify the spatial extent of these shifts. The systems analyzed include the North Atlantic subpolar gyre, Tibetan Plateau, land permafrost, Amazon rainforest, Antarctic sea ice, monsoon systems, Arctic summer sea ice, Arctic winter sea ice, and Barents sea ice. Except for the monsoon systems, we found abrupt shifts in all of these across multiple models. Despite large inter-model variations, higher levels of global warming consistently increase the risk of abrupt shifts in CMIP6 models. At a global warming of 1.5°C, six out of 10 studied climate subsystems already show large-scale abrupt shifts across multiple models.

Plain Language Summary

This study investigates abrupt shifts in climate subsystems such as sea ice, monsoon systems, and permafrost, which could have severe impacts on the planet. It quantifies where and when these shifts might occur by analyzing the latest earth system models under a simulation of increasing CO_2. Using edge detection—a method to detect abrupt shifts—we identified which subsystems are vulnerable to abrupt shifts and under what levels of global warming. This helps evaluating the risks that specific climate subsystems undergo abrupt shifts under the effect of global warming. At a global warming of 1.5°C—which is a target set by the Paris climate agreement—six out of 10 studied climate subsystems showed large-scale abrupt shifts across multiple models.

1. Introduction

Research led by PhD Candidate

Sjoerd Terpstra

Abrupt shifts in CMIP6

Assessment of multiple subsystems:

1. Do abrupt shifts occur?
2. In how many models do they occur?
3. At which global warming level do they occur?

- 57 earth system models
- High forcing scenario → 1% CO_2
  - Pre-industrial to 4x CO_2
  - 150 years
- Two-dimensional ocean, atmosphere and land variables

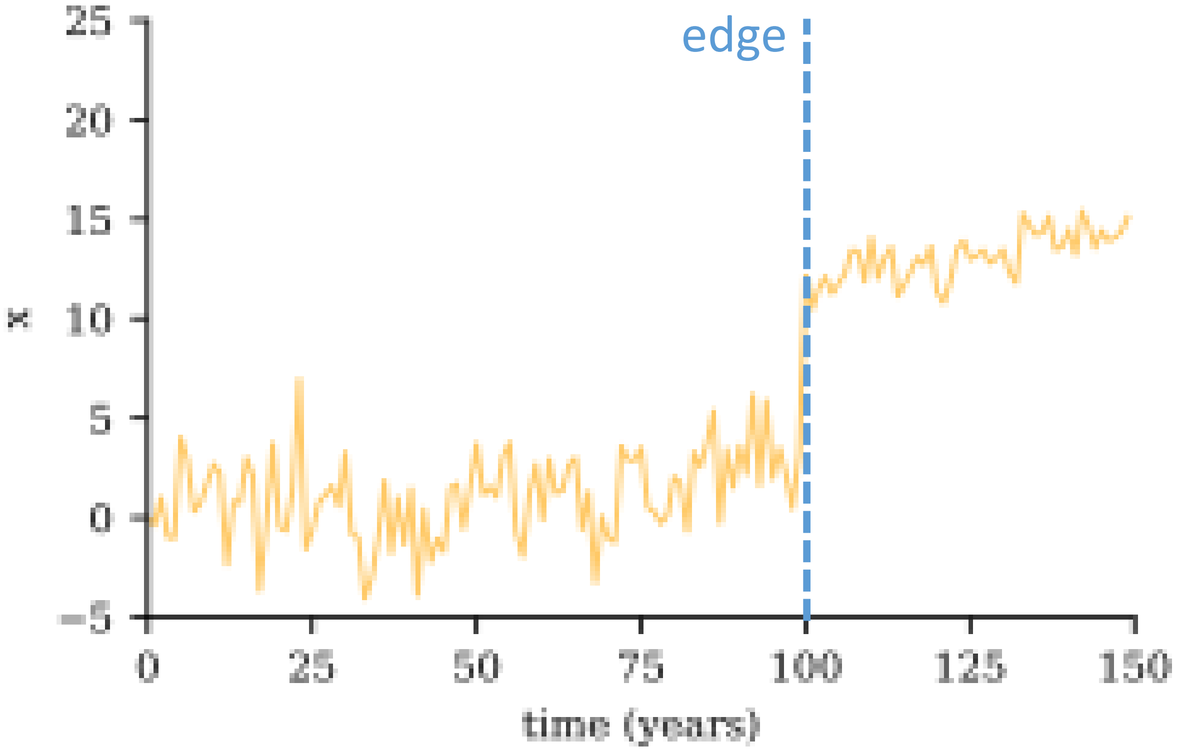
CMIP
Coupled Model Intercomparison Project

Edge detection

Canny **edge detection** adapted for spatiotemporal climate data (Bathiany et al. 2020)

- (longitude, latitude, time)

Edges are points in time and space at which the gradient is locally large

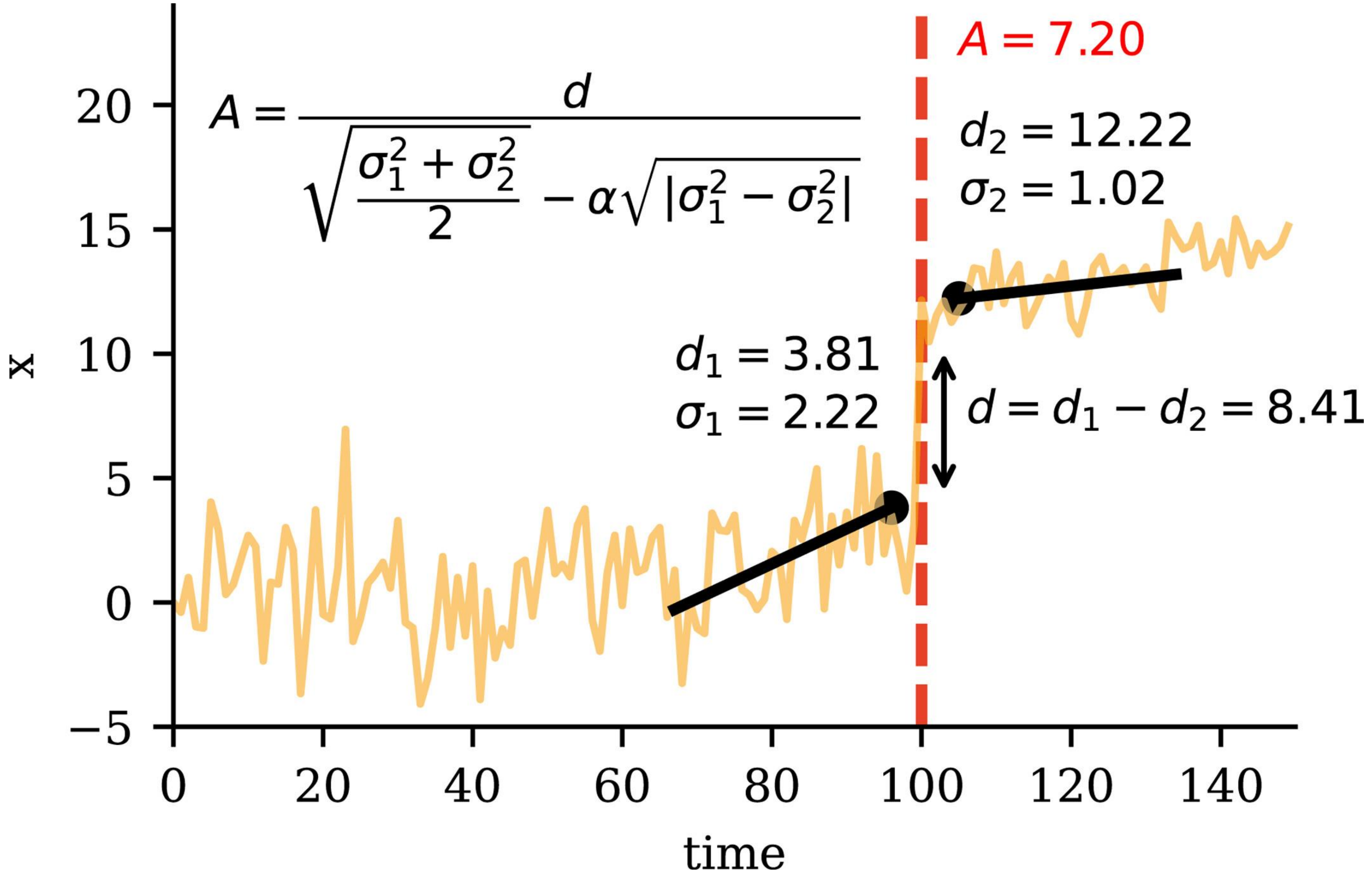


Abruptness measure

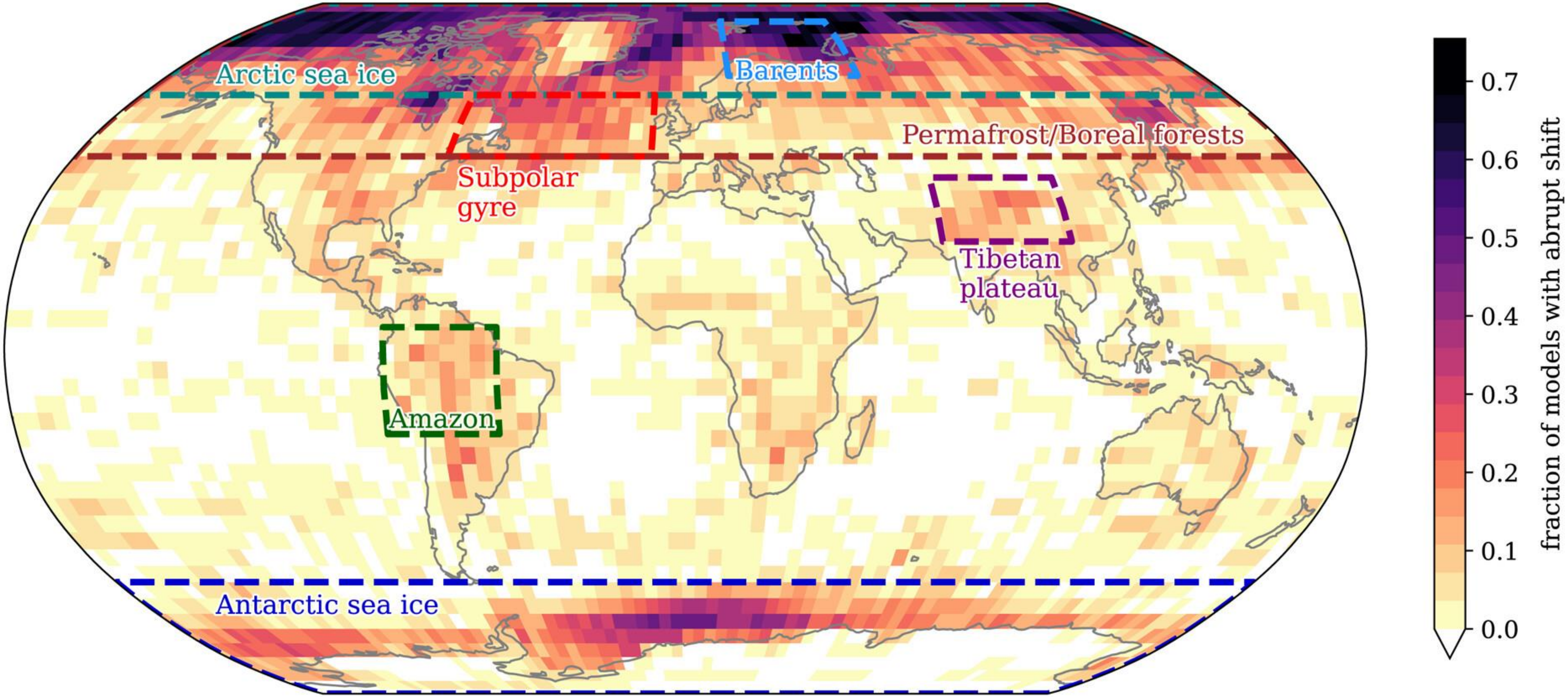
Via effect size metric

Depends on

- Change in variable
- Change in variability

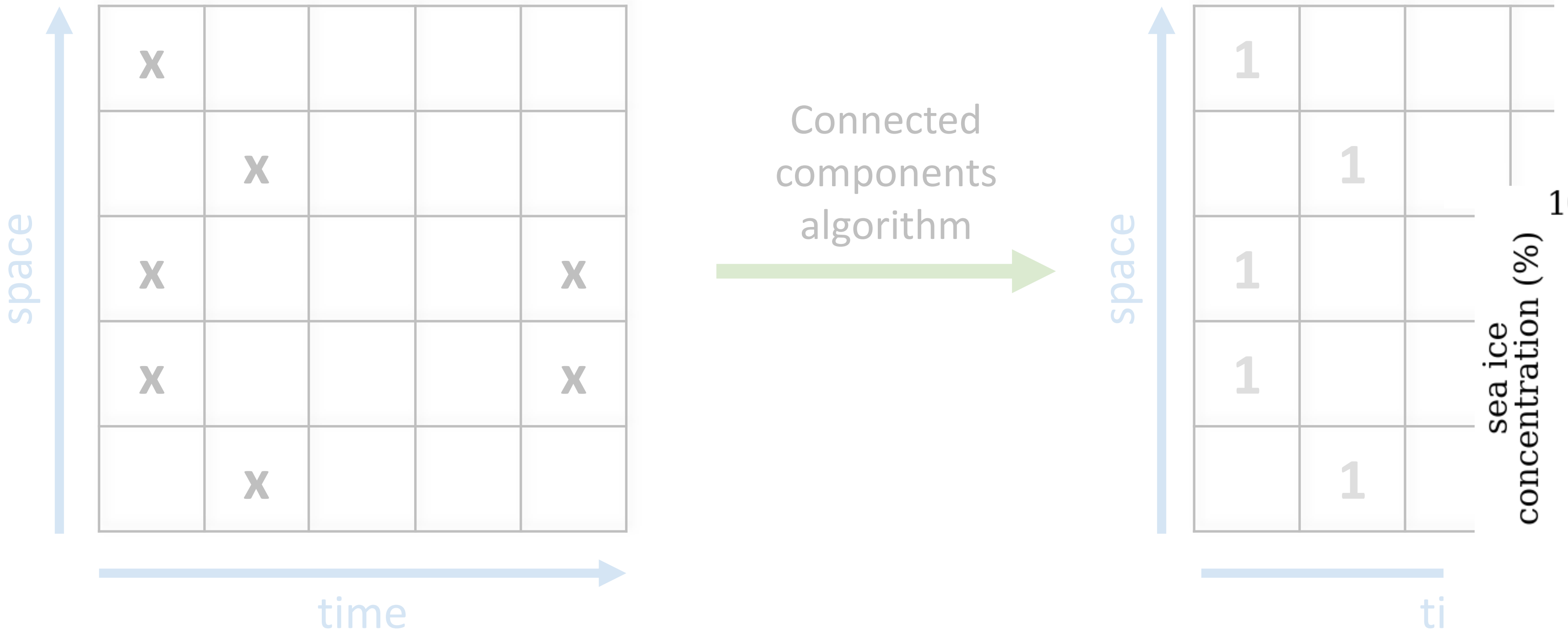


Fraction of Models with Abrupt Shift

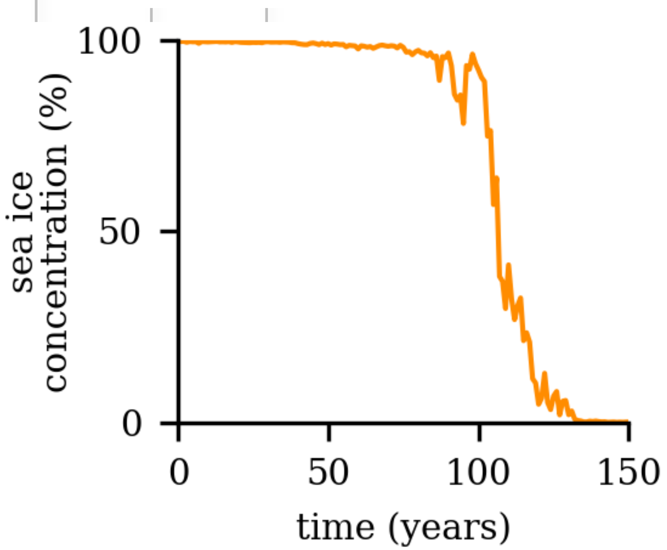


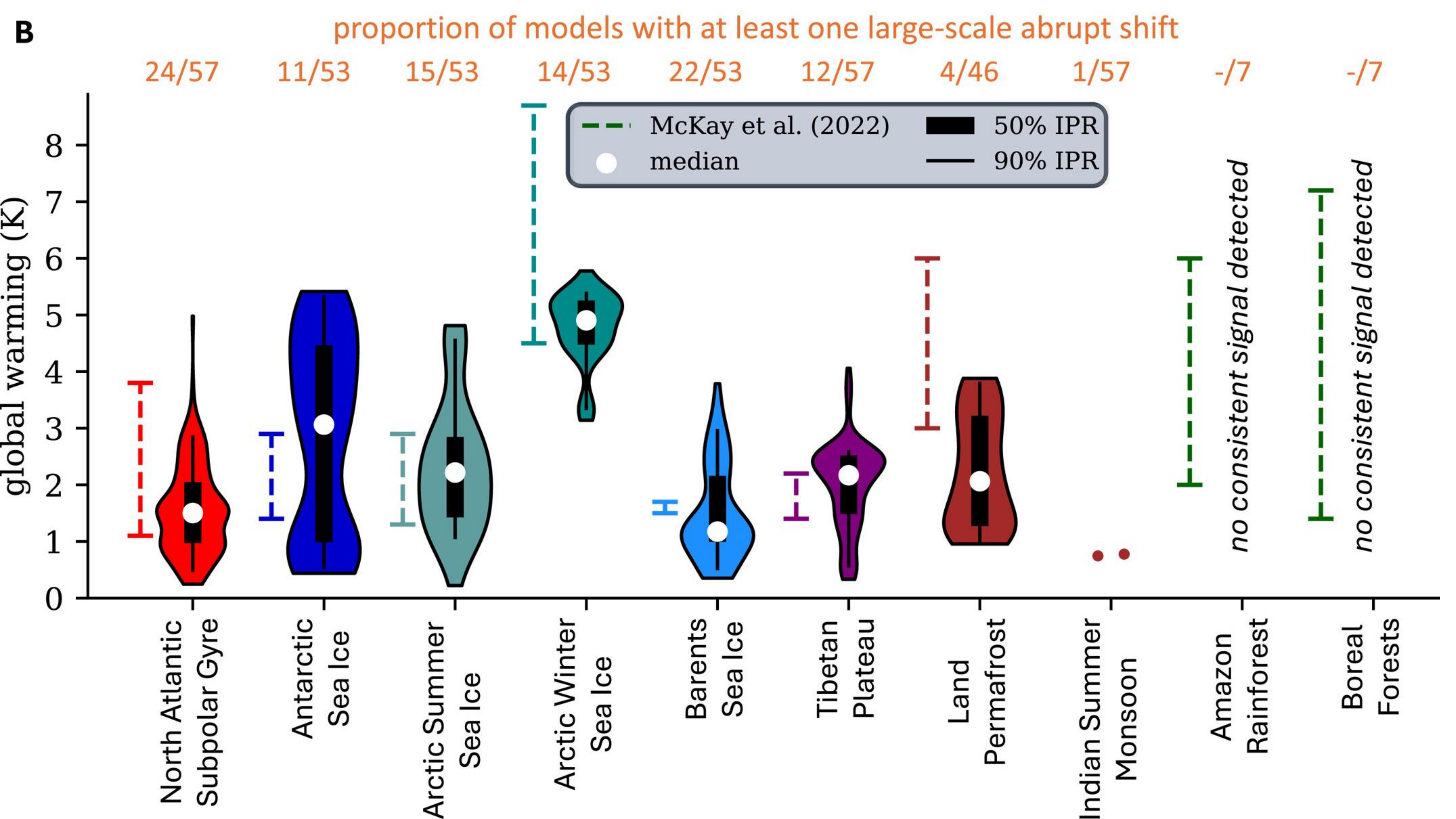
Large-scale abrupt shifts

Dynamically build regions within the subsystems that undergo an **abrupt shift** simultaneously → **large-scale abrupt shift**



Arctic winter sea ice




B
proportion of models with at least one large-scale abrupt shift
24/57
11/53
15/53
14/53
22/53
12/57
4/46
1/57
-/7
-/7
McKay et al. (2022)
median
50% IPR
90% IPR
global warming (K)
0
1
2
3
4
5
6
7
8
no consistent signal detected
no consistent signal detected
North Atlantic Subpolar Gyre
Antarctic Sea Ice
Arctic Summer Sea Ice
Arctic Winter Sea Ice
Barents Sea Ice
Tibetan Plateau
Land Permafrost
Indian Summer Monsoon
Amazon Rainforest
Boreal Forests

Tipping
in
Spatially Extended Systems:
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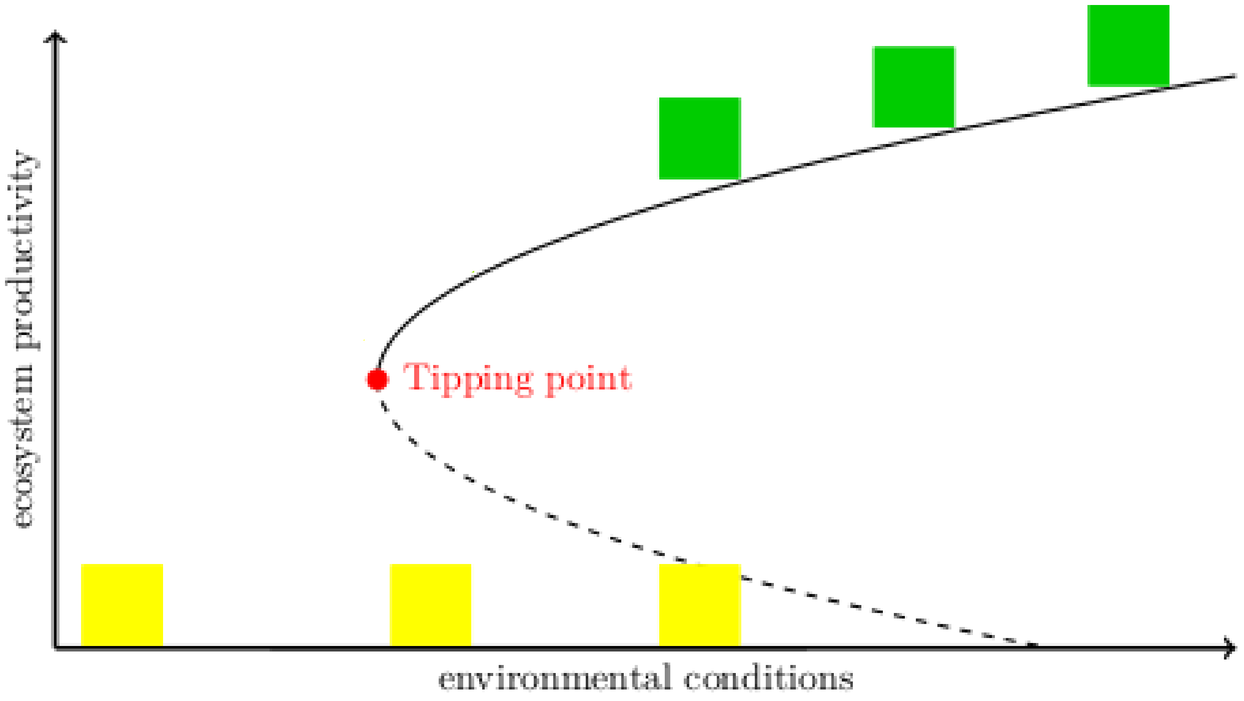
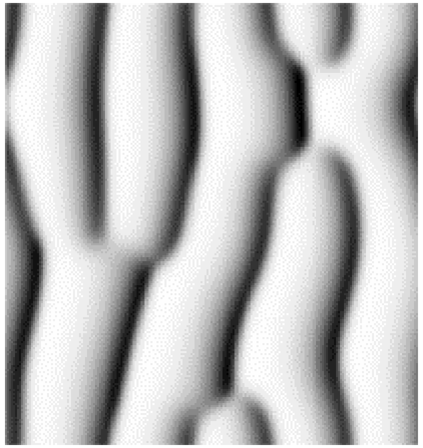
Patterns in models

Add spatial transport:

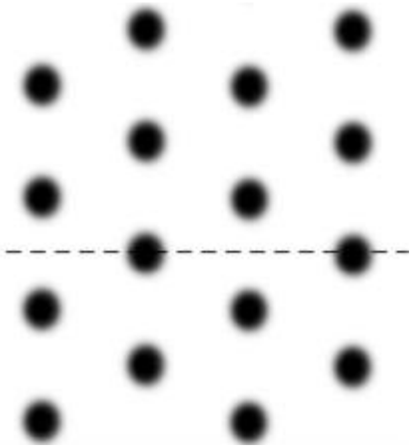
Reaction-Diffusion equations:

$$\begin{cases} \dfrac{du}{dt} = f(u,v) + D_u\, \Delta u \\ \dfrac{dv}{dt} = g(u,v) + D_v\, \Delta v \end{cases}$$



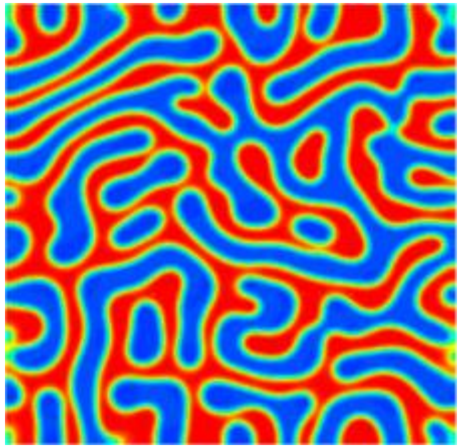


[Klausmeier, 1999]



[Gilad et al, 2004]

[Rietkerk et al, 2002]



[Liu et al, 2013]

Turing patterns

$$\begin{cases} \dfrac{du}{dt} = f(u,v) + D_u\,\Delta u \\ \dfrac{dv}{dt} = g(u,v) + D_v\,\Delta v \end{cases}$$

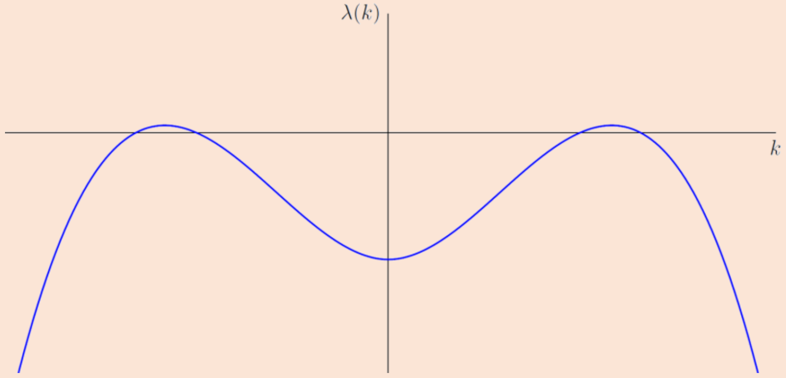
Turing bifurcation

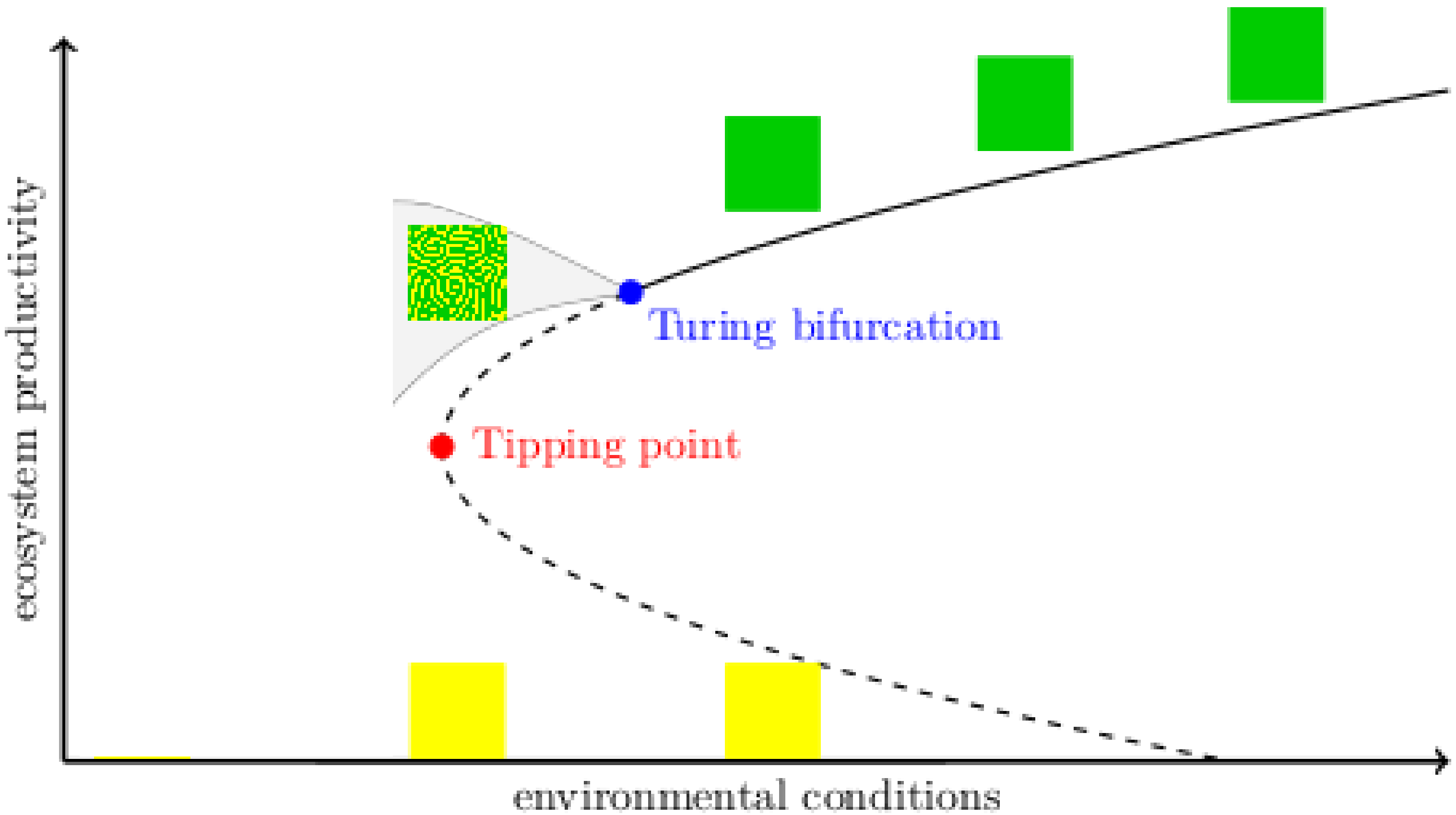
Instability to non-uniform perturbations

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_* \\ v_* \end{pmatrix} + e^{\lambda t} e^{ikx} \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}$$

→ Dispersion relation

$$\lambda(k) = \cdots$$





Weakly non-linear analysis

Ginzburg-Landau equation / Amplitude Equation

& Eckhaus/Benjamin-Feir-Newel criterion

[Eckhaus, 1965; Benjamin & Feir, 1967; Newell, 1974]

Busse balloon

$$\begin{cases} \frac{du}{dt} = f(u,v) + D_u \Delta u \\ \frac{dv}{dt} = g(u,v) + D_v \Delta v \end{cases}$$

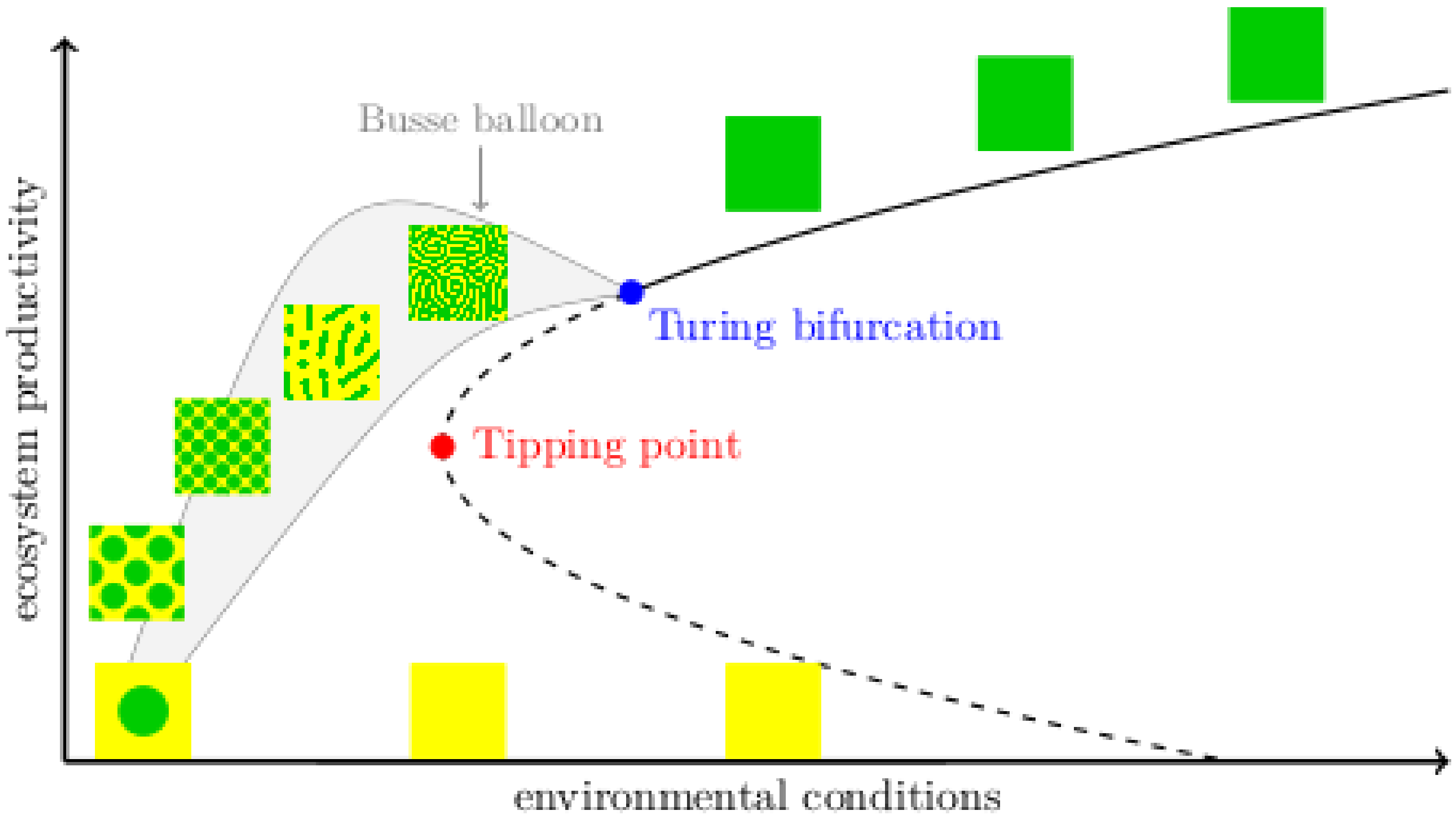
Busse balloon

A model-dependent shape in *(parameter, observable)* space that indicates all stable patterned solutions to the PDE.

Construction Busse balloon

Via numerical continuation

few general results on the shape of Busse balloon

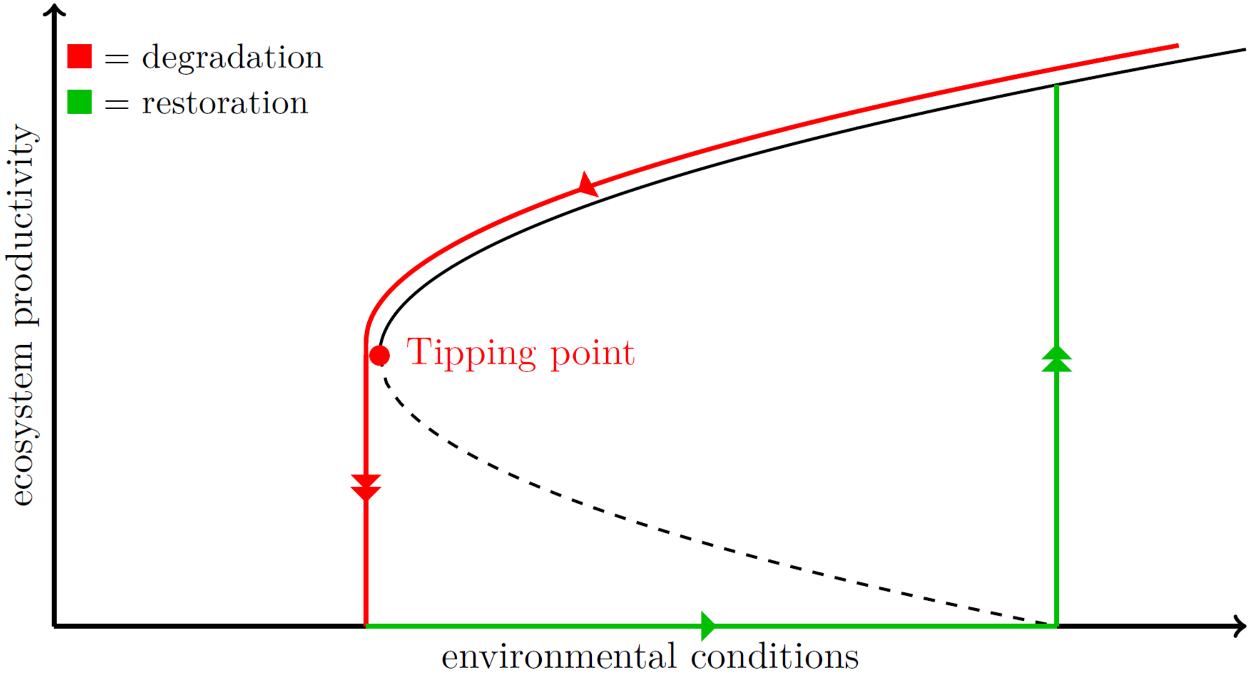


Busse balloon

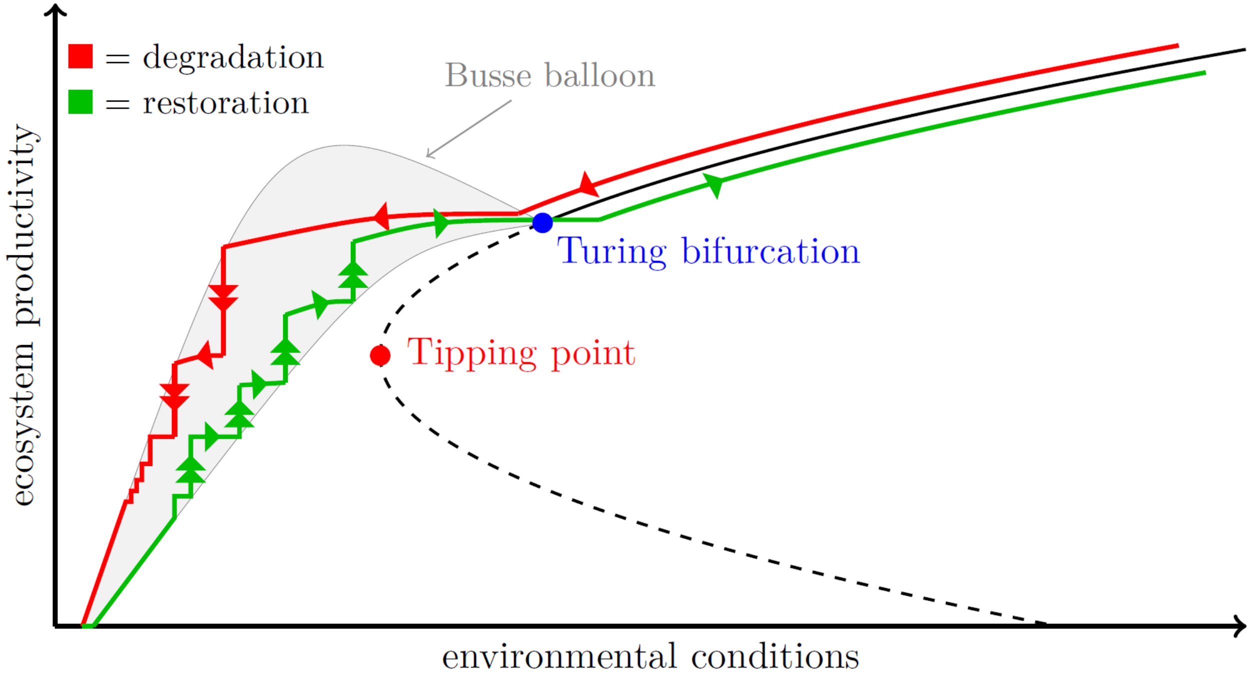
Idea originates from thermal convection

[Busse, 1978]

Tipping of patterned systems

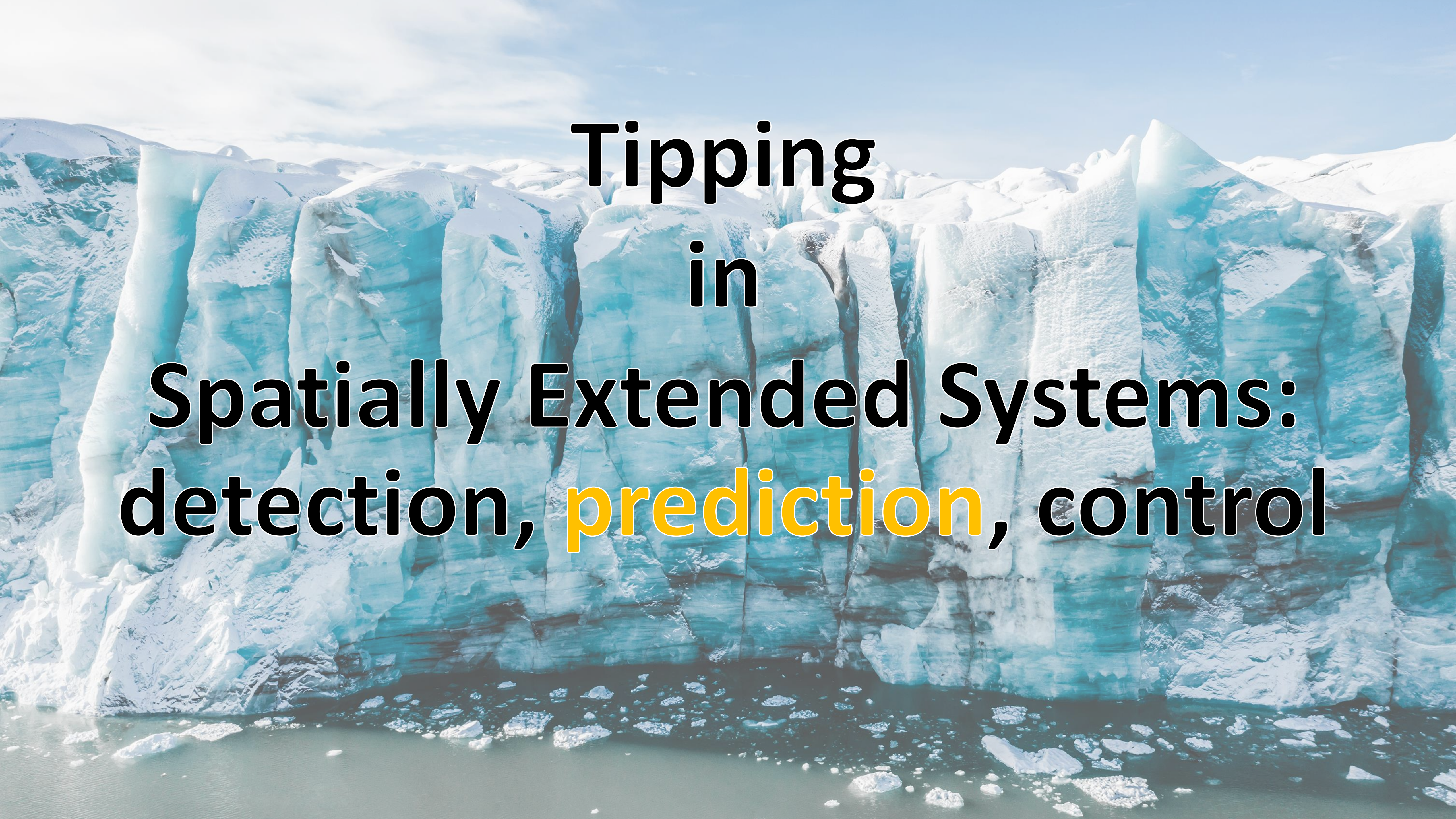


Classic tipping



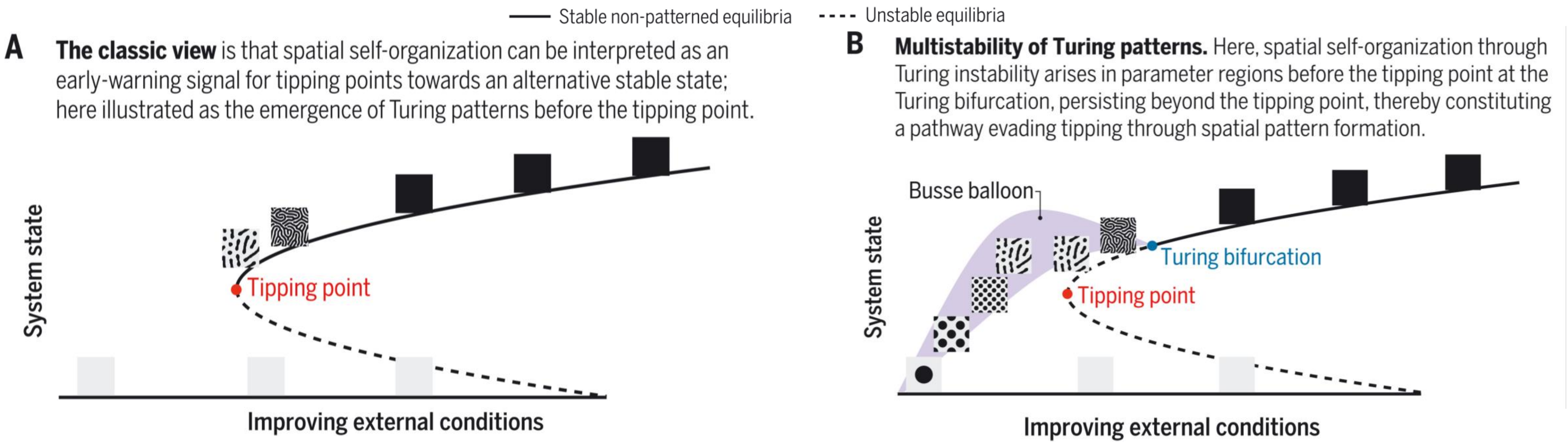
Tipping of patterns

Rietkerk, M., Bastiaansen, R., Banerjee, S., van de Koppel, J., Baudena, M., & Doelman, A. (2021). Evasion of tipping in complex systems through spatial pattern formation. Science, 374(6564), eabj0359.



Tipping
in
Spatially Extended Systems: detection, prediction, control

Early warning of Critical transitions: distinguishing tipping points from Turing destabilizations



Research by Phd Candidate

Paul Sanders

preprint on arXiv

Recall: dispersion relations

Turing patterns

$$\begin{cases} \frac{du}{dt} = f(u,v) + D_u \, \Delta u \\ \frac{dv}{dt} = g(u,v) + D_v \, \Delta v \end{cases}$$

Turing bifurcation

Instability to non-uniform perturbations

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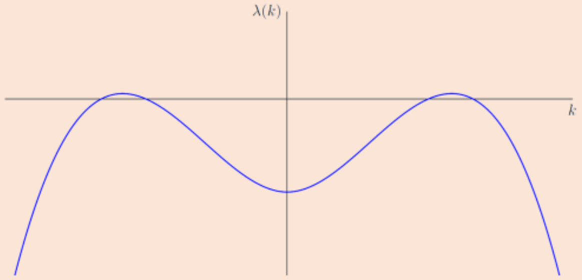
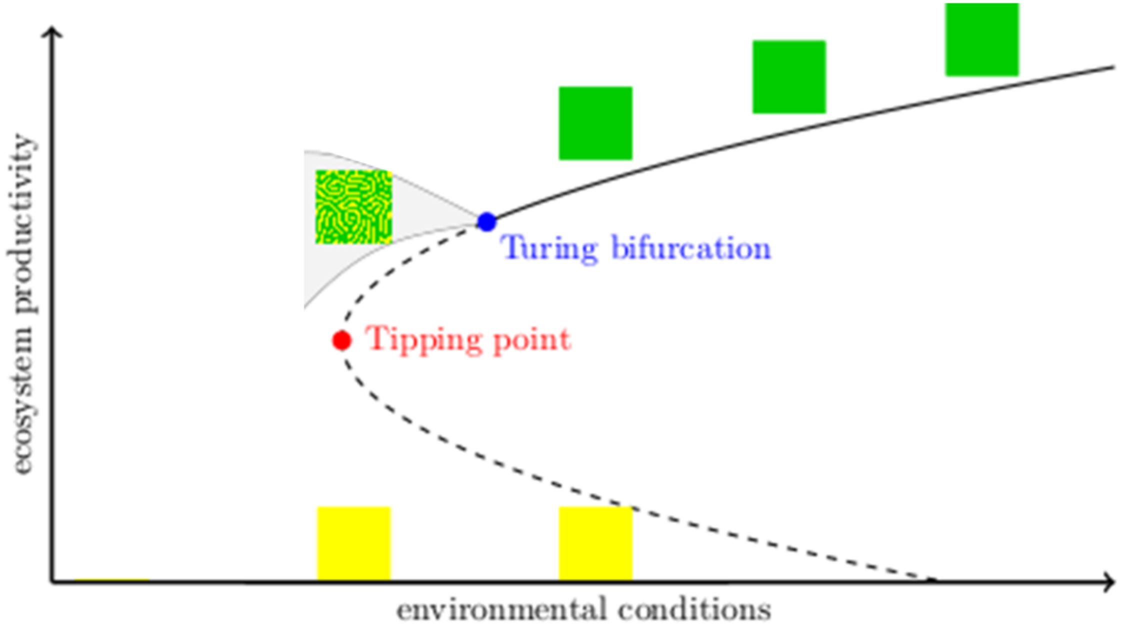




Weakly non-linear analysis

Ginzburg-Landau equation / Amplitude Equation

& Eckhaus/Benjamin-Feir-Newel criterion

[Eckhaus, 1965; Benjamin & Feir, 1967; Newell, 1974]

General stability Analysis Method

Data Y(x, t)

Perturbed Data

$$\delta Y(x,t) = Y(x,t) - \overline{Y}(t)$$

fit perturbed data to a **linearized spatial stability model**

$$\delta Y_t = A\delta Y + B\delta Y$$

Spatial differential operator

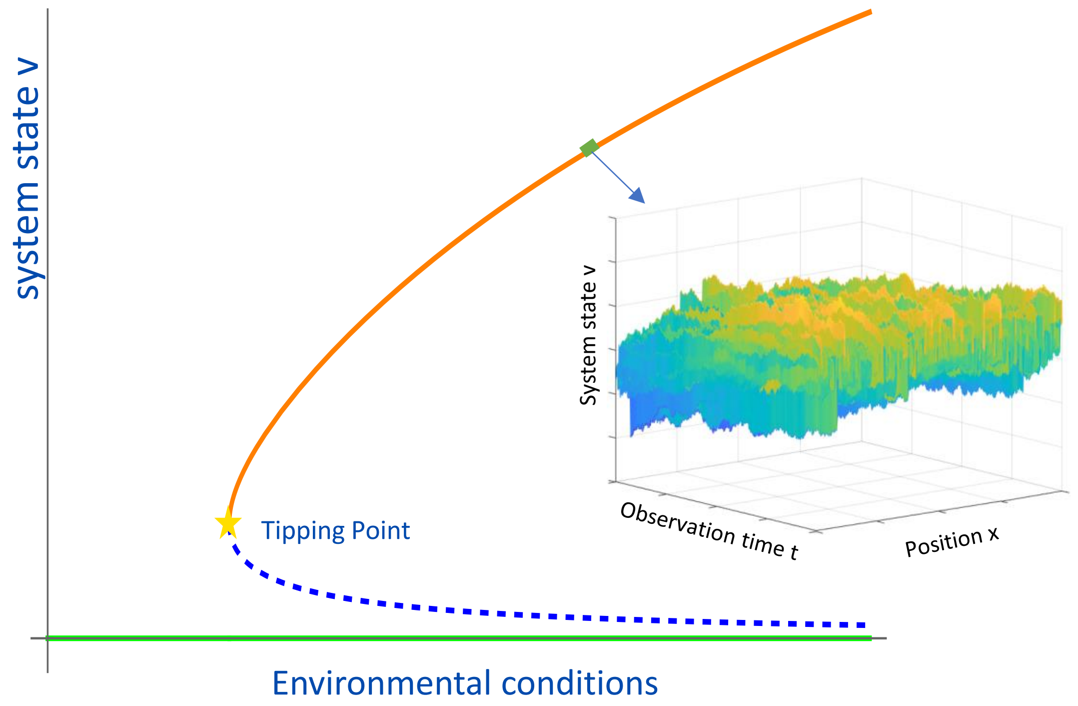
Retrieve **dispersion relation** to understand the **stability of different spatial patterns**

$$\lambda(k)\delta Y = A\delta Y + \hat{B}(k)\delta Y$$

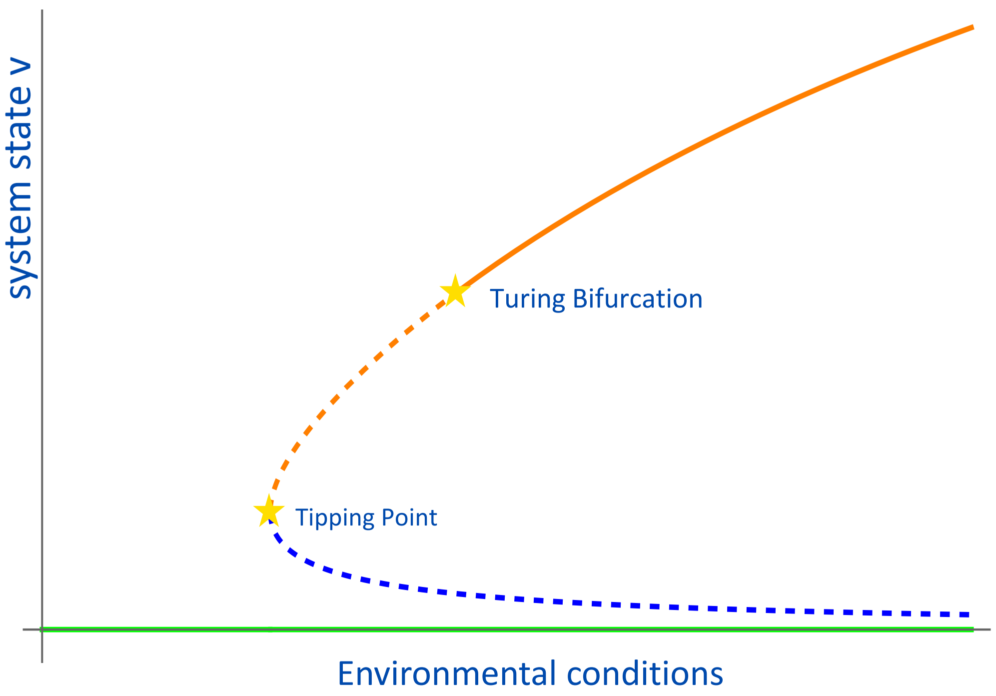
Synthetic data test: Extended Klausmeier model

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + p - u - uv^2 + Noise_1(x,t)$$
$$\frac{\partial v}{\partial t} = \delta\frac{\partial^2 v}{\partial x^2} + uv^2(1 - hv) - mv + Noise_2(x,t)$$

Case 1: Saddle-node bifurcation



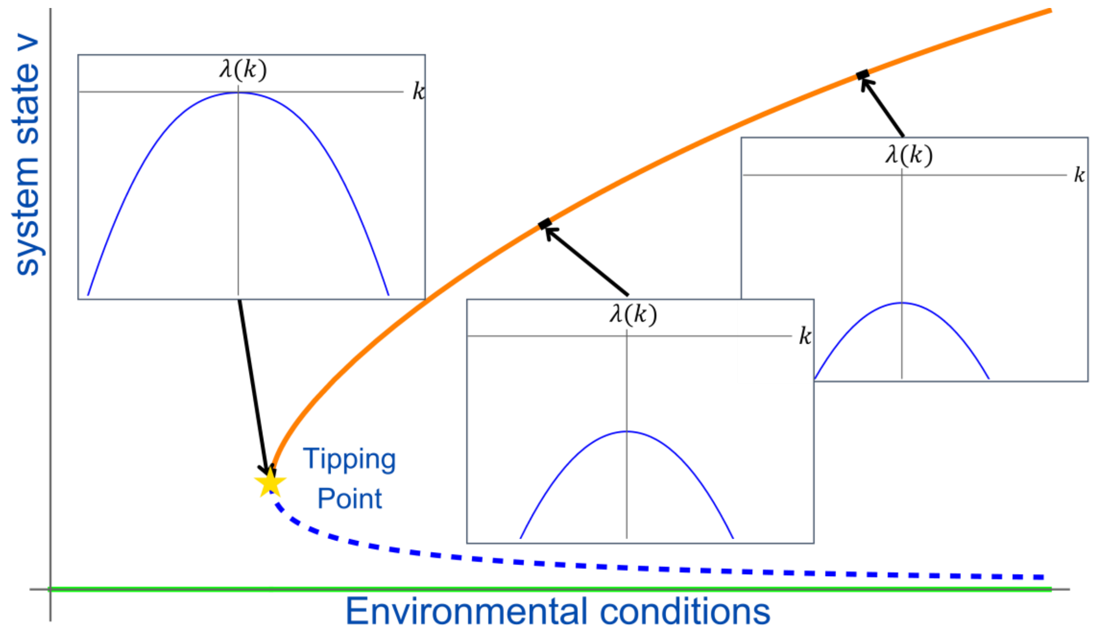
Case 2: Turing bifurcation



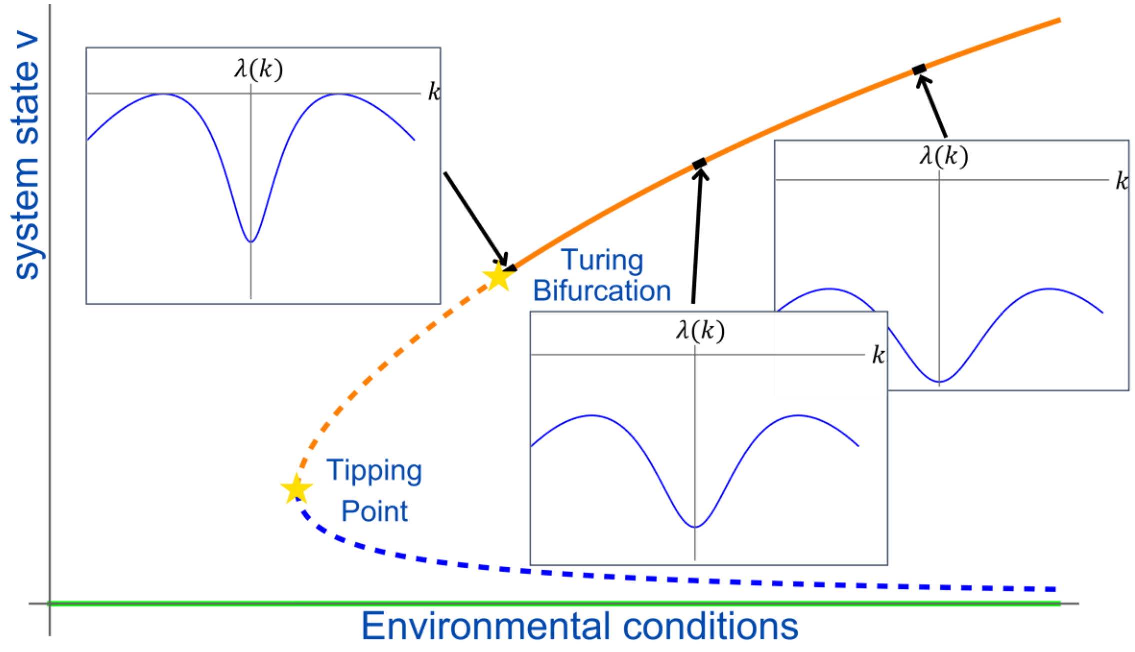
Theoretical outcome

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + p - u - uv^2 + Noise_1(x,t)$$
$$\frac{\partial v}{\partial t} = \delta \frac{\partial^2 v}{\partial x^2} + uv^2(1 - hv) - mv + Noise_2(x,t)$$

Case 1: Saddle-node bifurcation

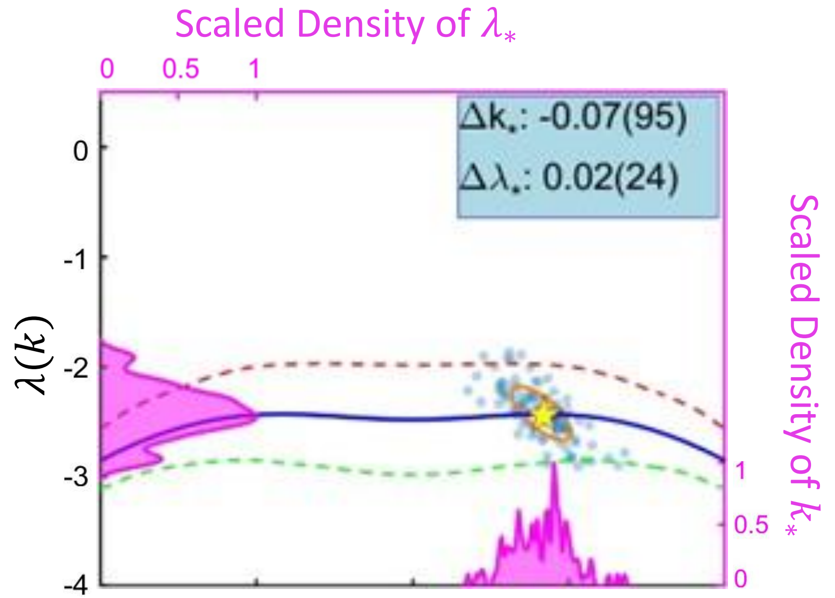


Case 2: Turing bifurcation

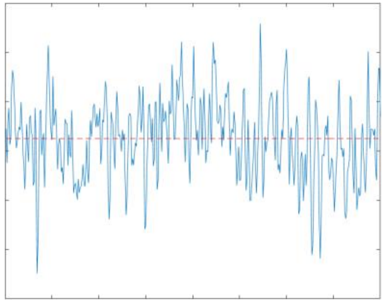


Testing the method on Critical behaviour

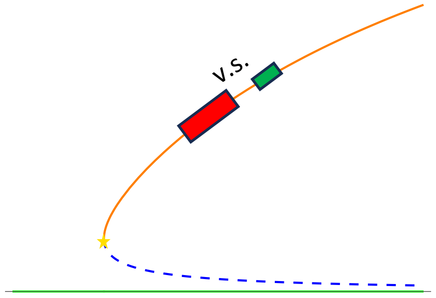
- 100 Distinct datasets
- Spatially Correlated Noise
- Average and extreme dispersion relation outcomes plotted
- Density plotted for found dominant eigenmode k_* and dominant eigenvalue λ_*
- $\Delta k_* = k_{*,real} - k_*$



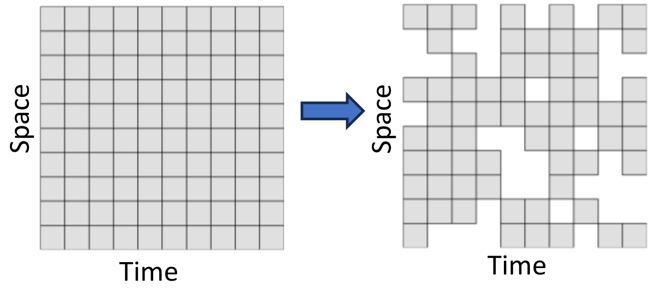
1. Dynamical Noise



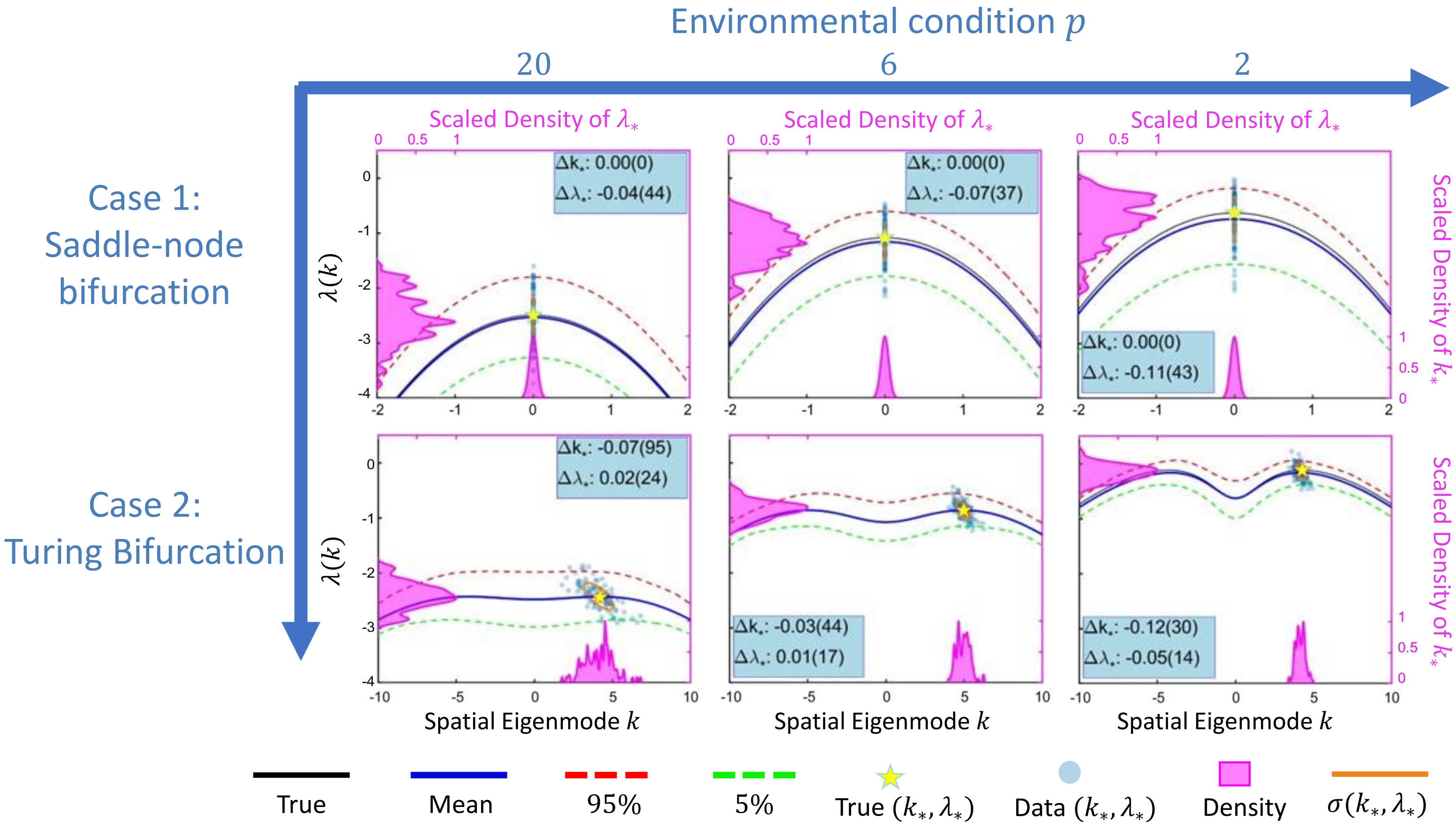
2. Observation time

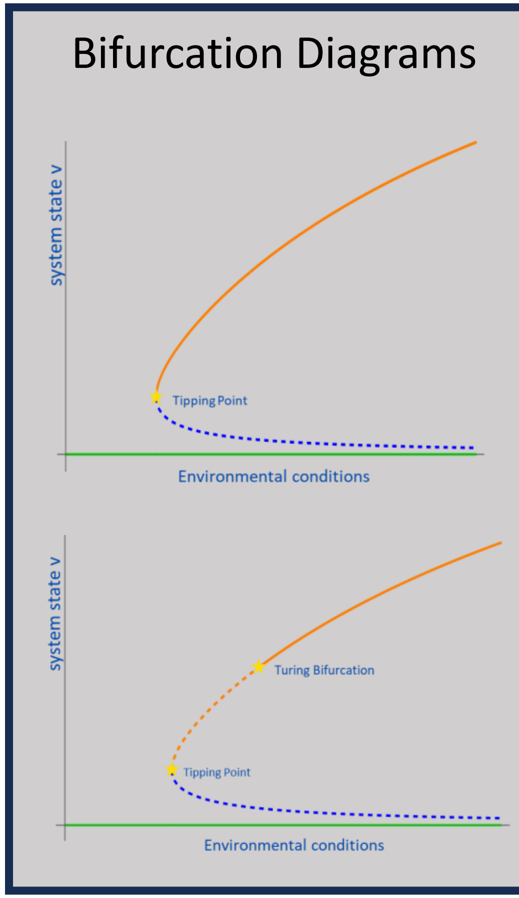


3. Data Sampling



Constant forcing numerical experiment

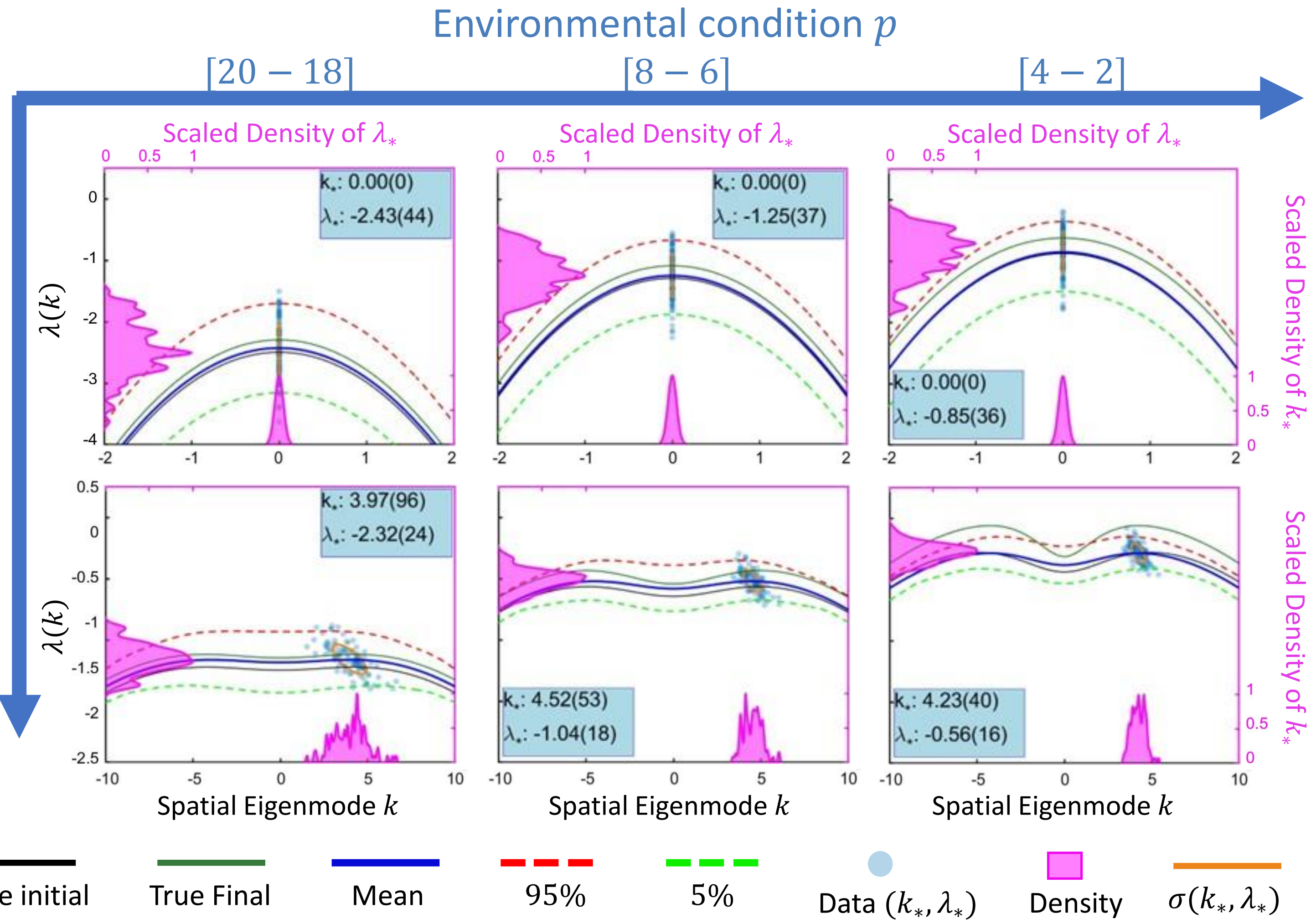


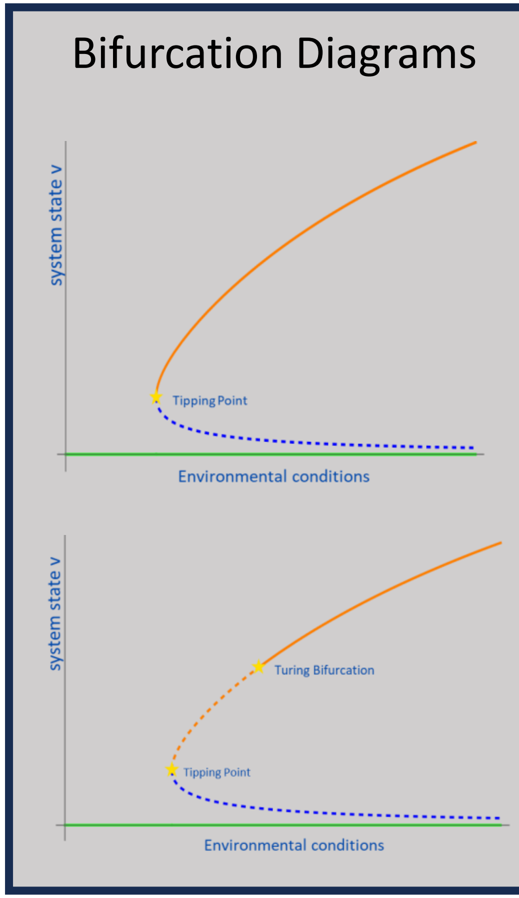


Time-varying forcing numerical experiment

Case 1:
Saddle-node bifurcation

Case 2:
Turing Bifurcation



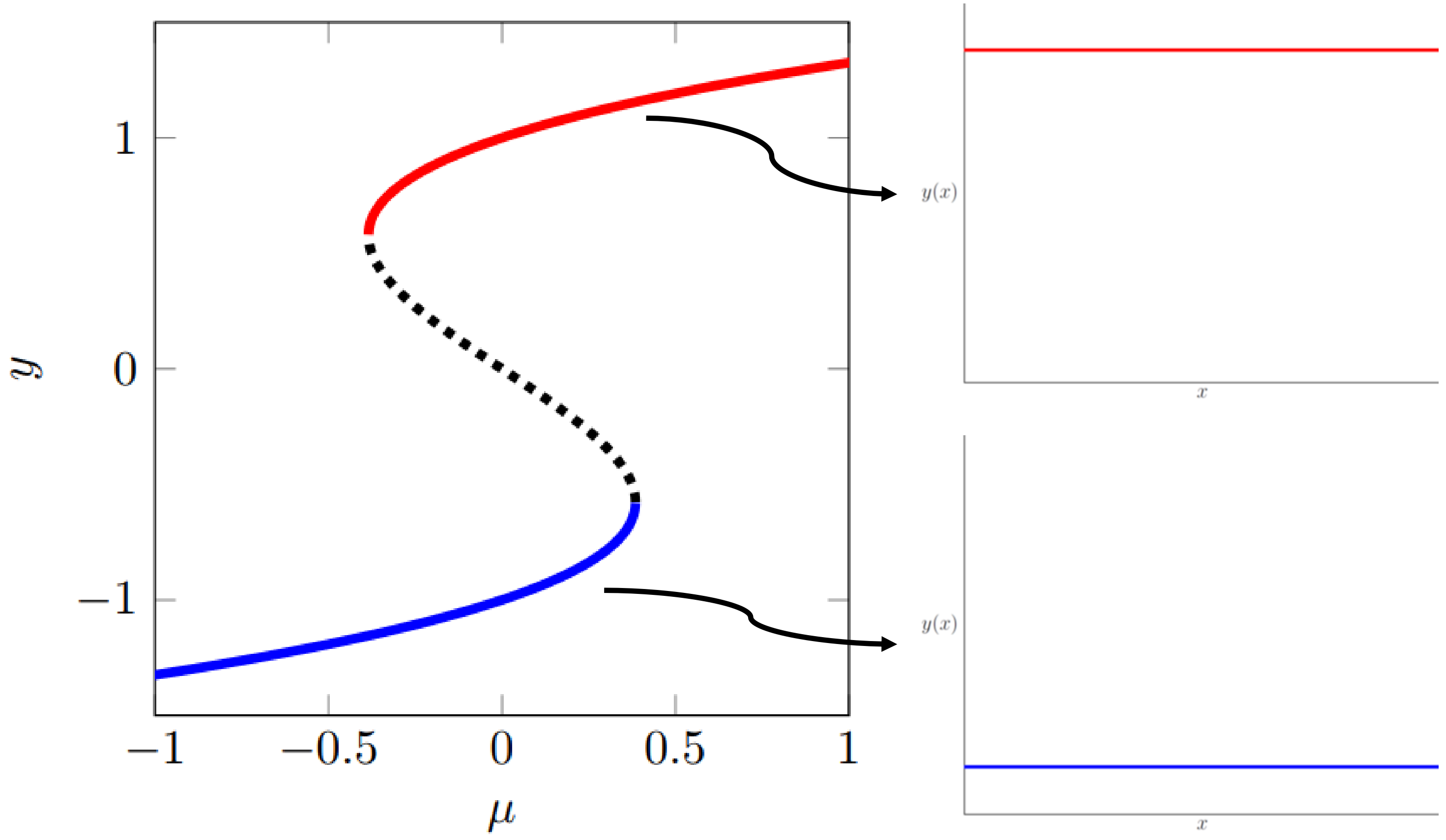


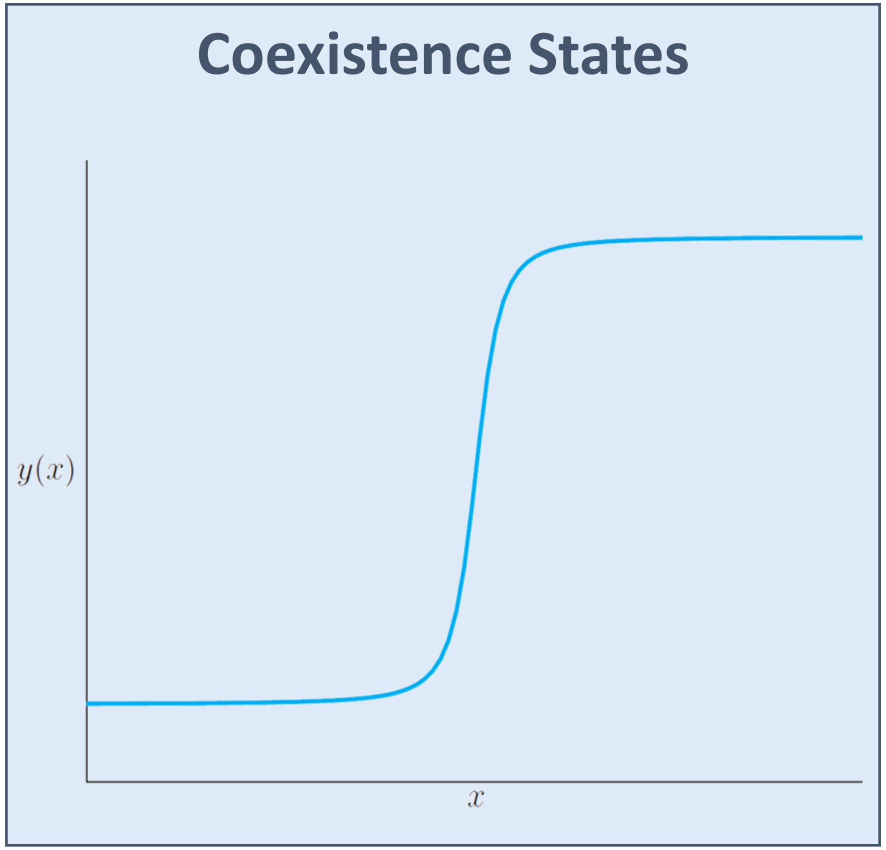
Tipping in Spatially Extended Systems: detection, prediction, control

Coexistence states

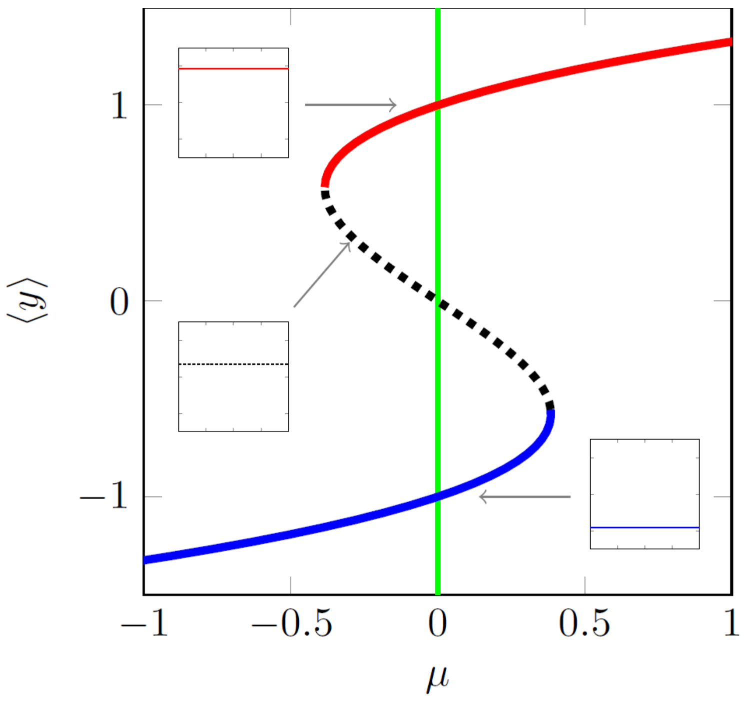
Bistable (Allen-Cahn/Nagumo) equation:

$$\frac{\partial y}{\partial t} = y(1 - y^2) + \mu + D\,\frac{\partial^2 y}{\partial x^2}$$





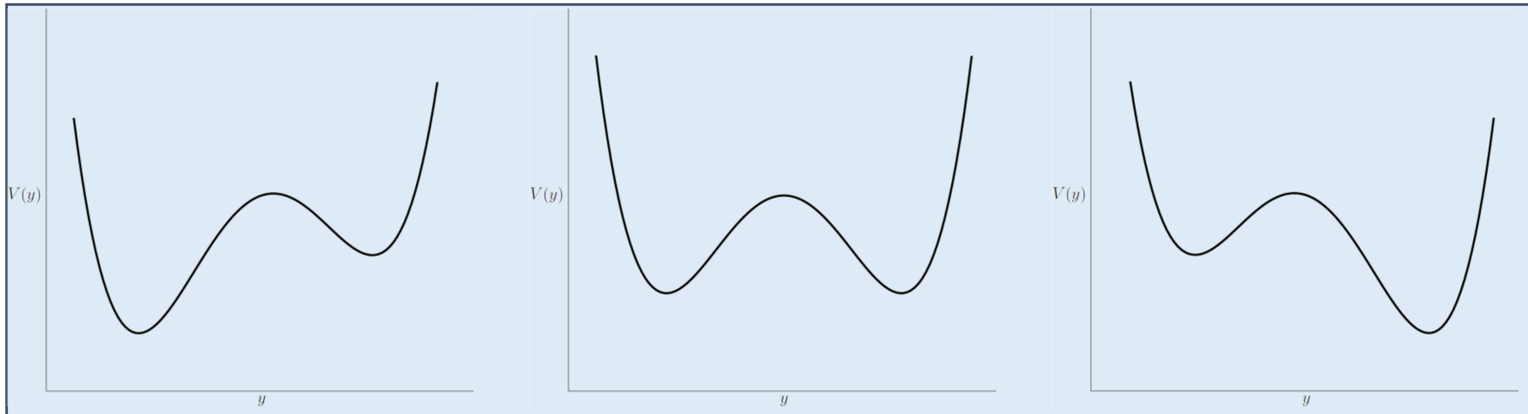
Front Dynamics



$$\frac{\partial y}{\partial t} = D\,\frac{\partial^2 y}{\partial x^2} + f(y;\mu)$$

Potential function $V(y;\mu)$:

$$\frac{\partial V}{\partial y}(y;\mu) = -f(y;\mu)$$

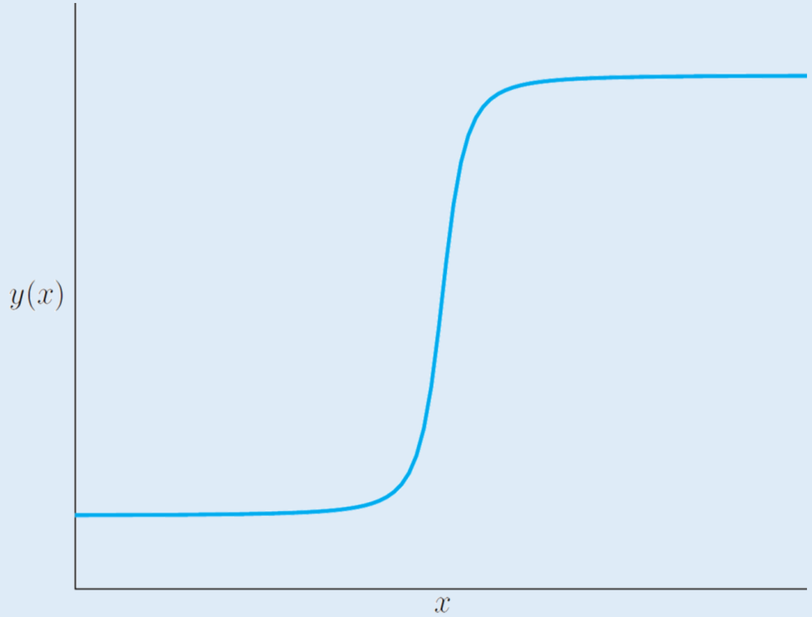


moves right

stationary

moves left

Maxwell Point $\mu_{maxwell}$



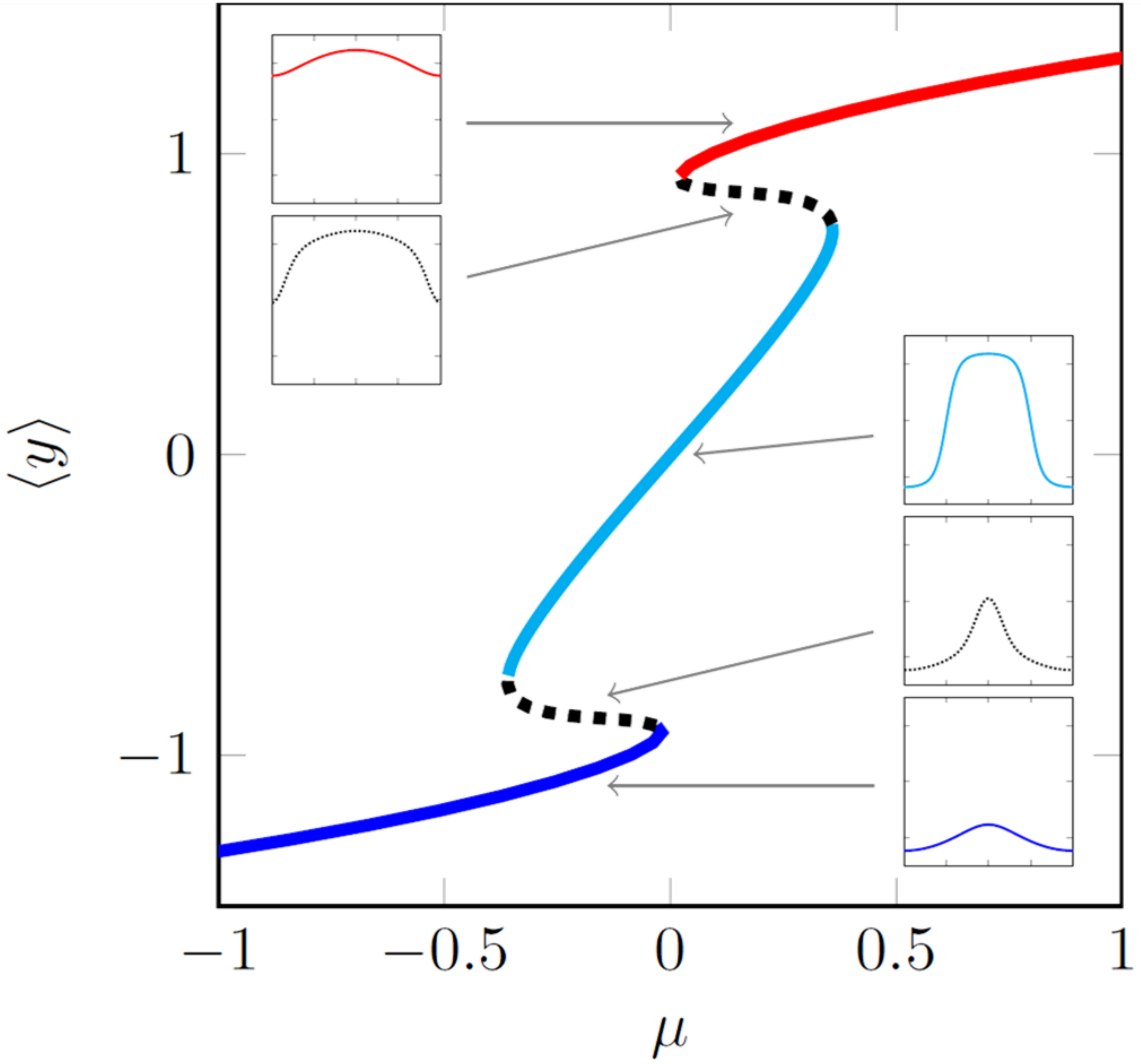
Adding Spatial Heterogeneity

$$\frac{\partial y}{\partial t} = D\frac{\partial^2 y}{\partial x^2} + f(y, {\color{red}x}; \mu)$$

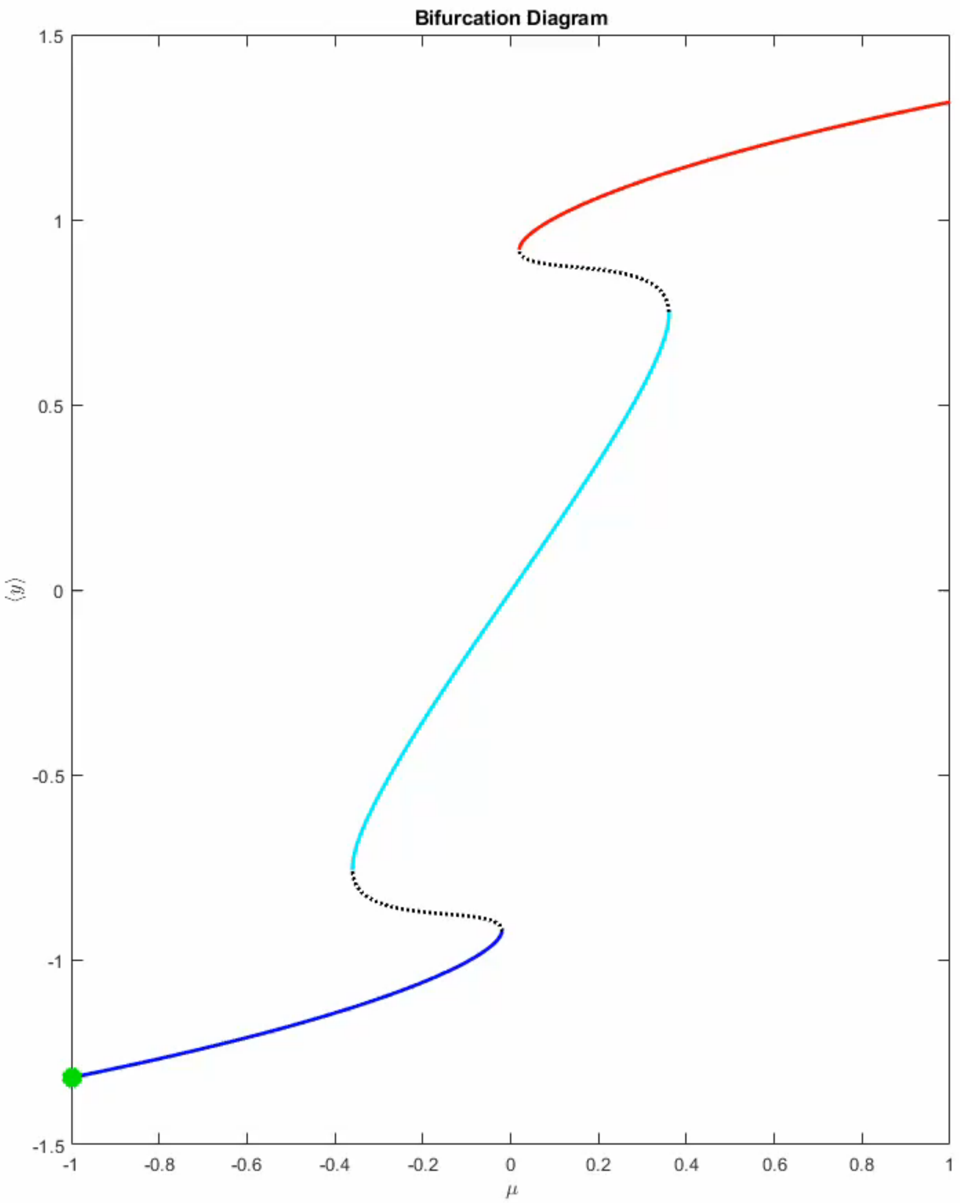
Now, the **local** difference in potentials determines the front movement

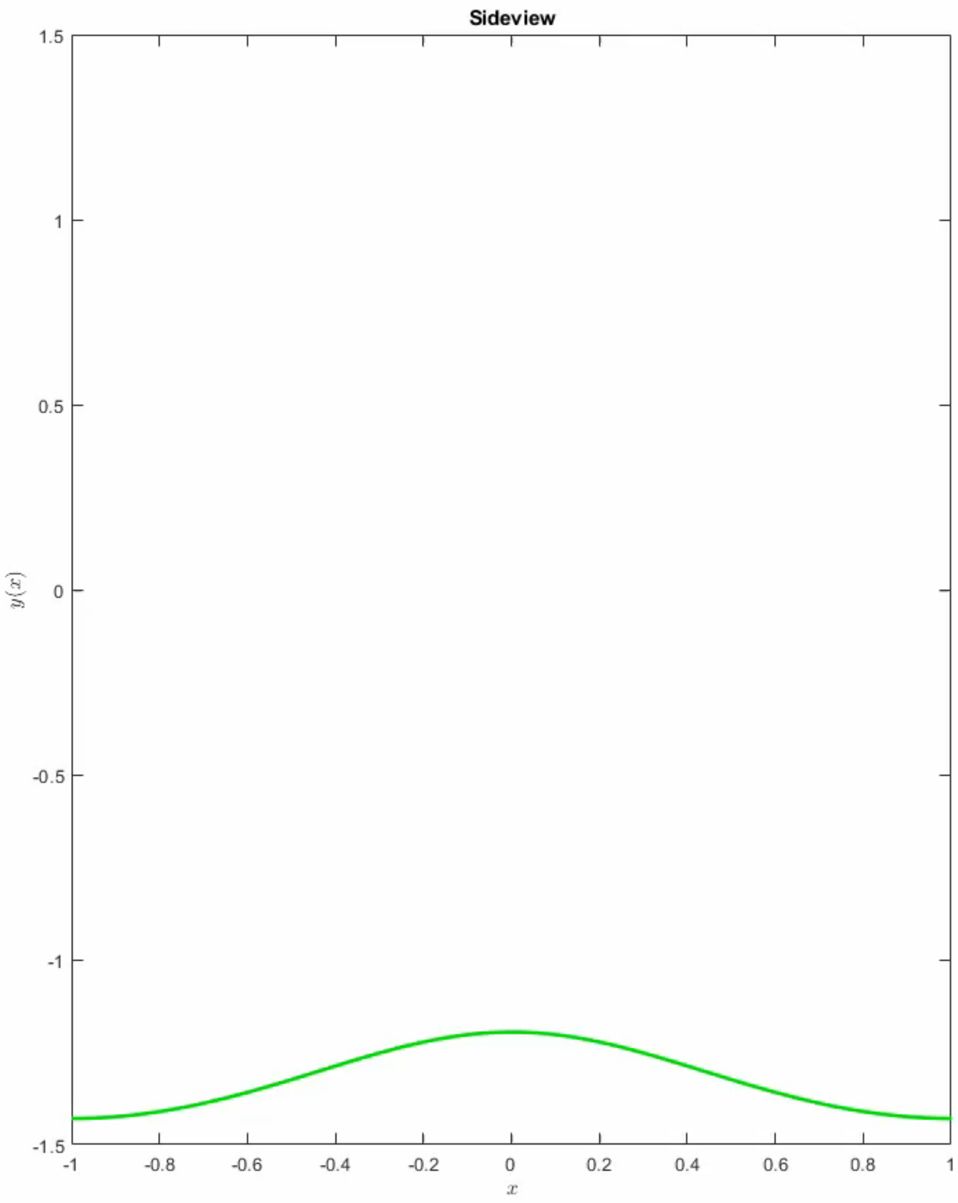
New behaviour:

- Multi-fronts can be stationary [Bastiaansen, Doelman, Kaper, 2025, *preprint*]
- Maxwell point is smeared out

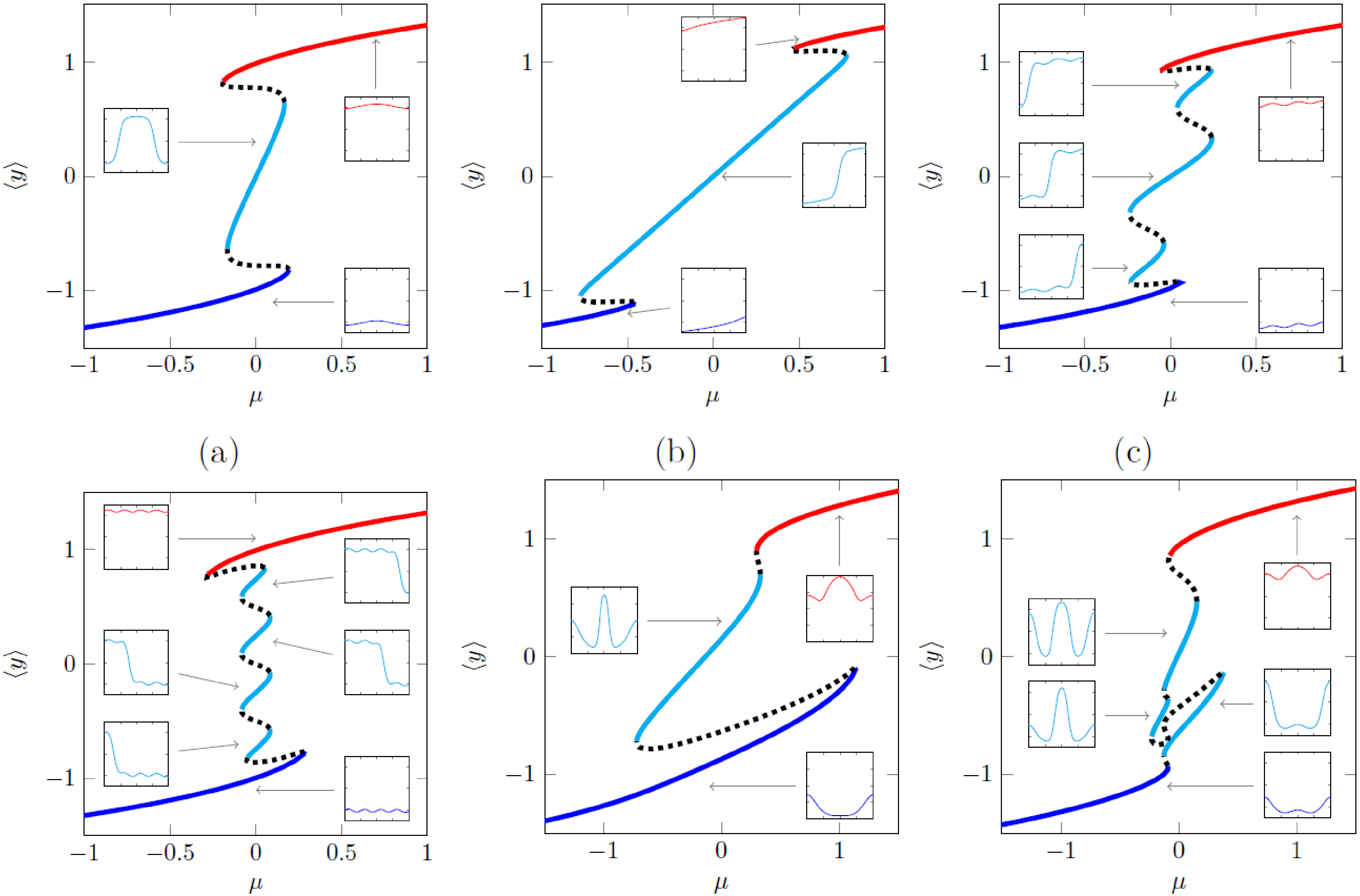


Fragmented Tipping





Other Spatial Heterogeneities



[Bastiaansen, Dijkstra, Von der Heydt, 2022]

Tipping

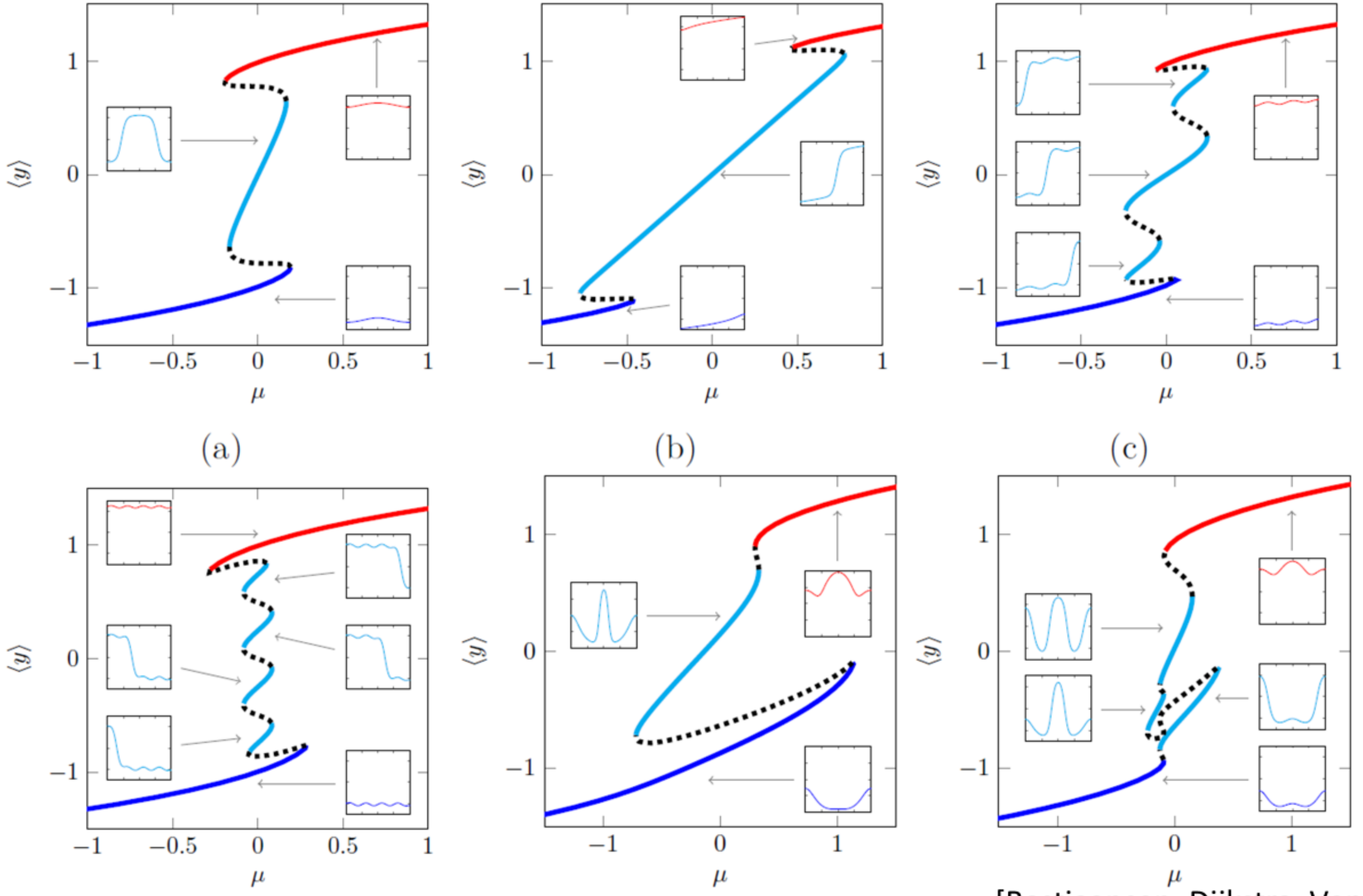
in

Spatially Extended Systems:

detection, prediction, control

Optimisation of heterogeneity in Allen Cahn equation

Other Spatial Heterogeneities



[Bastiaansen, Dijkstra, Von der Heydt, 2022]

Research by Phd Candidate

Aurora Faure Ragani

research in progress

Can’t we just pick and choose our favourite!?

Idea: with local action, prevent larger-scale change

Source: Andrew Millison on Youtube (see his videos on Great Green Wall)

Approach: make it an optimisation problem

Mathematical Optimisation Problem

Minimize	$f(z)$
subject to	$g(z) = 0$
	$h(z) \leq 0$

Approach 1: “simulation-like”

Mathematical Optimisation Problem

Minimize $f(z)$

subject to $g(z) = 0$

$h(z) \leq 0$

COST FUNCTION

For example:

Minimize the globally averaged state at time t_e

$$f(z) := \int y(x, t_e; z)\, dx$$

Model-Constraint

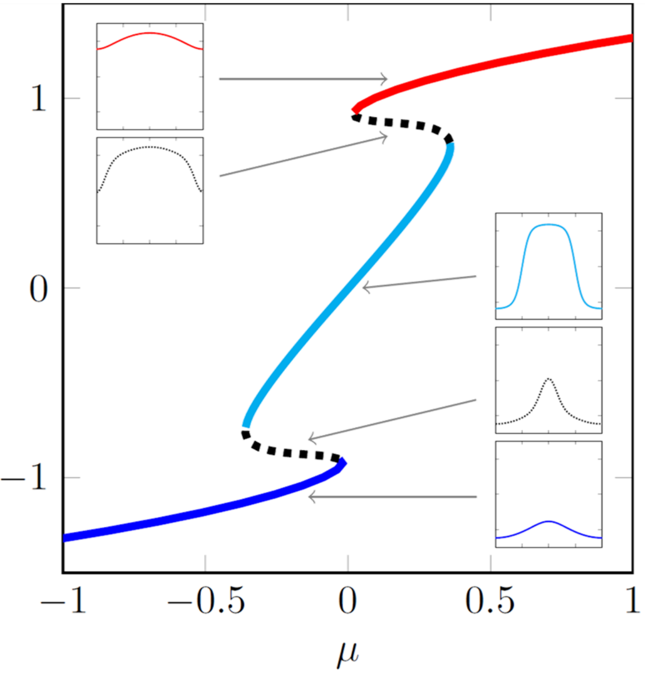
Some model with specified scenario:

$$g(z) := -\frac{\partial y}{\partial t} + D\,\frac{\partial^2 y}{\partial x^2} + y(1 - y^2) + \mu(x,t) + z(x)$$

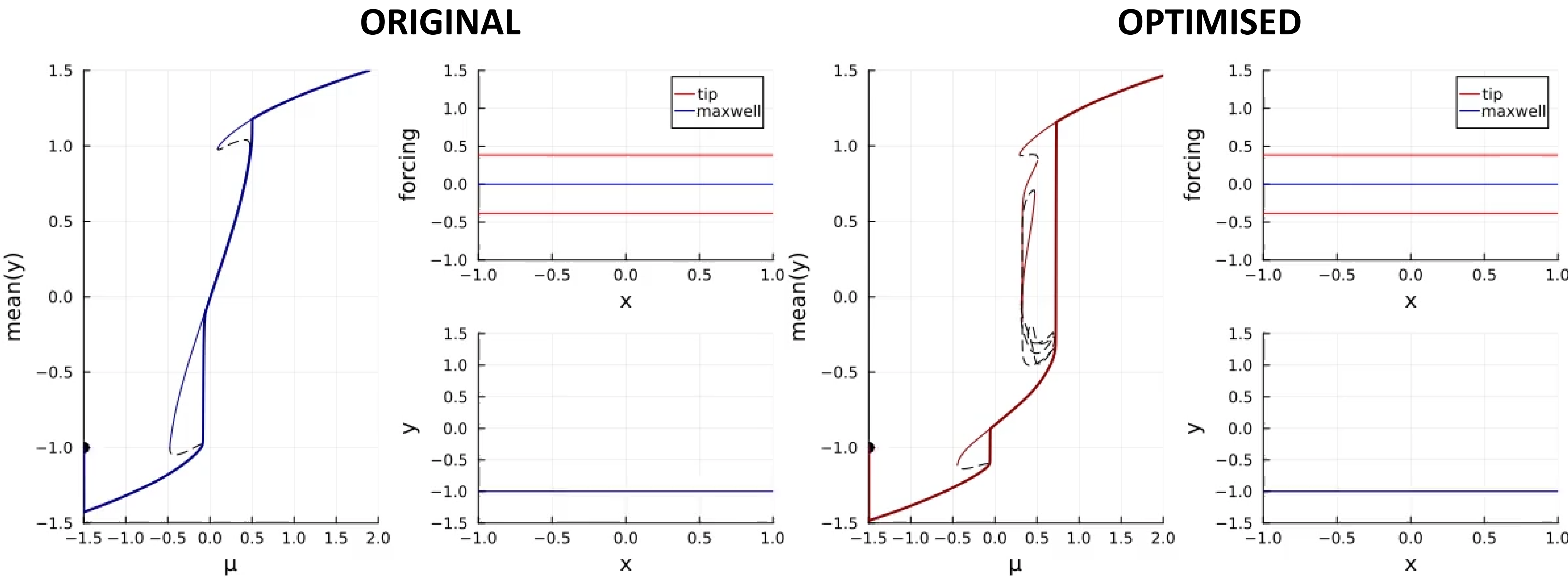
Perturbation size restriction

The applied perturbation cannot be too large

$$h(z) := \|z\| - \delta \leq 0$$



Example optimisation



Details

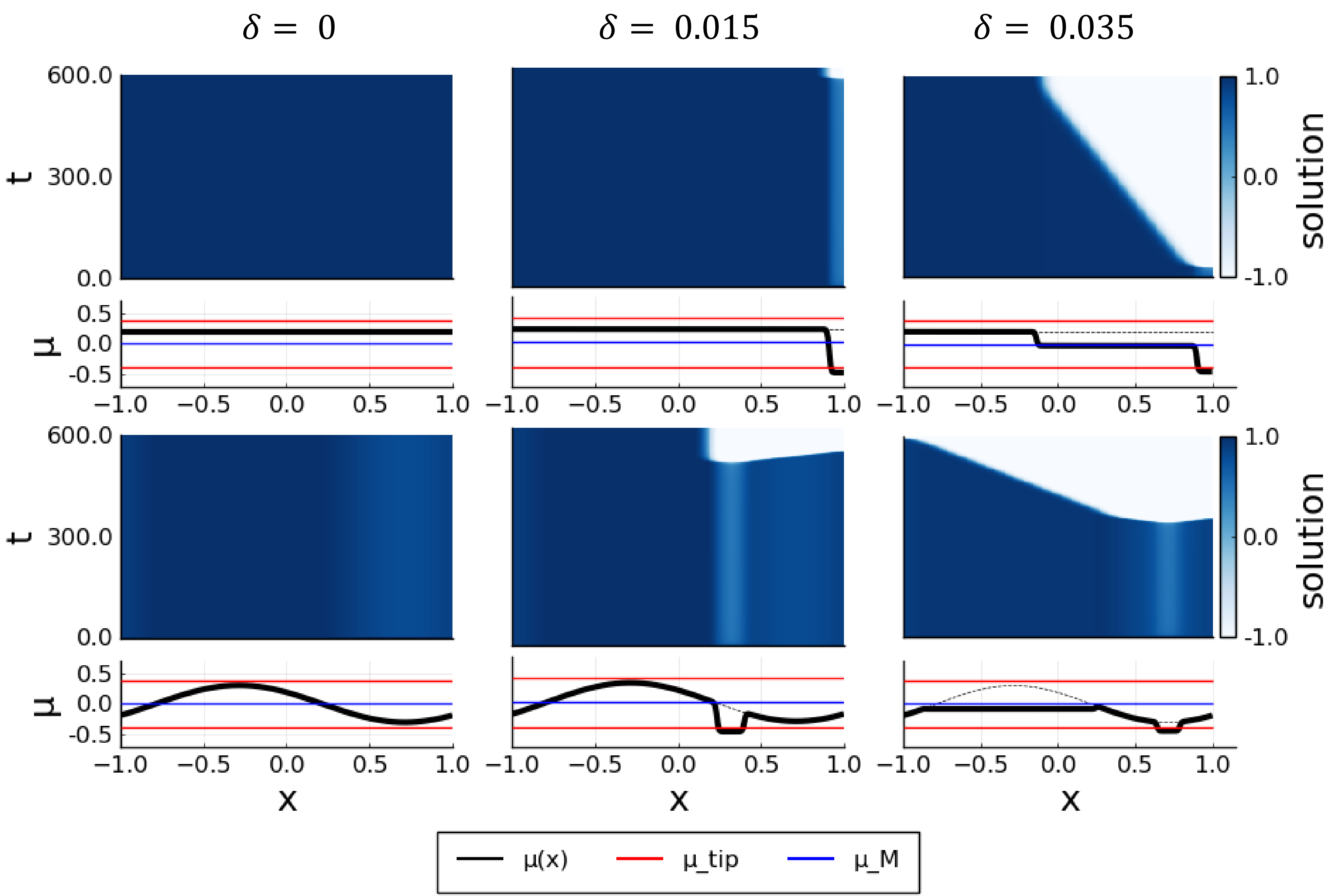
Objective: minimise the mean of the final state

$\mu(t_e) = 0.7$

$\delta = 0.3$

Pre-existing heterogeneity: $\frac{1}{2}\cos(\pi x)$

Example optimisation



Approach 2: “continuation-like”

Mathematical Optimisation Problem

Minimize $f(z)$

subject to $g(z) = 0$

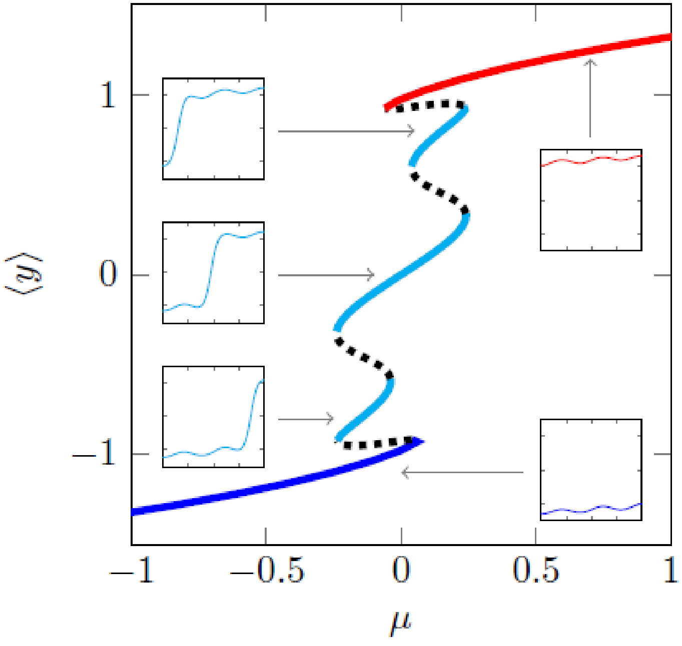
$h(z) \leq 0$

COST FUNCTION

For example:

Location of bifurcation as far as possible to the right

$$f(z) \coloneqq -\mu_{BIF}(z)$$



Bifurcation constraint

Needs to be a solution at a saddle-node bifurcation

$$g_1(z) \coloneqq D\,\frac{\partial^2 y}{\partial x^2} + y(1 - y^2) + \mu(x) + z(x)$$

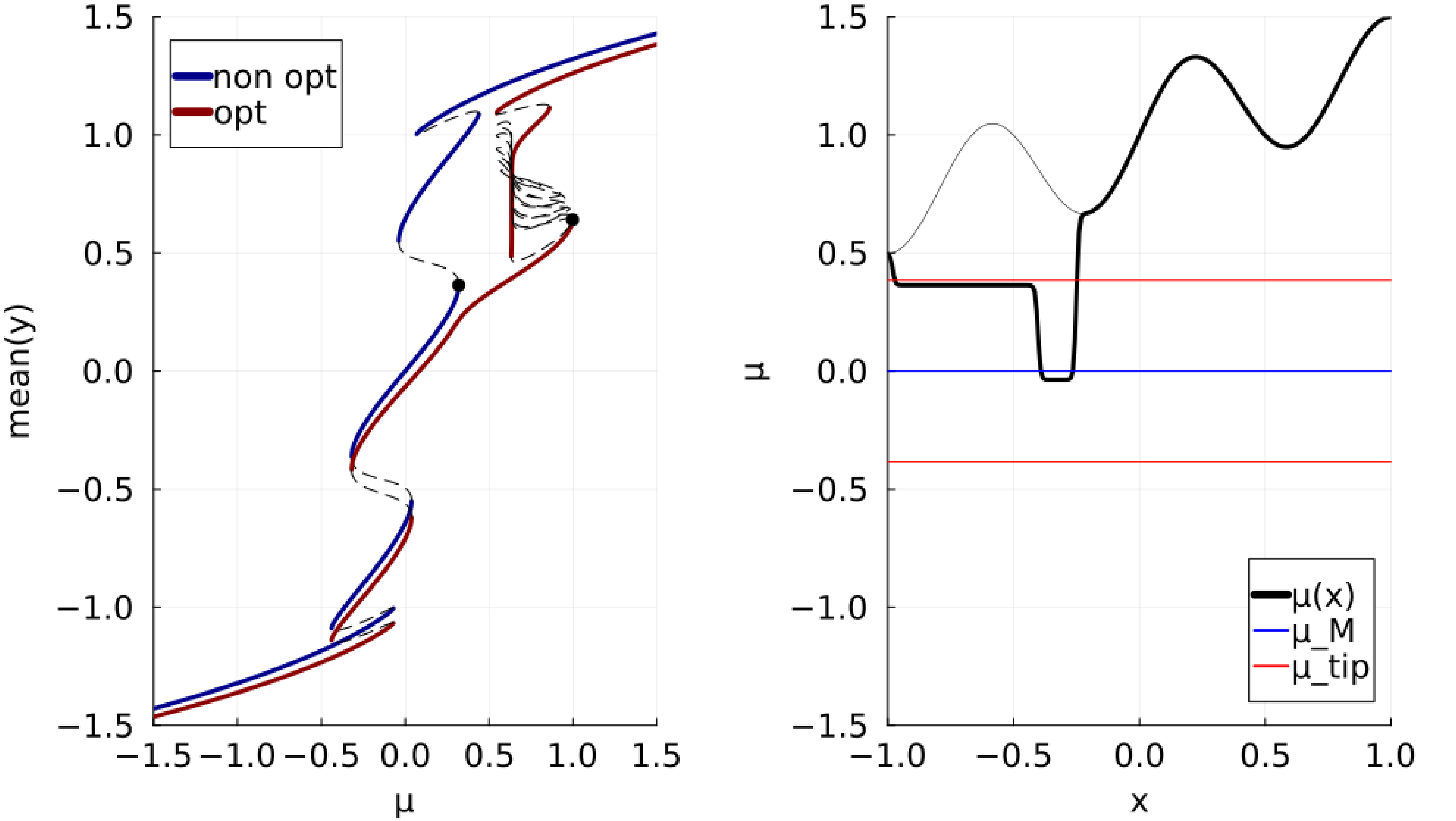
$$g_2(z) \coloneqq \text{“}eigenvalue\ at\ zero\text{”}$$

Perturbation size restriction

The applied perturbation cannot be too large

$$h(z) \coloneqq \|z\| - \delta \leq 0$$

Example optimisation



Details

Objective: move bifurcation to the right as far as possible

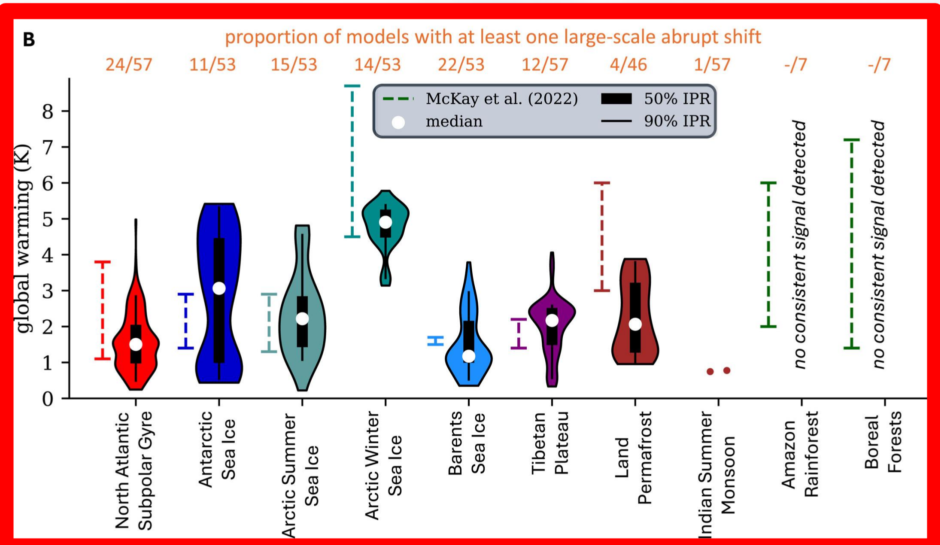
$\delta = 0.3$

Pre-existing heterogeneity: $\frac{1}{2}\cos(\pi x)\sin(2\pi x)$

Tipping
in
Spatially Extended Systems: detection, prediction, control

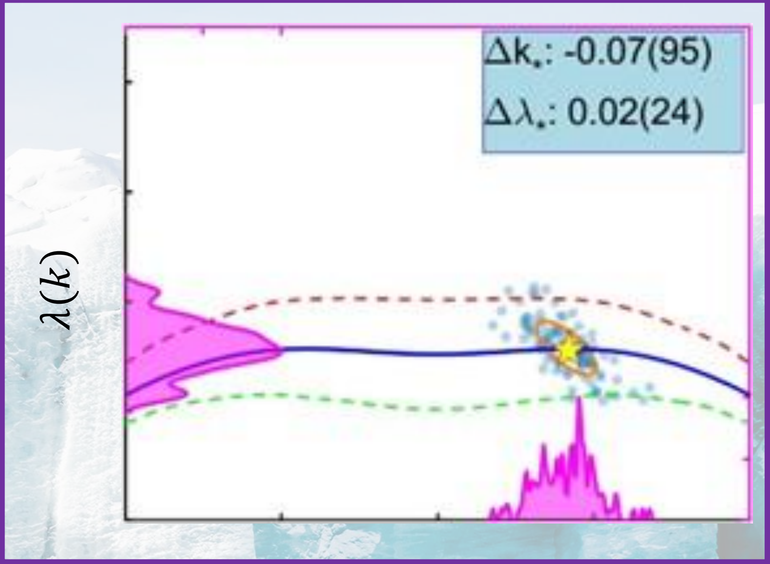
Summary

Summary

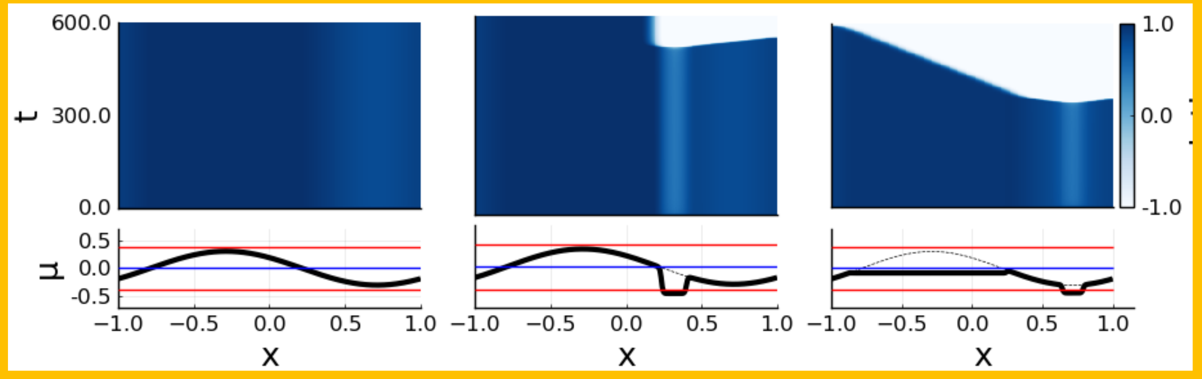


1. Abrupt shifts present in global climate models

2. Refined spatial early warning signs can signal not only for *when* but also *what* transition happens



3. Optimising heterogeneity can postpone, halt, prevent or revert tipping



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Slides at bastiaansen.github.io