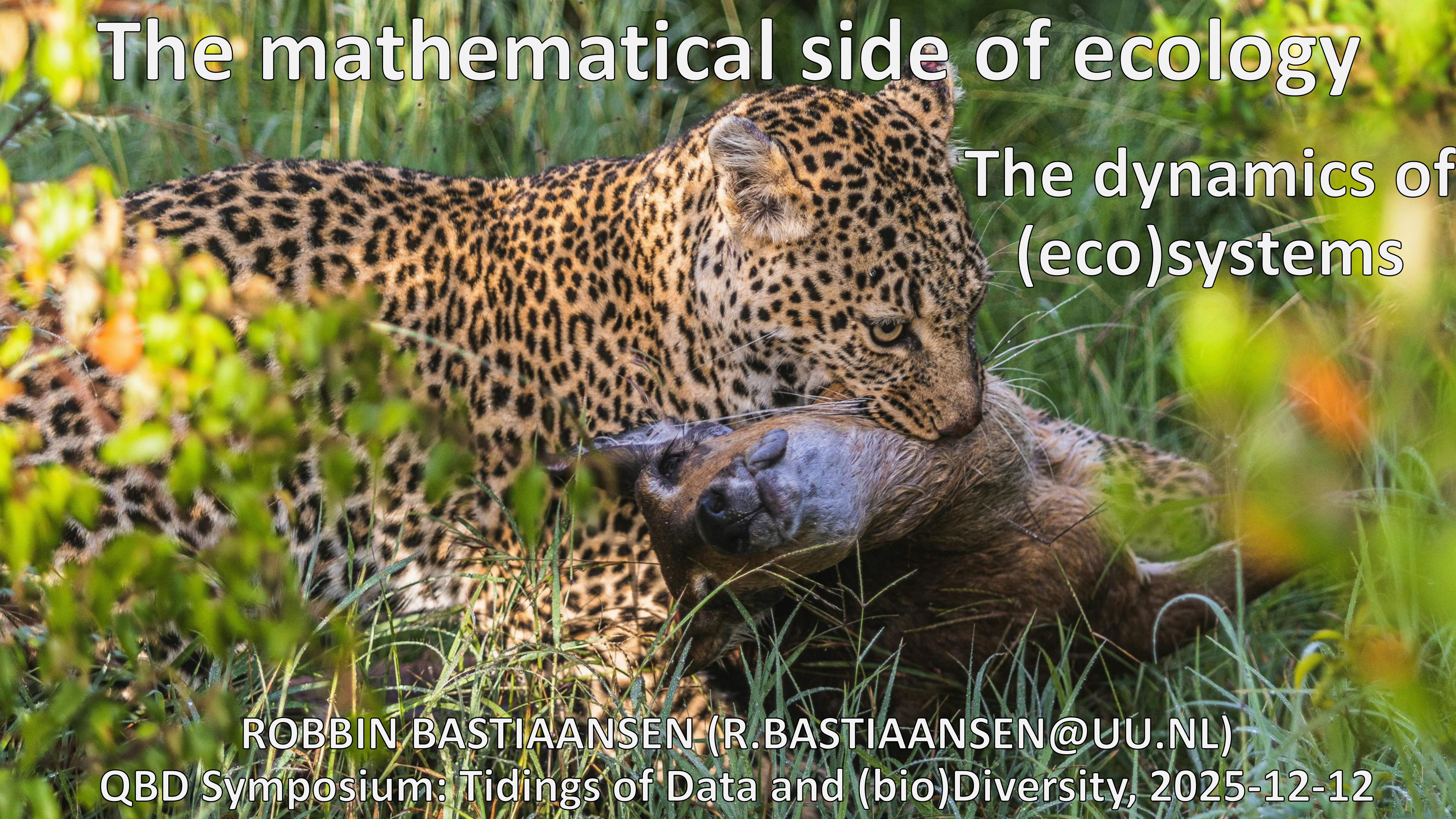

The mathematical side of ecology

The dynamics of (eco)systems

ROBBIN BASTIAANSEN (R.BASTIAANSEN@UU.NL)

QBD Symposium: Tidings of Data and (bio)Diversity, 2025-12-12

Shark Tale

Shark Tale

Year	Percentage Sharks
1914	11.9%
1915	21.4%
1916	22.1%
1917	21.2%
1918	36.4%
1919	27.3%
1920	16.0%
1921	15.9%
1922	14.8%
1923	10.7%

Shark Tale

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1914	11.9%
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1918	36.4%
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1921	15.9%
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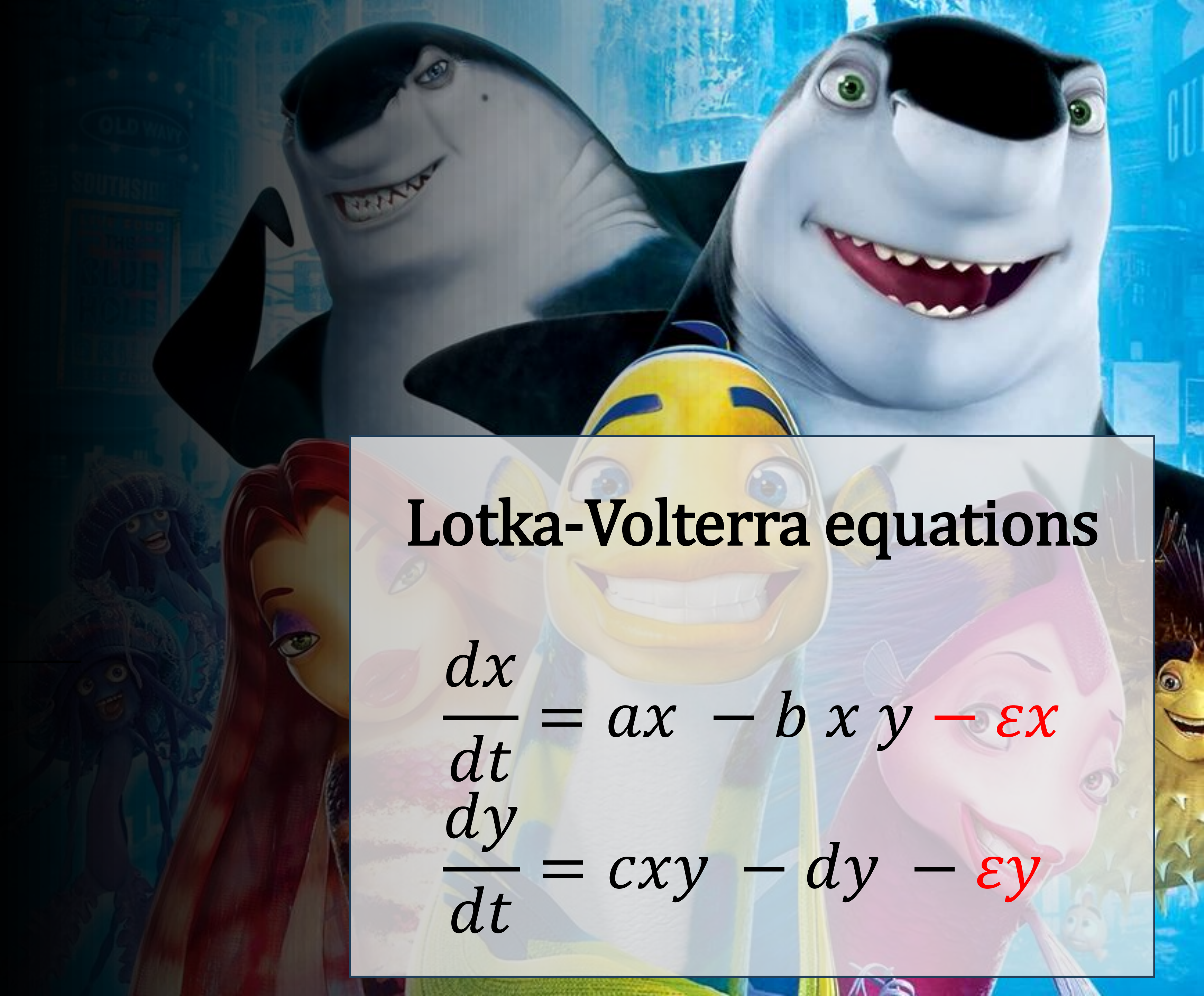
1923	10.7%

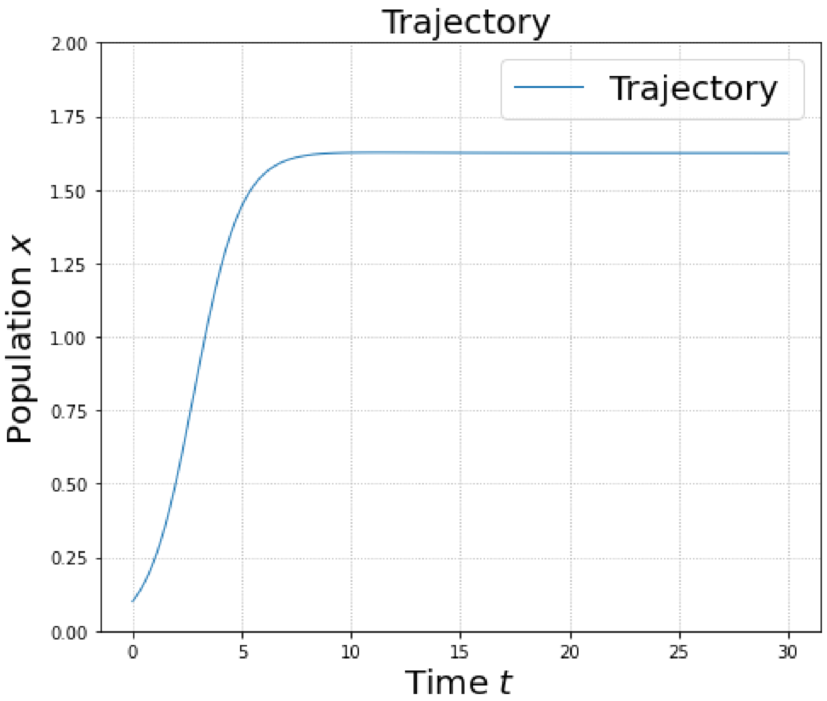


Lotka-Volterra equations

$$\frac{dx}{dt} = ax - b\,x\,y - \varepsilon x$$

$$\frac{dy}{dt} = cxy - dy - \varepsilon y$$

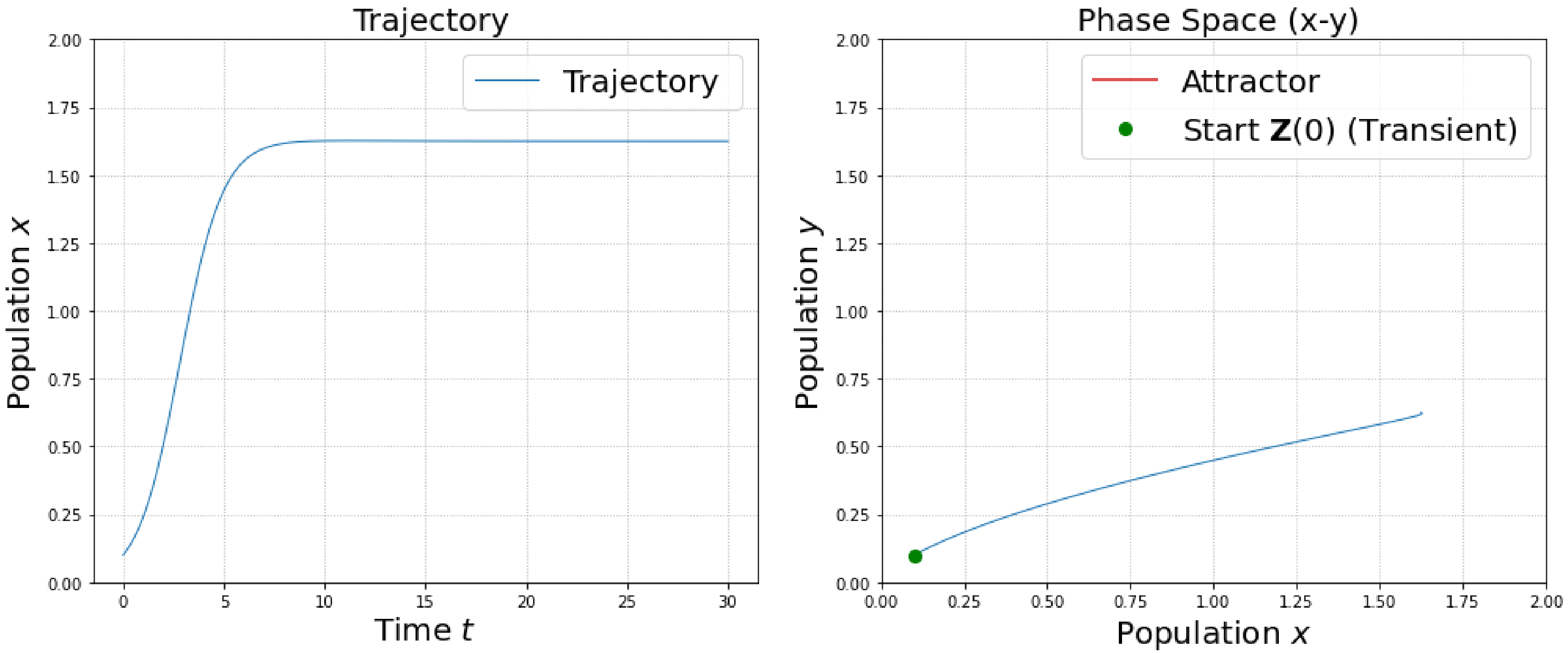
The long term behaviour – Fixed Point



$$\frac{dx}{dt} = x\,(r_1 - a_1 x - b_1 y)$$
$$\frac{dy}{dt} = y\,(r_2 - a_2 y - b_2 x)$$

Model, code and images by Gemini 2.5 Flash

The long term behaviour – Fixed Point



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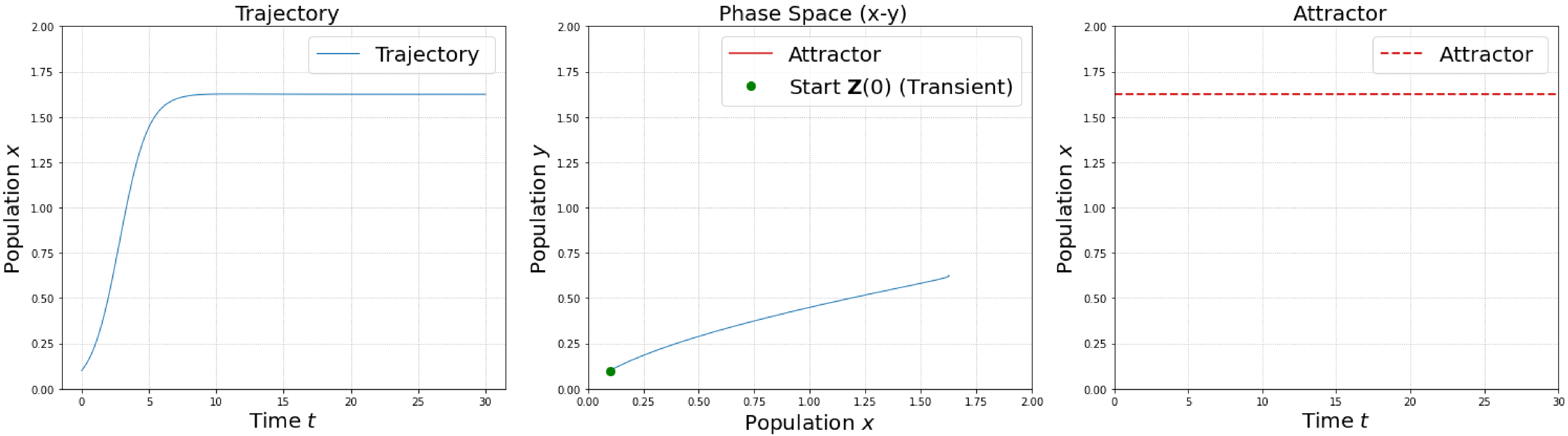
The long term behaviour – Fixed Point



Long term behaviour: fixed / no dynamics!

Future projections are clear

$$\frac{dx}{dt} = x\,(r_1 - a_1 x - b_1 y)$$
$$\frac{dy}{dt} = y\,(r_2 - a_2 y - b_2 x)$$

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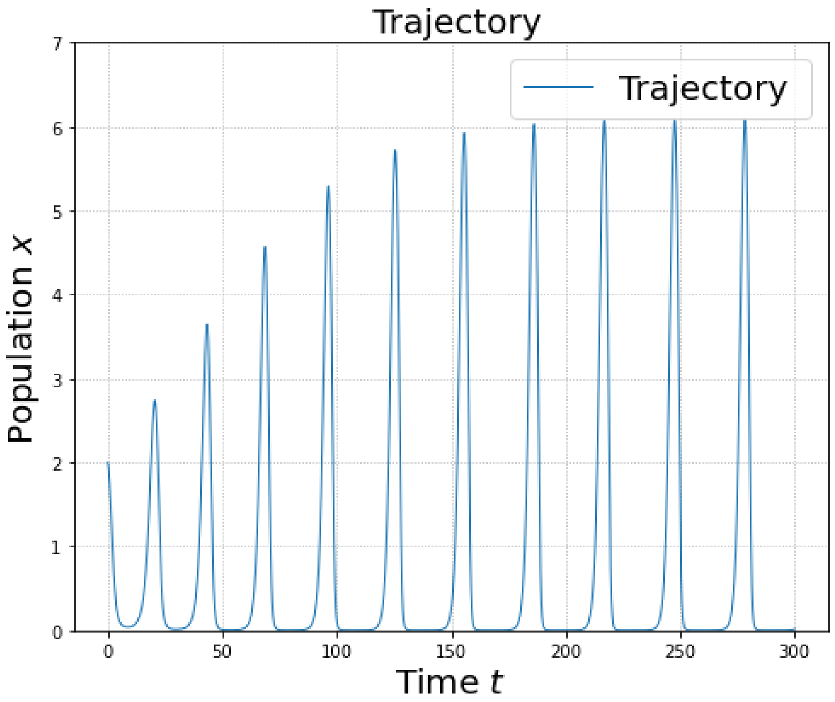
The long term behaviour – Limit Cycles



$$\frac{dx}{dt} = x\left(a\left(1 - {}^{x}/_{K}\right) - b\,\frac{y}{1+hx}\right)$$
$$\frac{dy}{dt} = y\left(-c + d\,\frac{x}{1+hx}\right)$$

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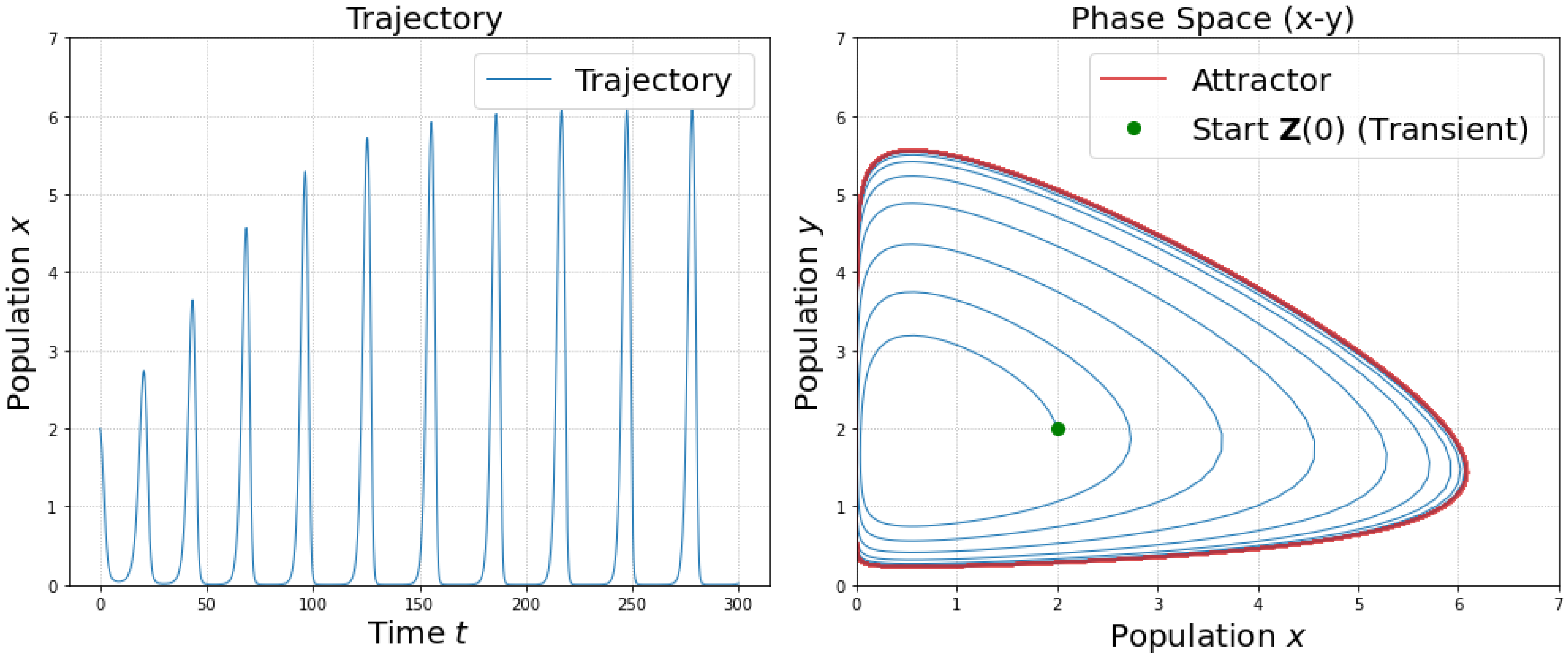
The long term behaviour – Limit Cycles



$$\frac{dx}{dt} = x\left(a\left(1 - {}^{x}\!/_{K}\right) - b\,\frac{y}{1 + hx}\right)$$
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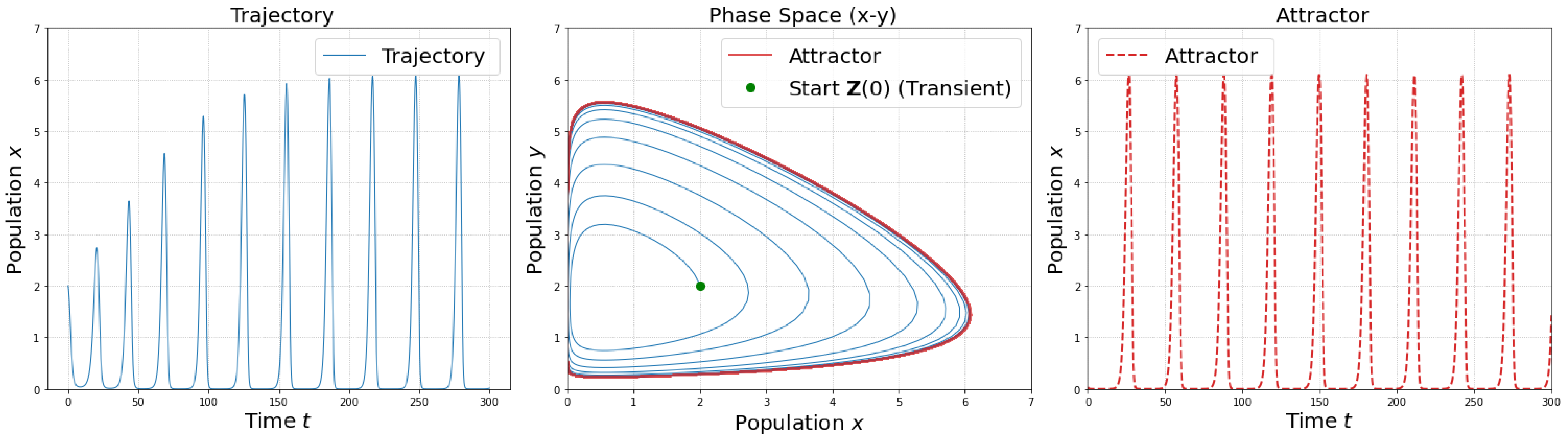
The long term behaviour – Limit Cycles



Long term behaviour: periodic dynamics!

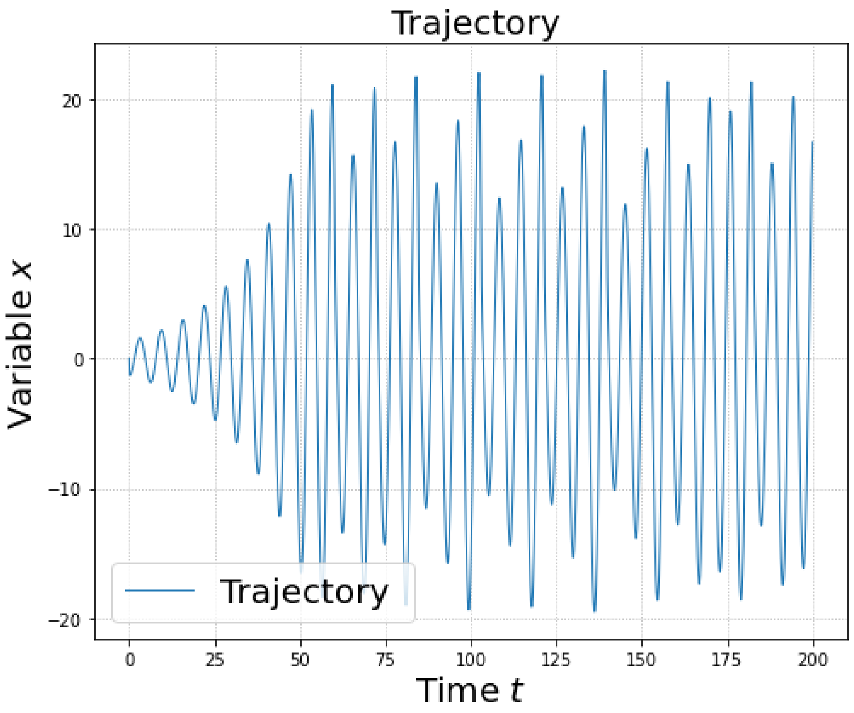
One can determine periodicity

Future projections are clear

$$\frac{dx}{dt} = x\left(a(1 - {}^{x}/_{K}) - b\,\frac{y}{1+hx}\right)$$
$$\frac{dy}{dt} = y\left(-c + d\,\frac{x}{1+hx}\right)$$

Model, code and images by Gemini 2.5 Flash

The long term behaviour – Strange Attractor



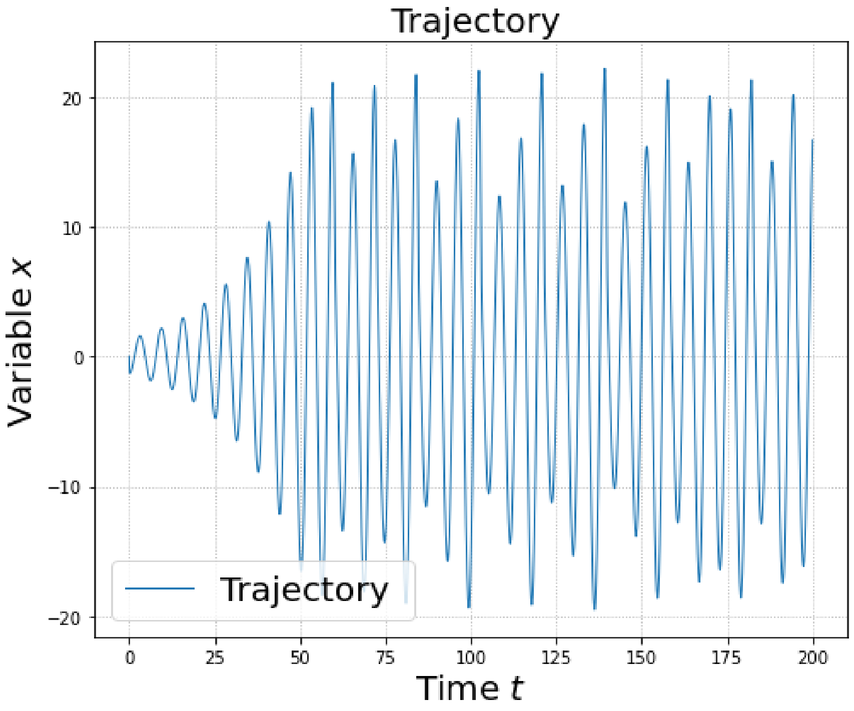
$$\frac{}{dt} = -yz$$

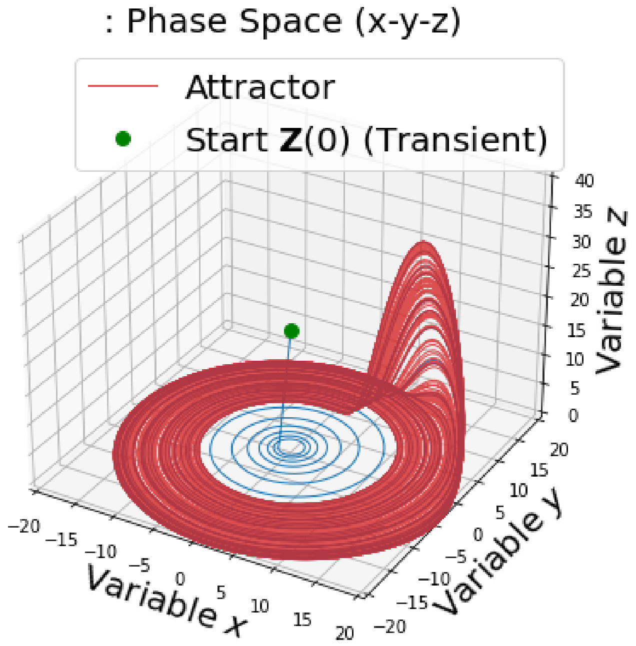
$$\frac{dy}{dt} = x + a\,y$$

$$\frac{dz}{dt} = b + z\,(x - c)$$

Model, code and images by Gemini 2.5 Flash

The long term behaviour – Strange Attractor





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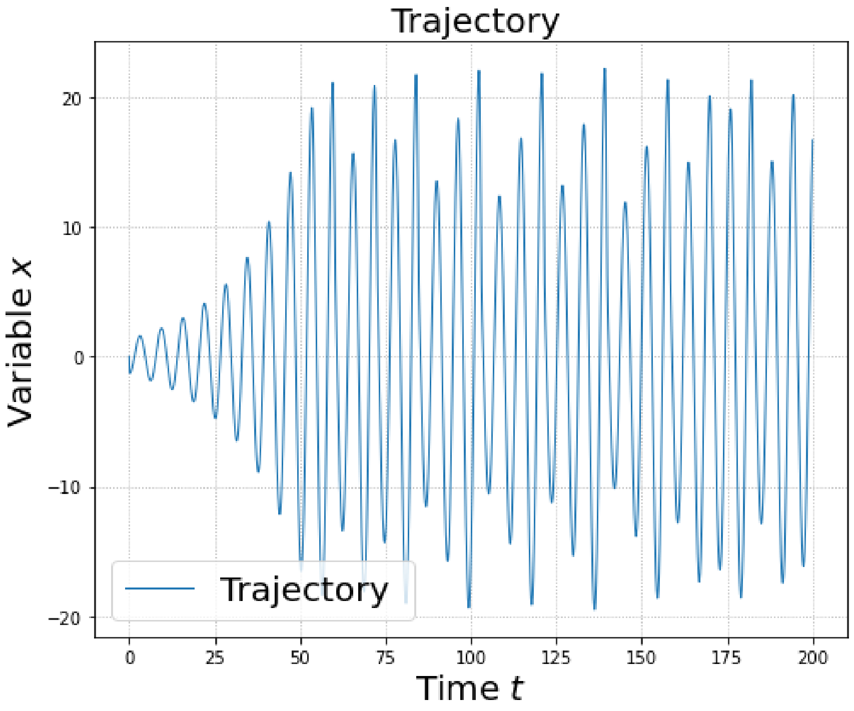
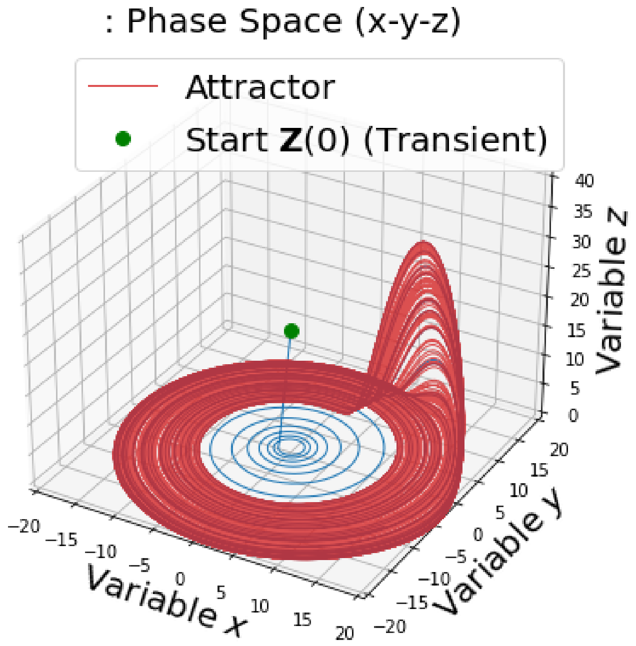
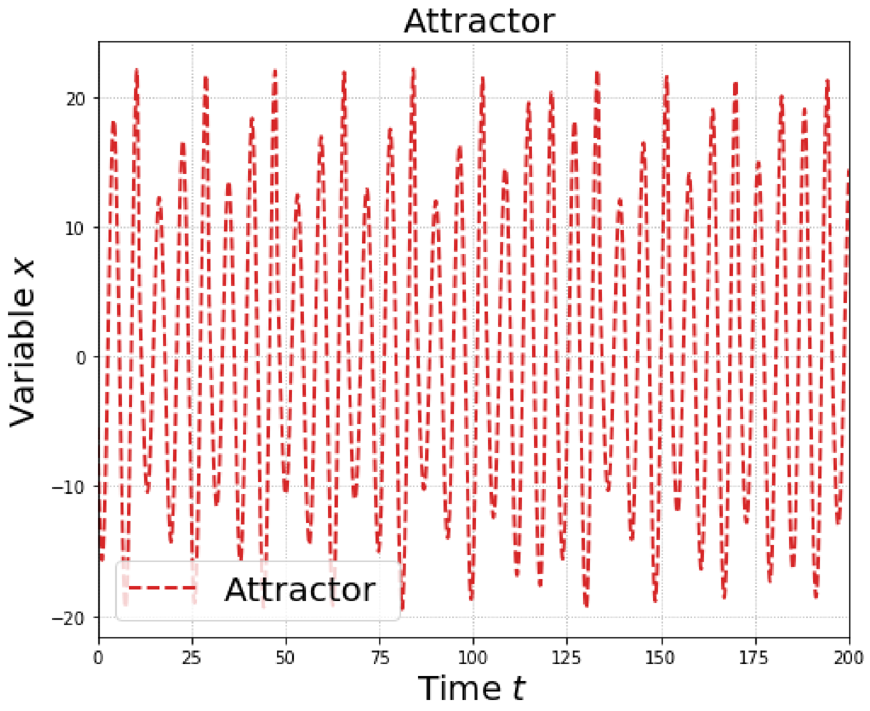
The long term behaviour – Strange Attractor







Long term behaviour: chaotic dynamics!

Not necessarily clear periodicity

Future projections are unclear

$$\frac{dx}{dt} = -yz$$
$$\frac{dy}{dt} = x + a\,y$$
$$\frac{dz}{dt} = b + z\,(x - c)$$

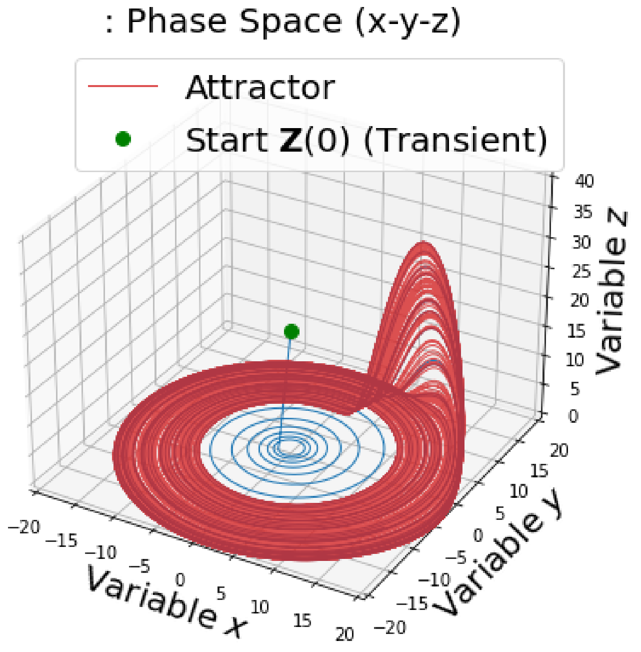
Model, code and images by Gemini 2.5 Flash

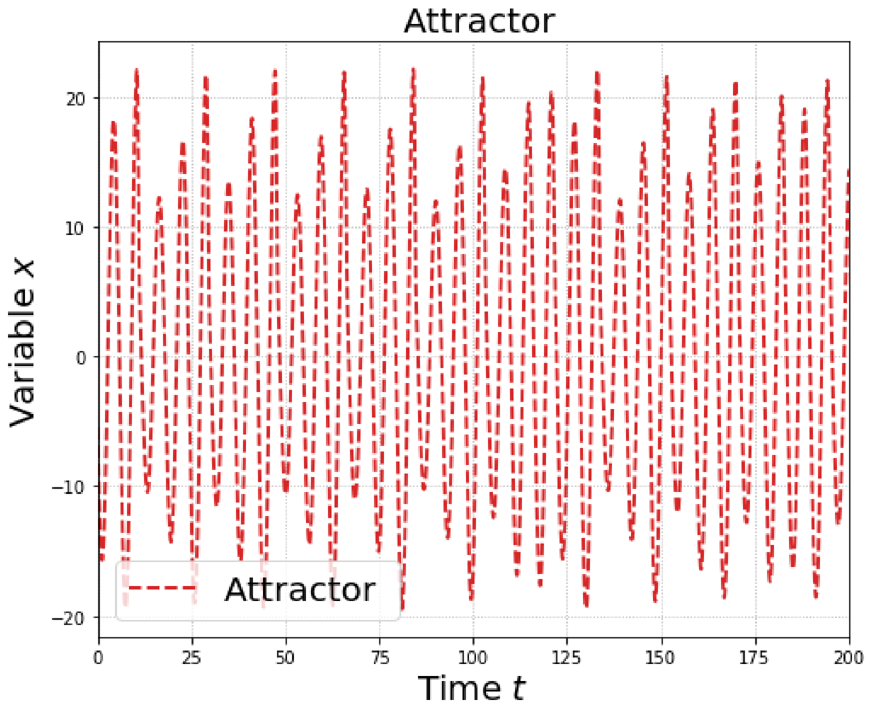
The long term behaviour – Strange Attractor

YET
there still is structure!

We can still make predictions

Properties can be estimated via e.g. long-term time averages or ensemble averages





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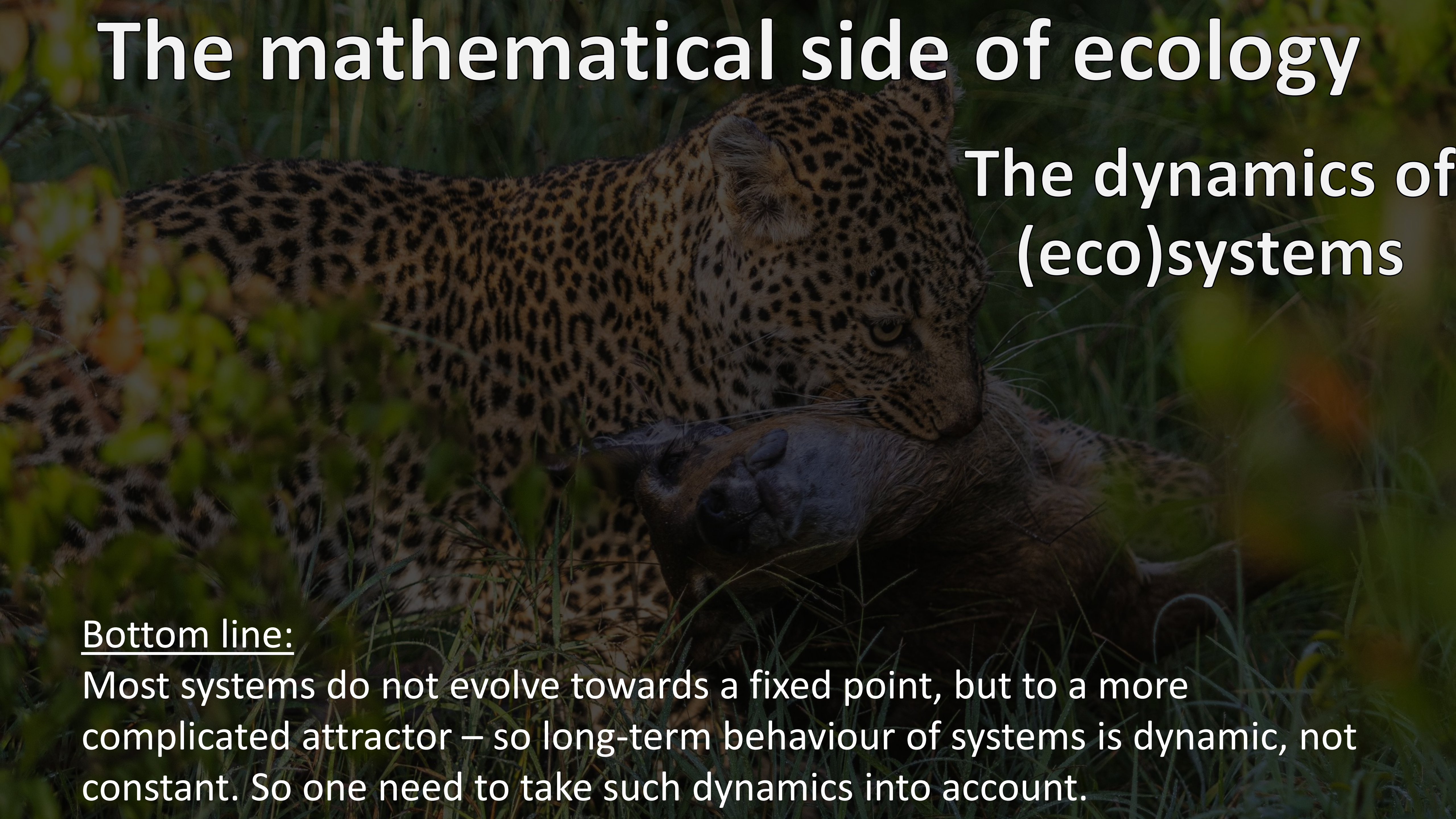

The mathematical side of ecology

The dynamics of (eco)systems

<u>Bottom line:</u>

Most systems do not evolve towards a fixed point, but to a more complicated attractor – so long-term behaviour of systems is dynamic, not constant. So one need to take such dynamics into account.

Go Fleur!