

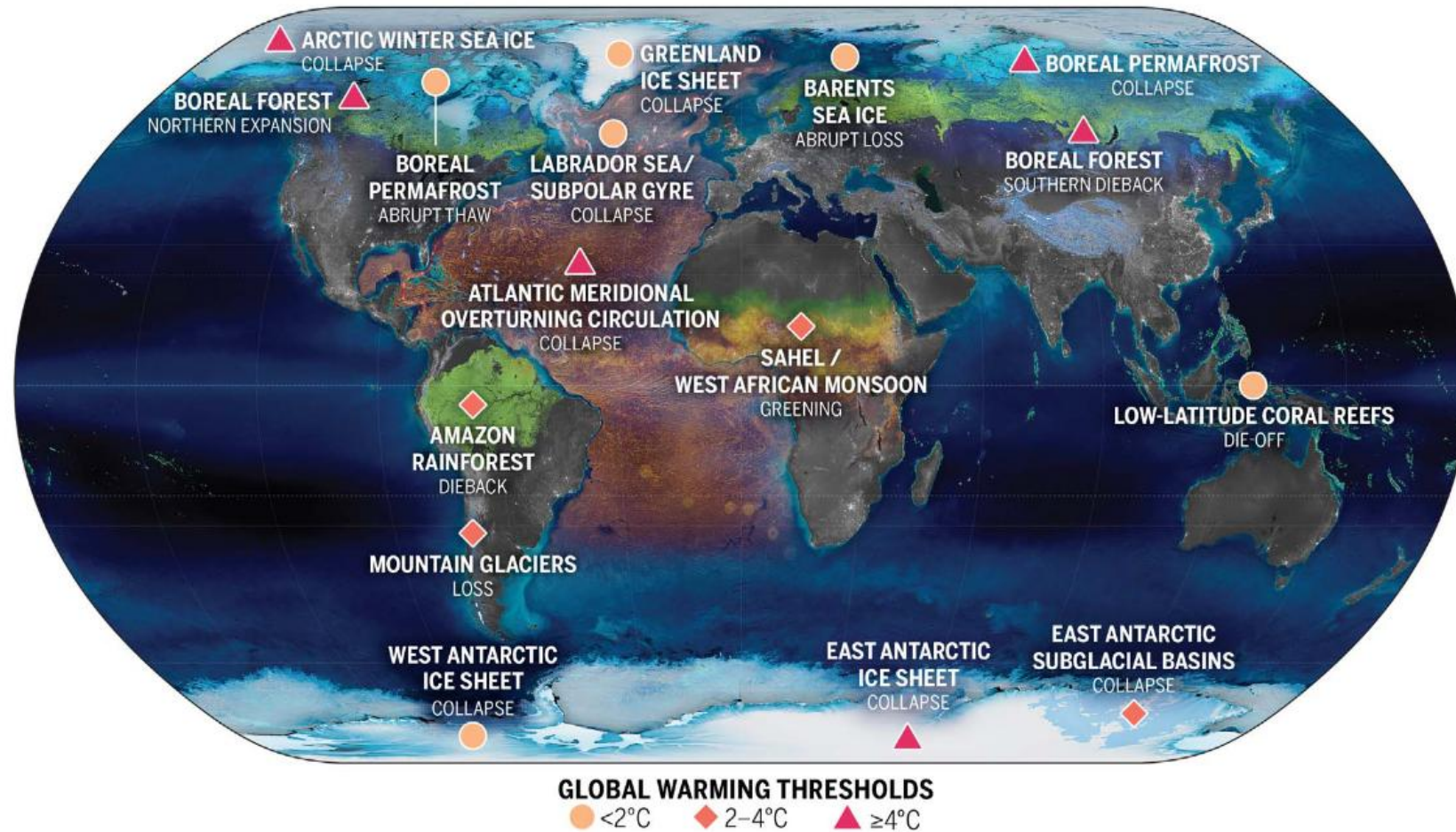
# TIPPING DYNAMICS IN MULTISCALE SYSTEMS



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DIEP seminar, 2026-05-28

# Tipping Points

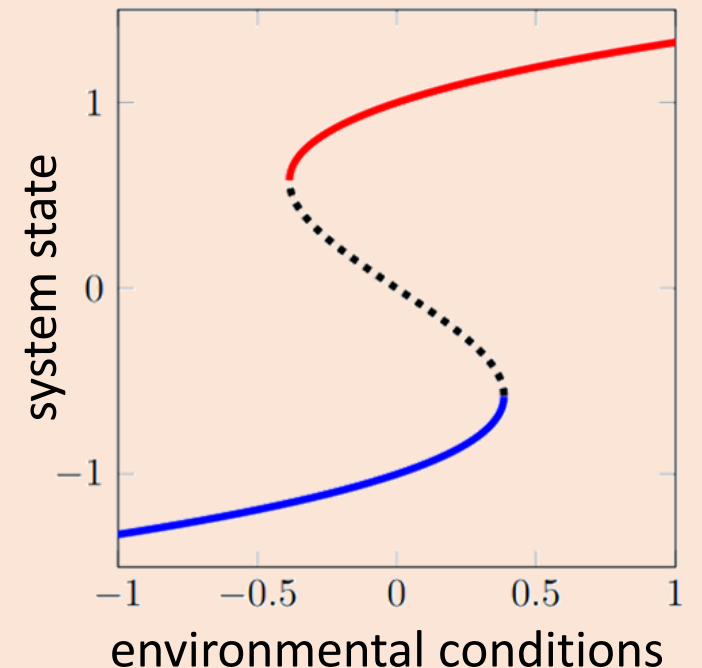
IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”



## Mathematics

Tipping points  $\leftrightarrow$  Bifurcations

$$\frac{dy}{dt} = f(y, \mu)$$



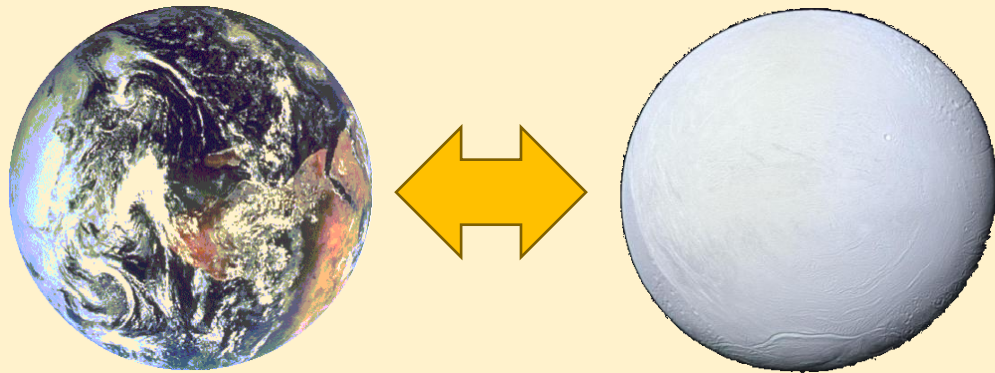
# Critical shifts

## Classic Literature

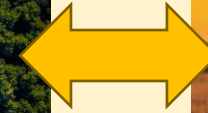
[Holling, 1973]

[Noy-Meier, 1975]

[May, 1977]

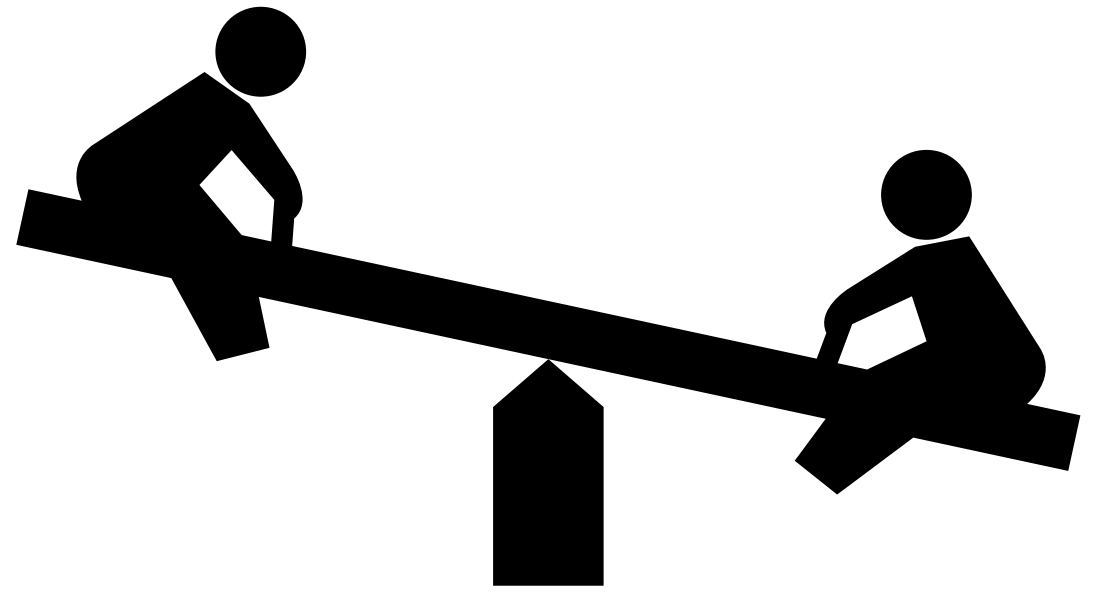
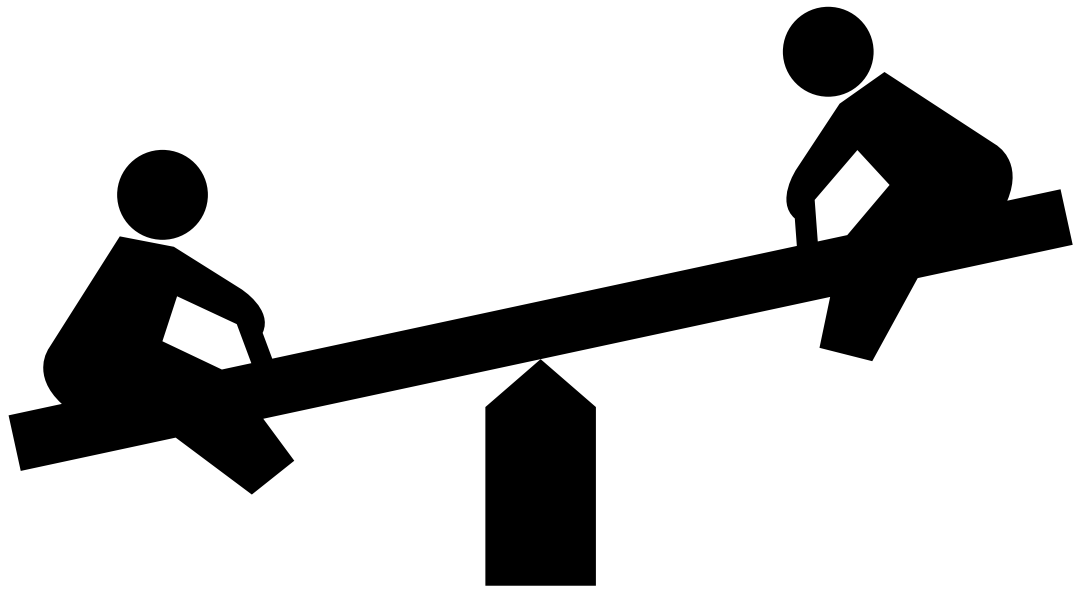


Planetary transitions



Ecosystem shifts





**more weight on left**

**more weight on right**



**more weight on left**

**more weight on right**

THE POCKET TELEPHONE: WHEN IT WILL RING!



The latest modern horror in the way of inventions is supposed to be the pocket telephone. We can imagine the moments this instrument will choose for action! —(By W. K. Haselden.)

# The diffusion of innovation



**critical mass achieved: it's everywhere**

# The diffusion of innovation



only early adaptors

critical mass achieved: it's everywhere

“How did you go bankrupt? Two ways: Gradually, then suddenly.”  
[The Sun Also Rises by Ernest Hemingway]



**few bad decisions**

**few too many**

'Intelligent, articulate, thought-provoking'  
OBSERVER



**THE  
TIPPING  
POINT**

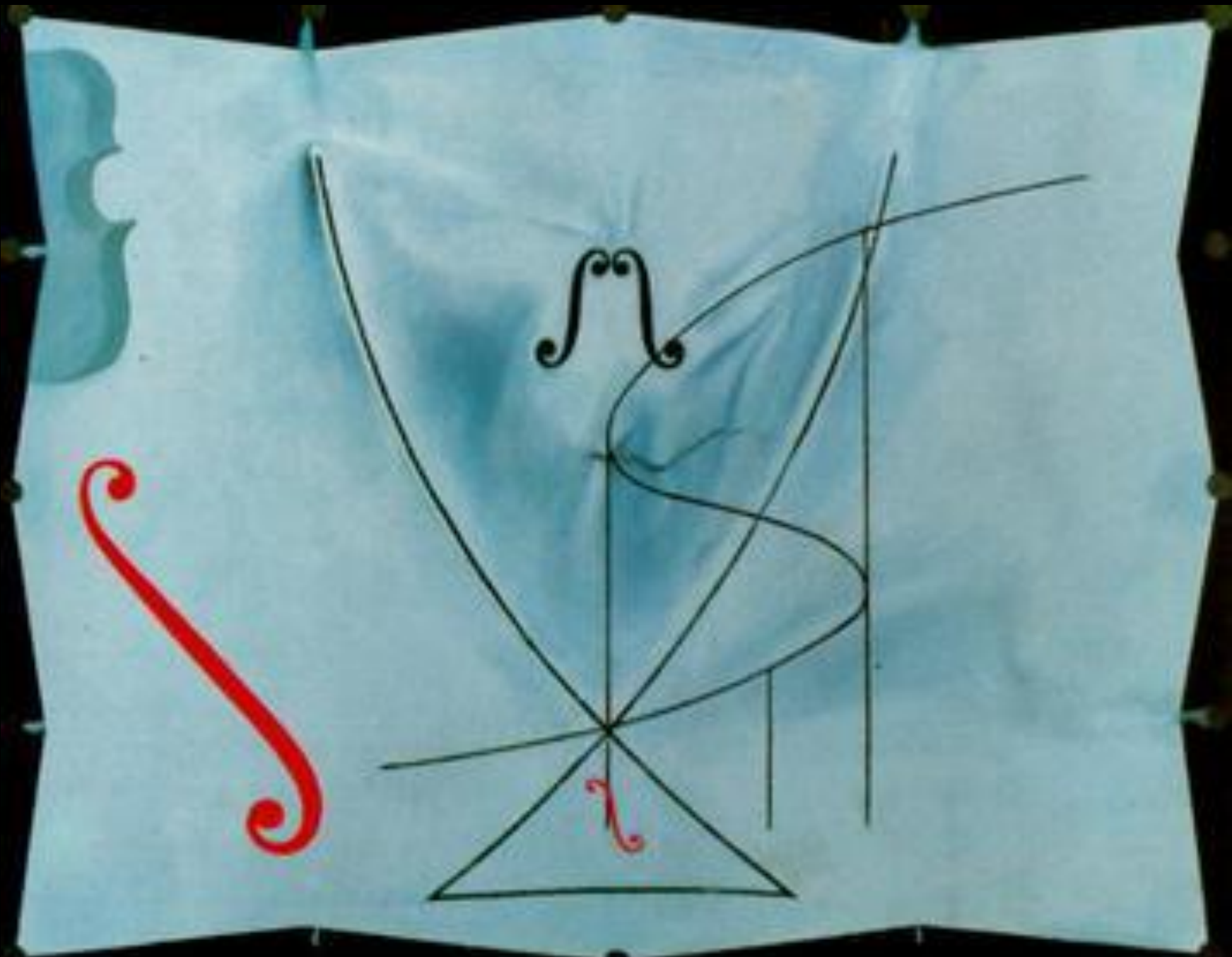
**MALCOLM  
GLADWELL**

HOW LITTLE THINGS CAN  
MAKE A BIG DIFFERENCE

The International Number One Bestseller

# THE CLASSIC THEORY OF TIPPING POINTS

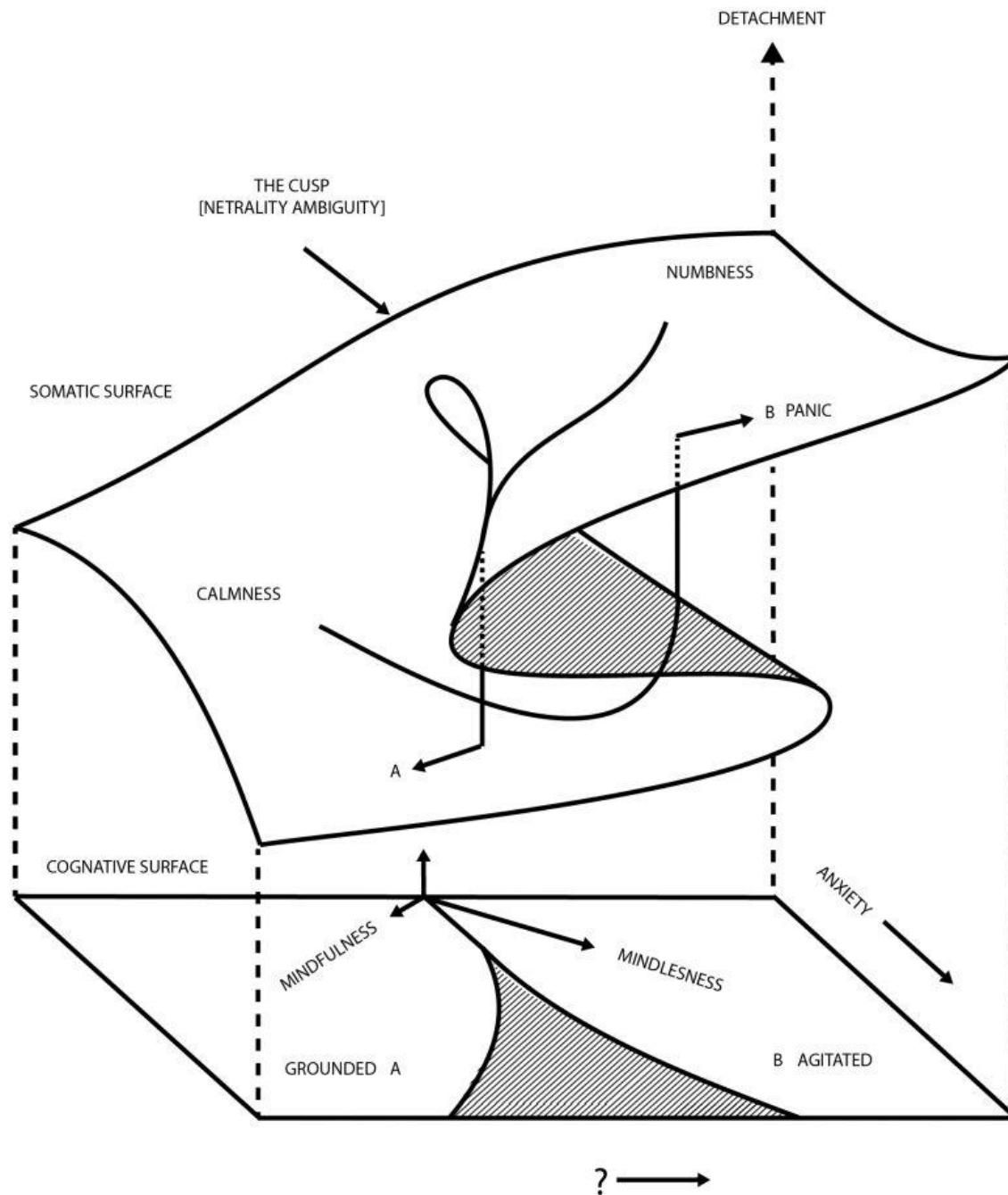




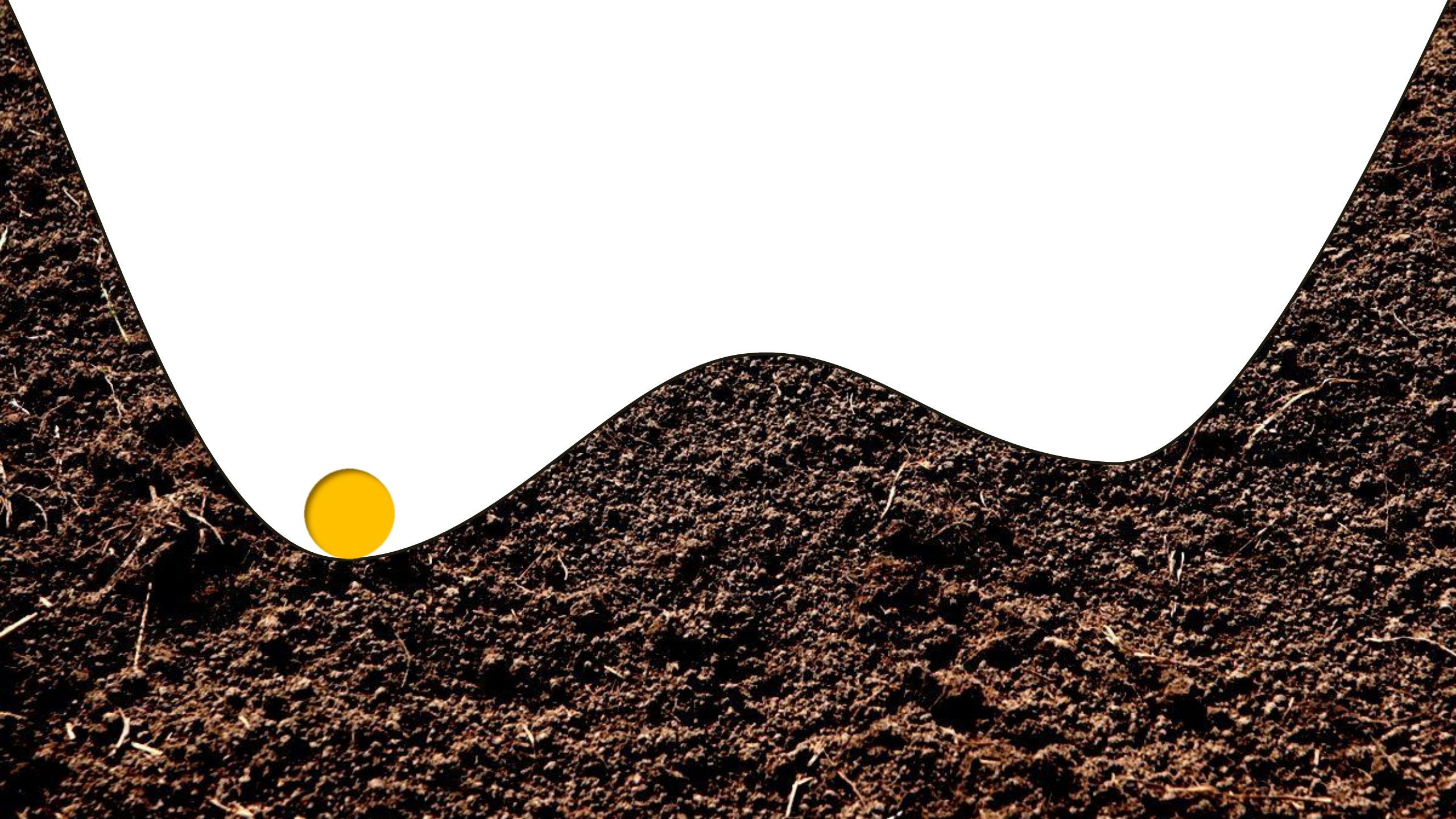
# Catastrophe theory

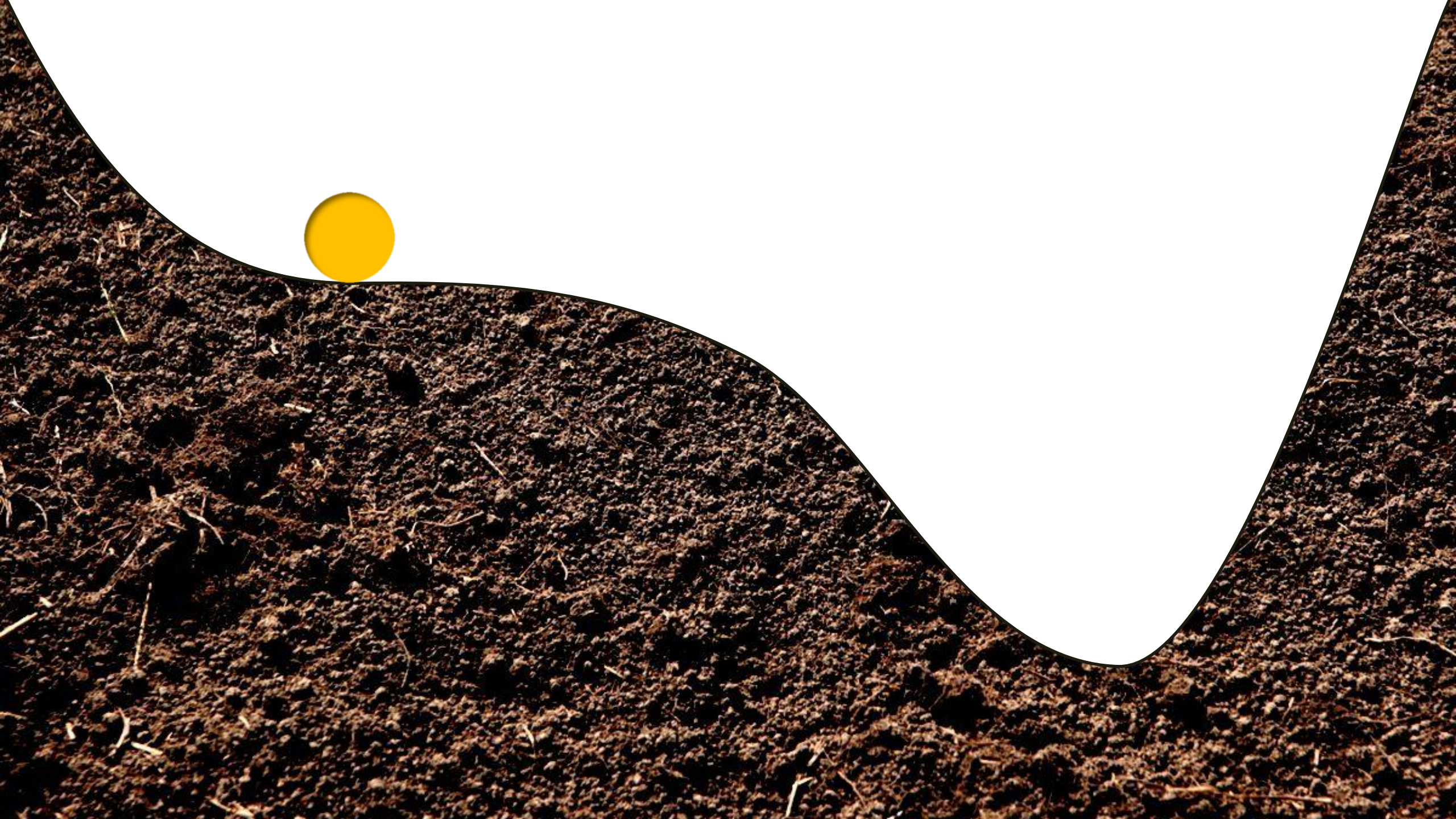
Studies structural changes in the potential function, Energy function or Lyapunov function of a dynamical system

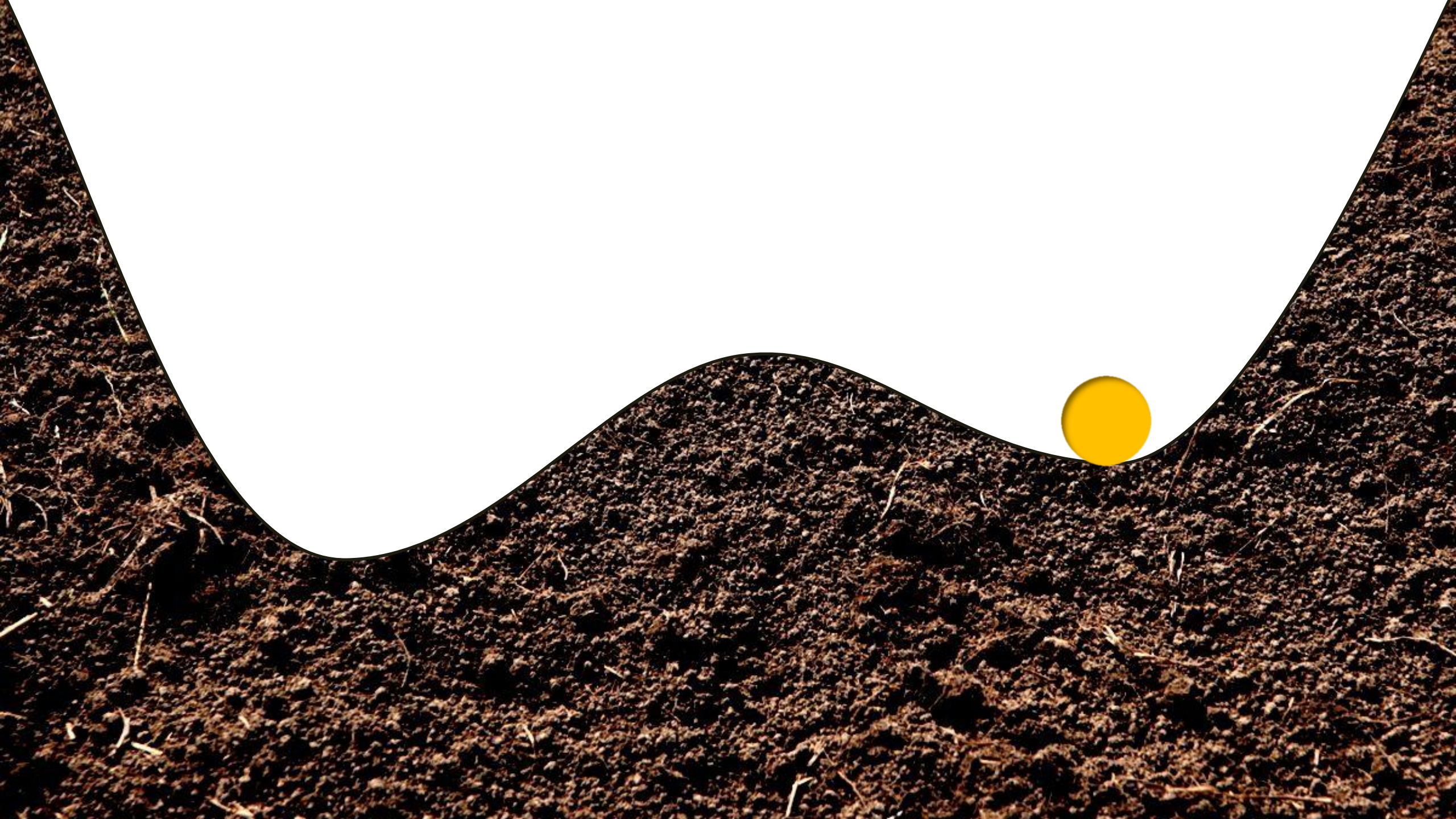
$$\frac{dy}{dt} = - \frac{\delta V(y)}{\delta y}$$













# Classic Theory of Tipping

$$\frac{d\vec{y}}{dt} = f(\vec{y}; \mu)$$

**Canonical example:**

$$\frac{dy}{dt} = y(1 - y^2) + \mu$$

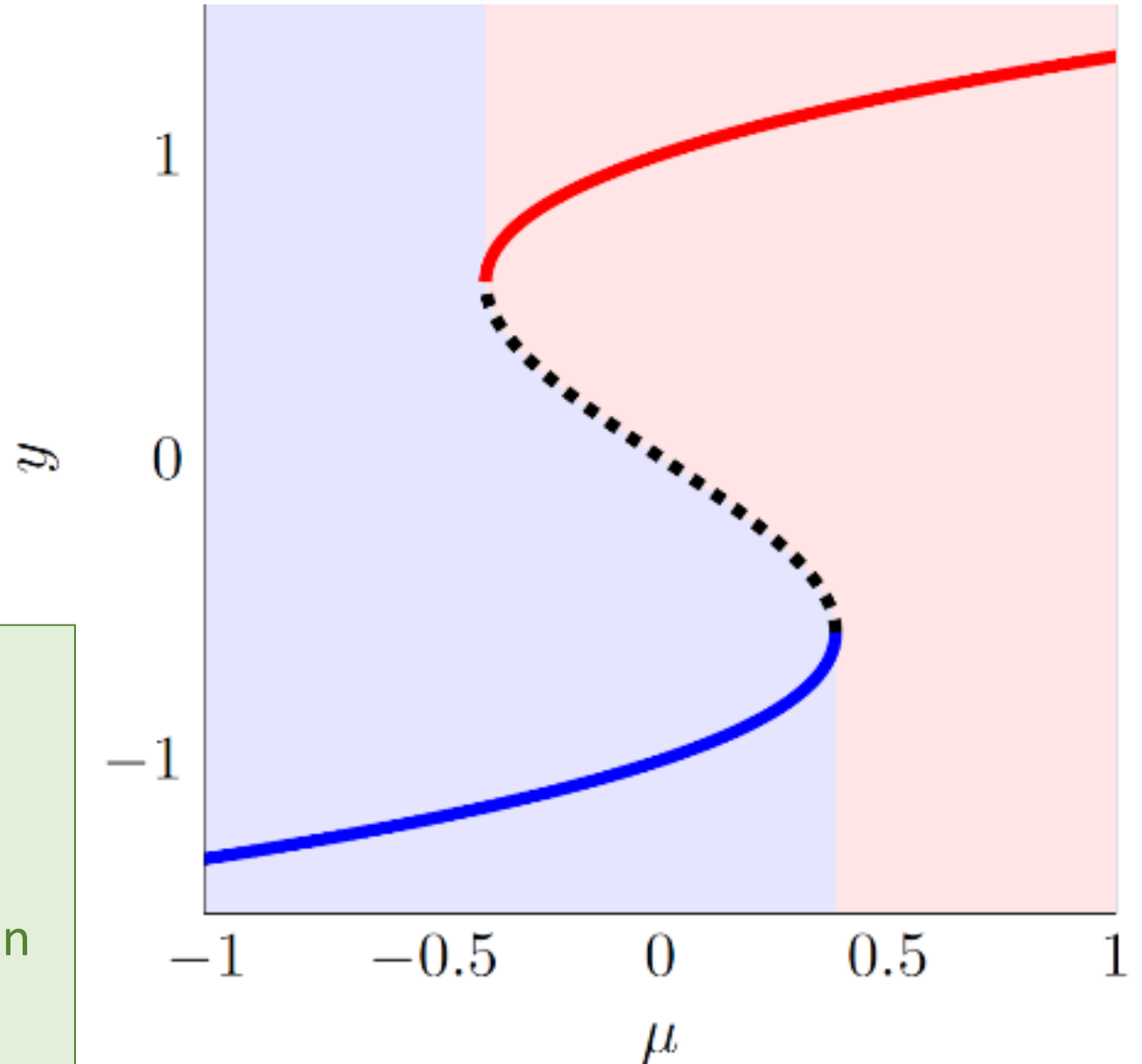
**Tipping**

[Ashwin et al, 2012]

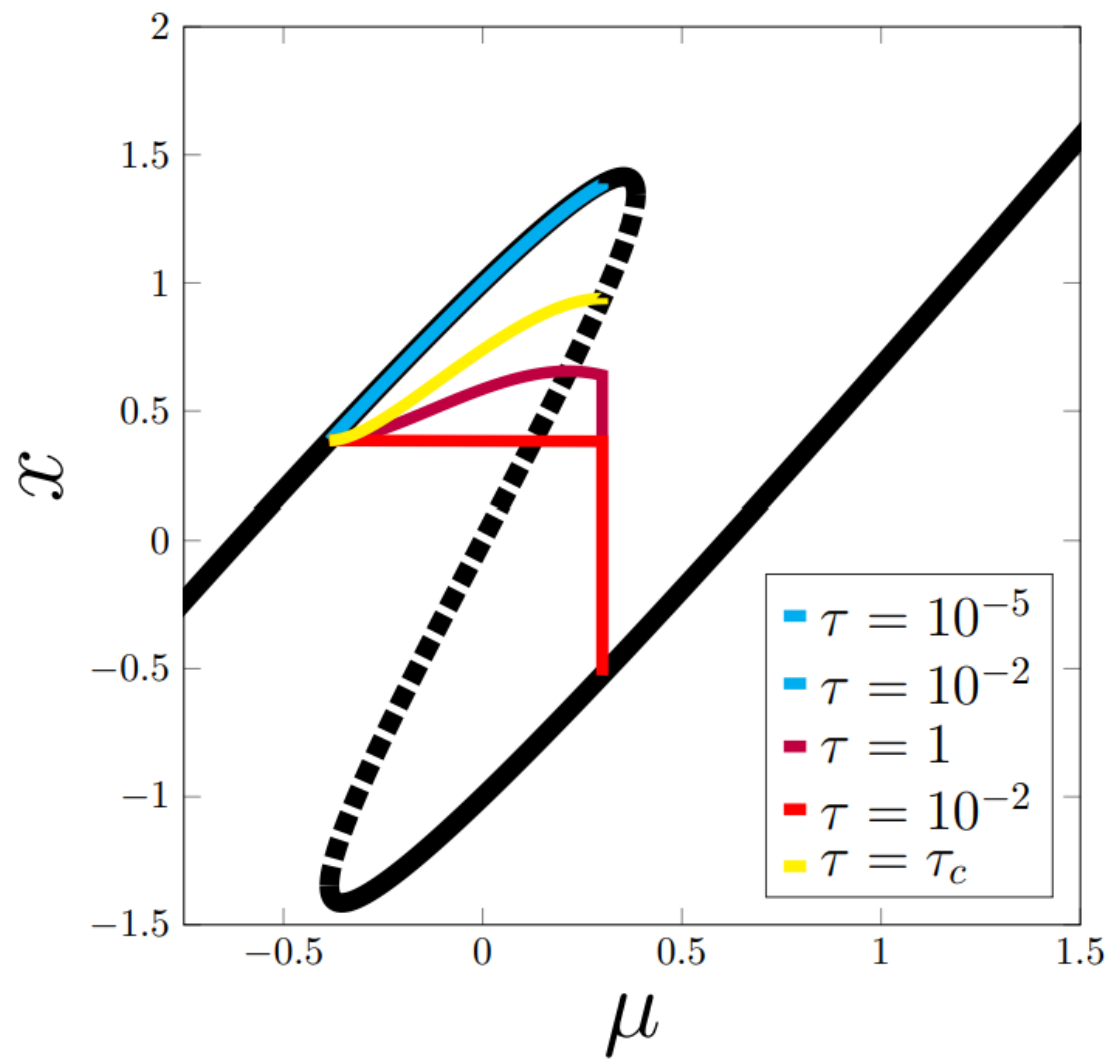
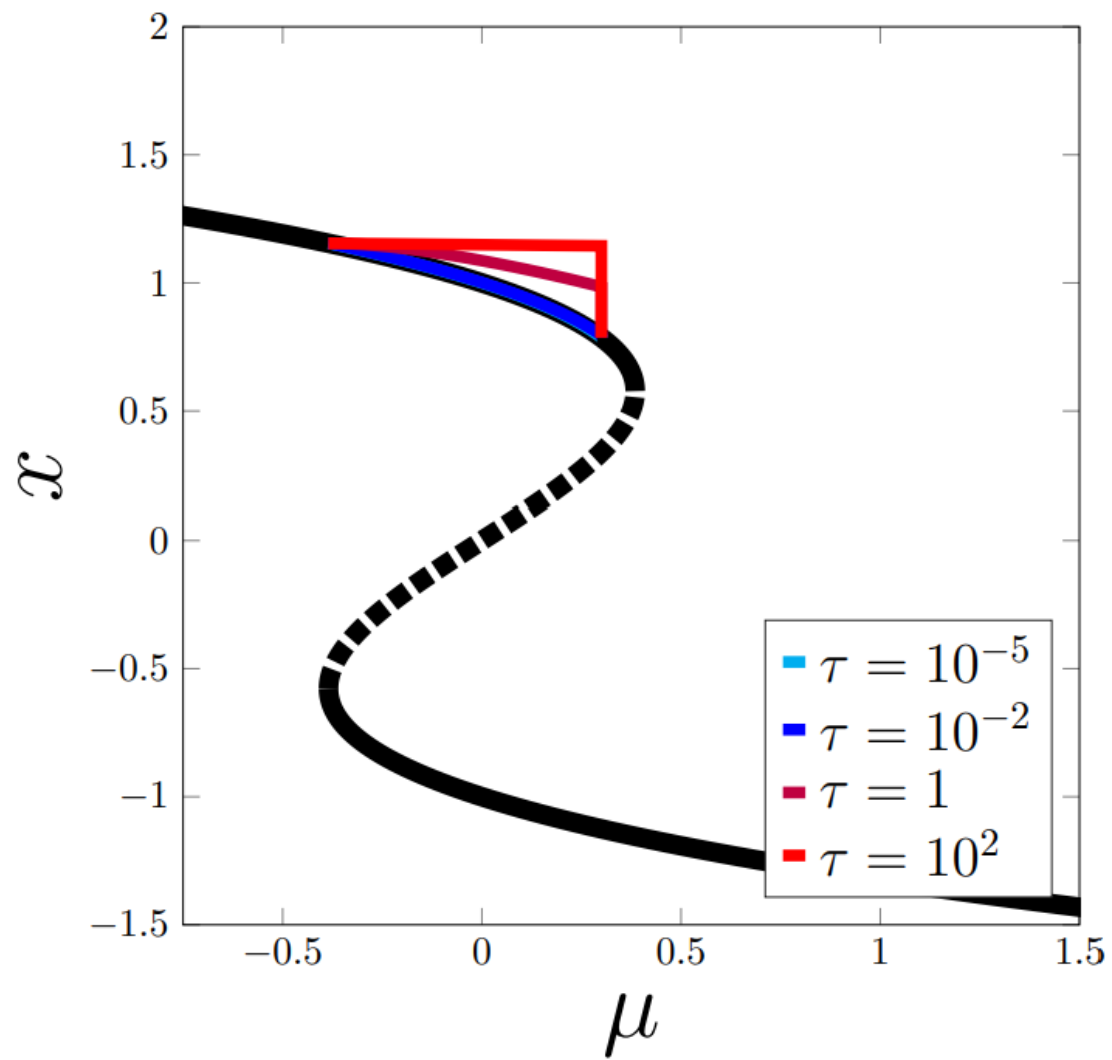
**Bifurcation-Tipping :** Basin disappears

**Noise-Tipping :** Forced outside Basin

**Rate-Tipping :** *(more complicated)*



# Rate-Tipping





# Timescales

The image features a dense, futuristic cityscape rendered in shades of blue and cyan. The buildings are complex, multi-layered structures with glowing windows and intricate patterns, suggesting a highly advanced digital or data environment. In the center of the scene, a silhouette of a person stands on a platform, looking out over the vast, sprawling city. The overall atmosphere is one of technological sophistication and scale. The word "Timescales" is prominently displayed in the upper-middle section of the image in a clean, white, sans-serif font.

# Systems with multiple time scales

$$\frac{dx}{dt} = f(x; \mu)$$

$$\frac{d\mu}{dt} = \tau g(\tau t)$$

Internal Dynamics

Parameter Drift

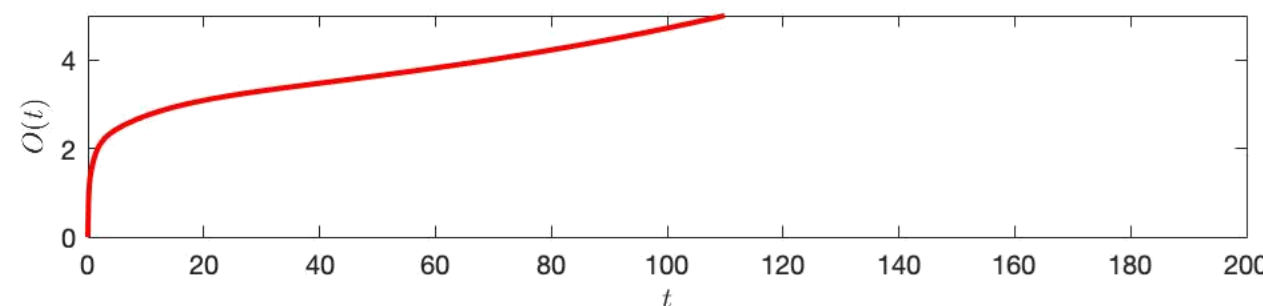
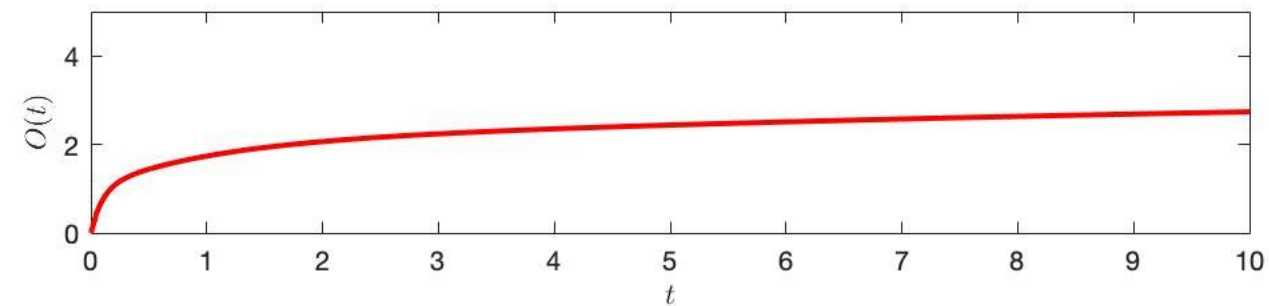
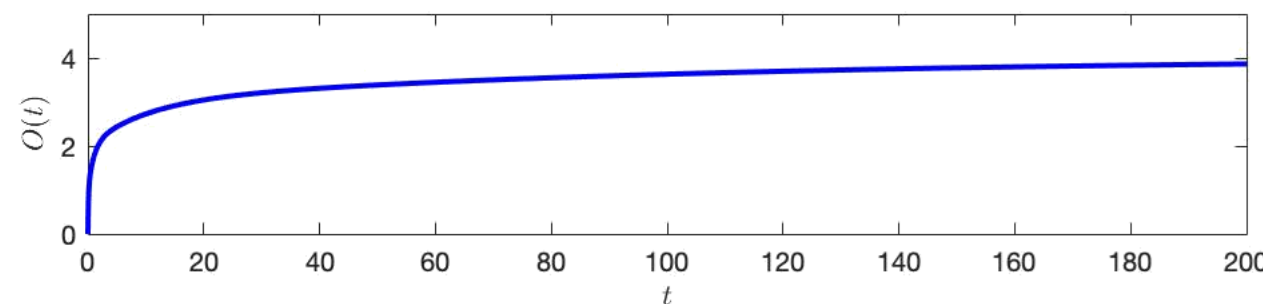
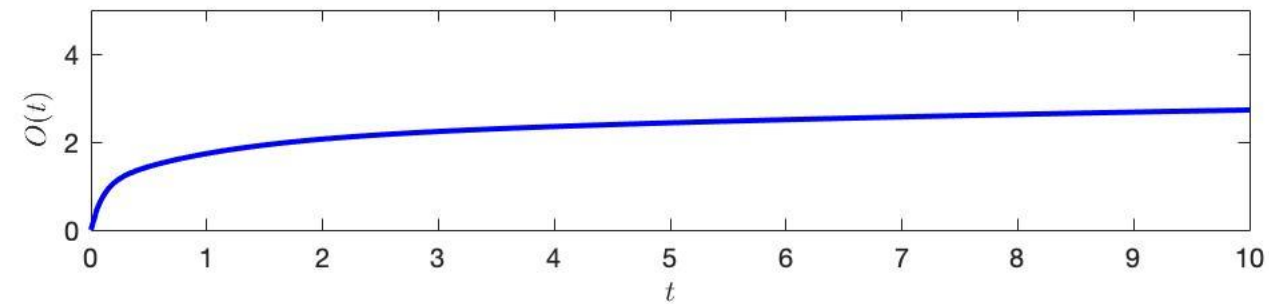
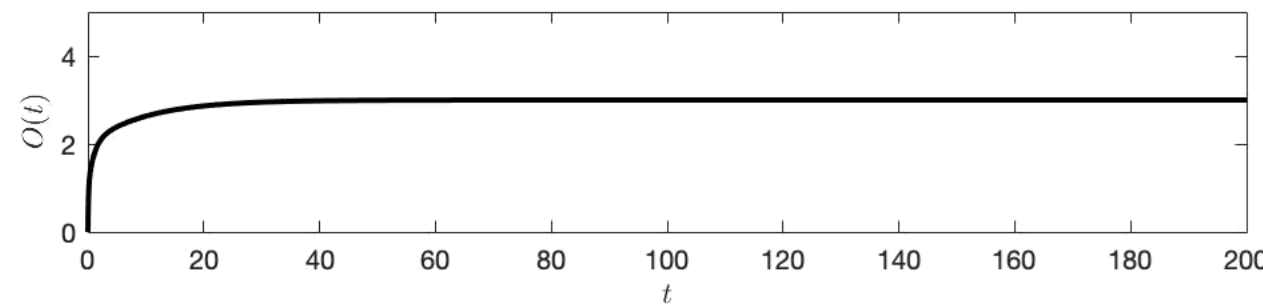
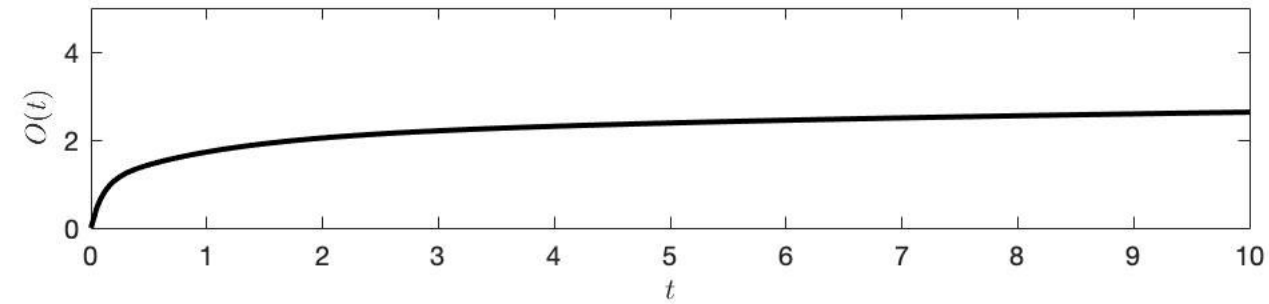
Time Scale Separation

$\tau \ll 1$  : forcing slow compared to system dynamics  $\rightarrow$  B-tipping

$\tau \gg 1$  : forcing fast compared to system dynamics  $\rightarrow$  S-tipping

$\tau = \mathcal{O}(1)$  : forcing comparable to system dynamics  $\rightarrow$  R-tipping

# Importance of timescales



The background is a complex, blue-toned digital cityscape or data landscape. It features numerous rectangular blocks, some appearing to float or be layered, with various lines and patterns suggesting a network or data flow. In the center, a dark silhouette of a person stands on a path that recedes into the distance. The overall atmosphere is high-tech and futuristic.

# Example 1

## Multiscale tipping dynamics

# Multiscale Global Energy Balance Model

$$C \frac{dT}{dt} = Q_0(1 - \alpha) - \epsilon(T)\sigma T^4 + \mu \quad (a)$$

Short-Wave

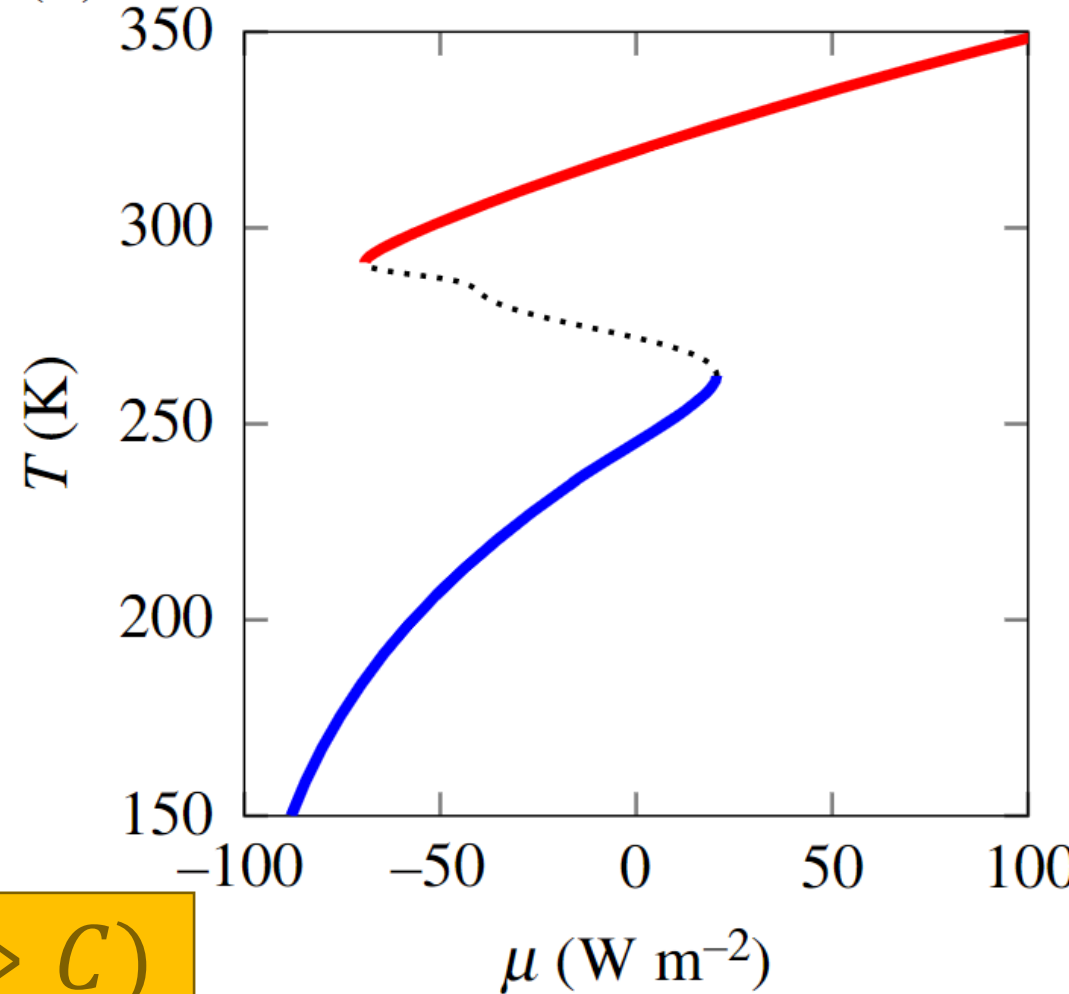
CO<sub>2</sub>

Long-Wave

$$\tau_\alpha \frac{d\alpha}{dt} = \alpha_0(T) - \alpha$$

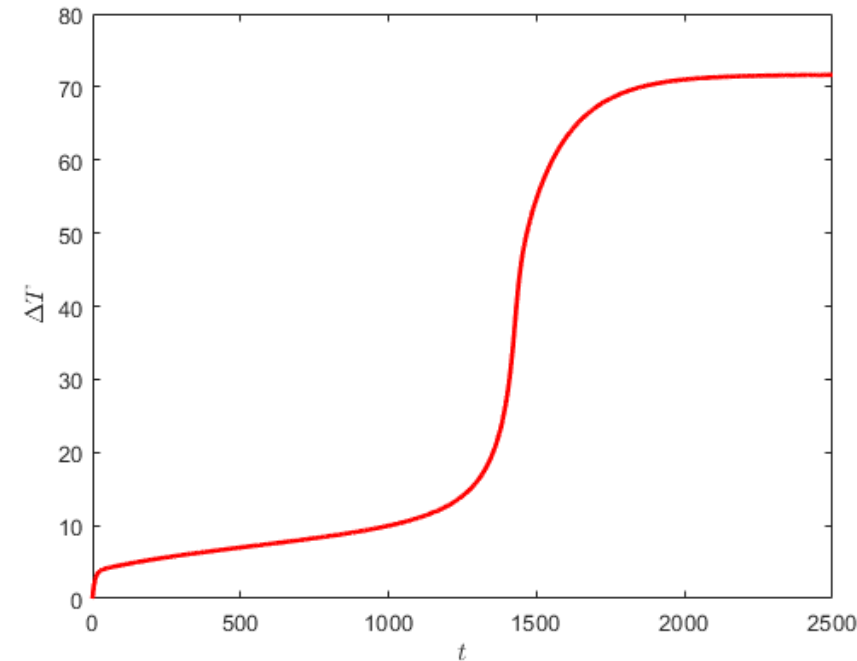
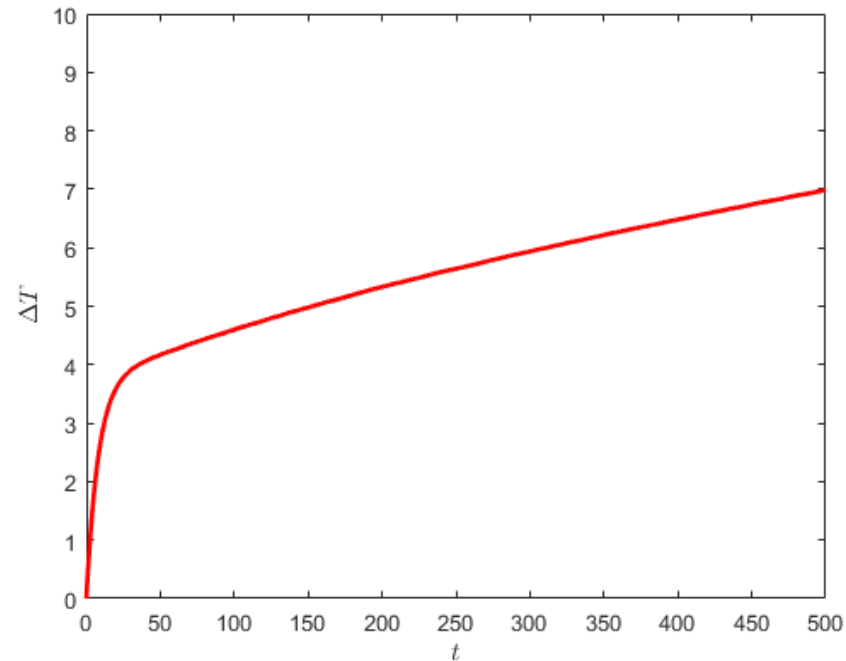
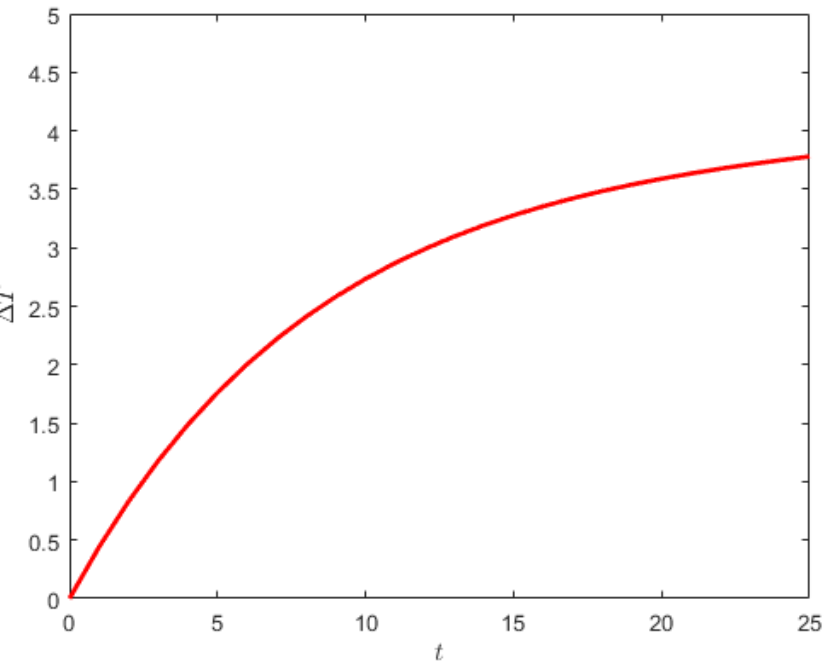
Dynamic albedo

Internal Time Scale Separation ( $\tau_\alpha \gg C$ )

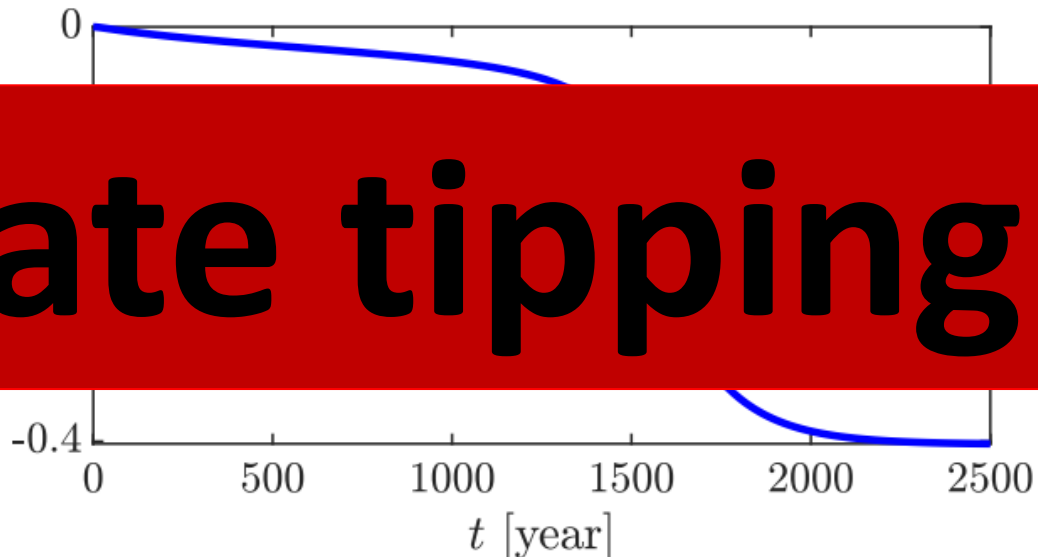
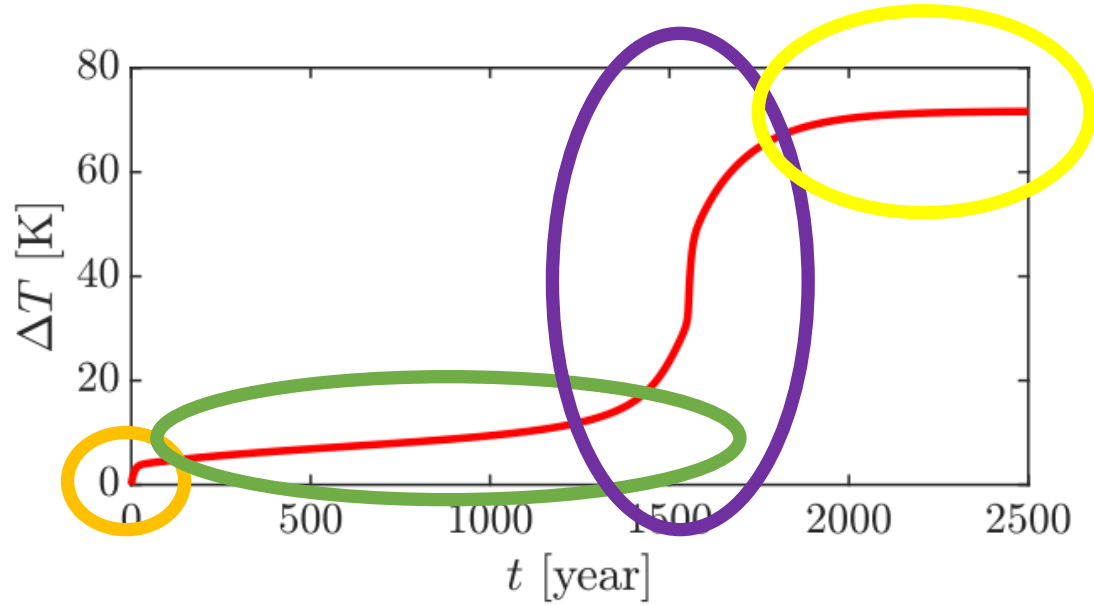


# Abrupt 4xCO2 forcing experiment

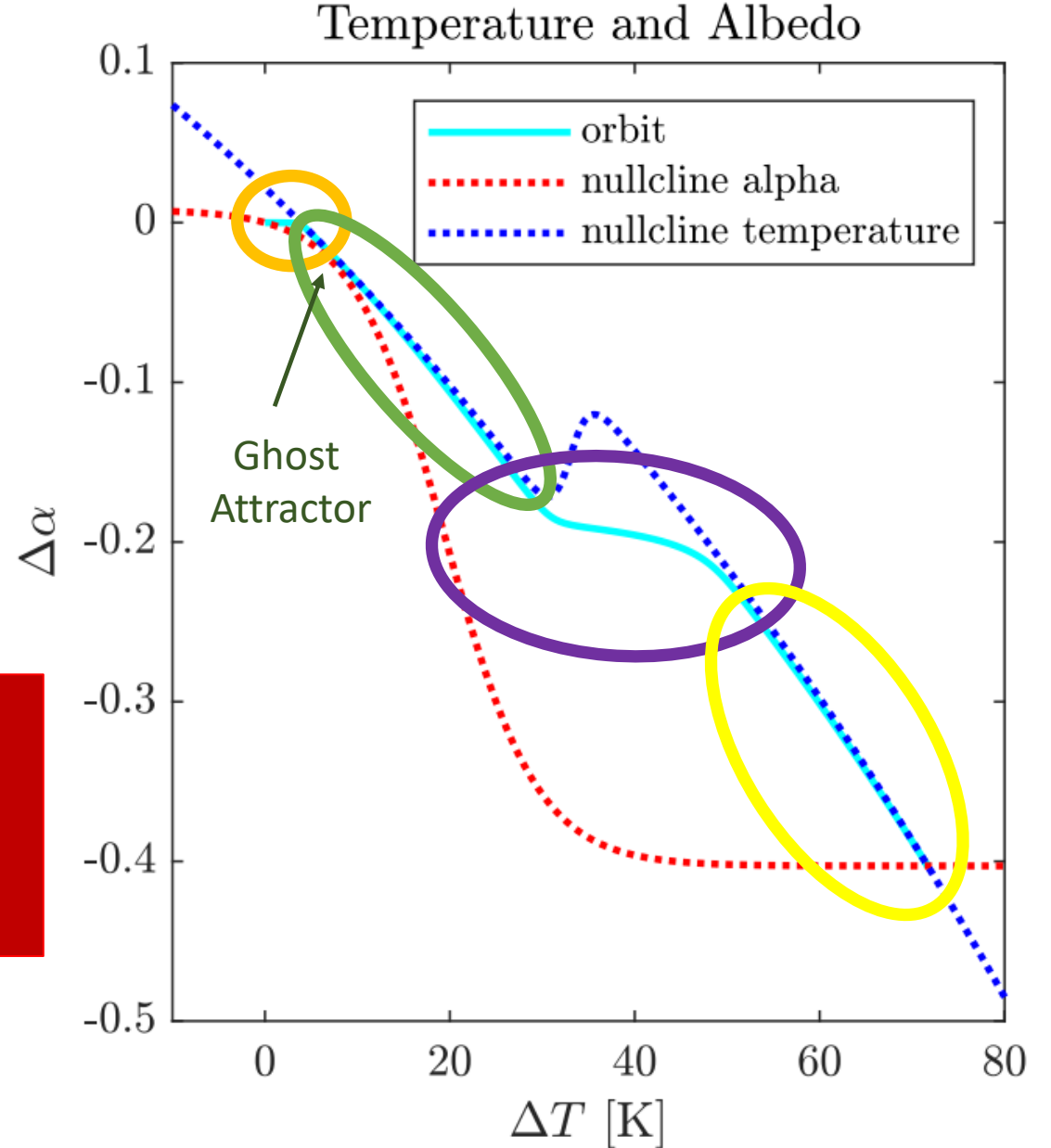
- Initialize for  $\mu_0$  (initial CO2-levels)
  - Change to  $\mu_1$  (4xCO2 levels)
- Look at dynamics



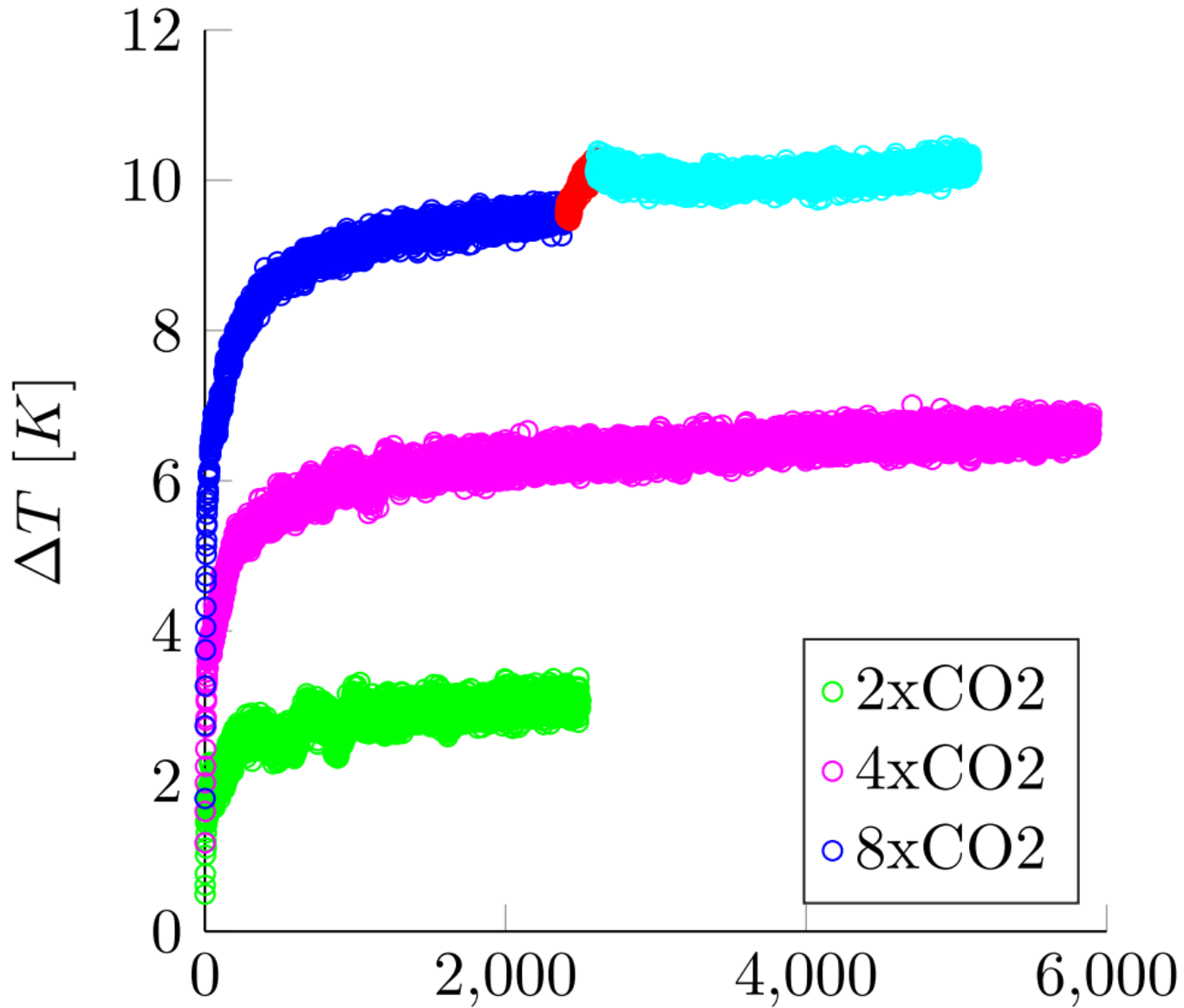
# How does this work?



**Late tipping!**



# Nonlinear Response in GCM



# Example 1-EXTRA

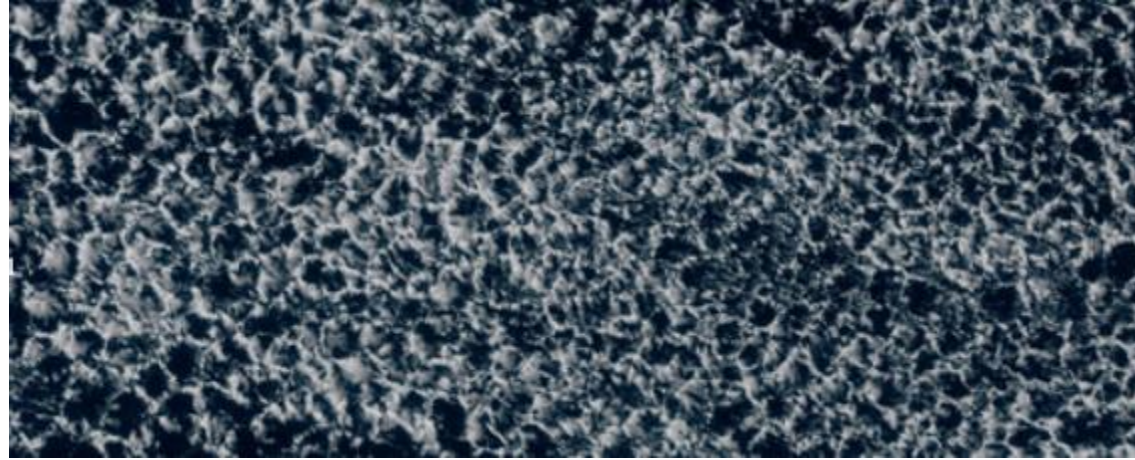
Multiscale tipping dynamics  
in spatially extended systems



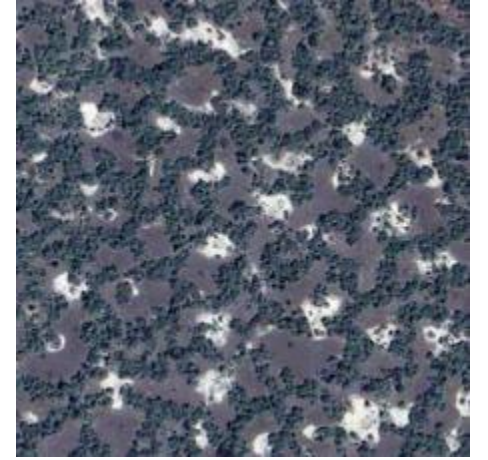
# Examples of spatial patterning – regular patterns



mussel beds



clouds



savannas



melt ponds



drylands

# Examples of spatial patterning – spatial interfaces

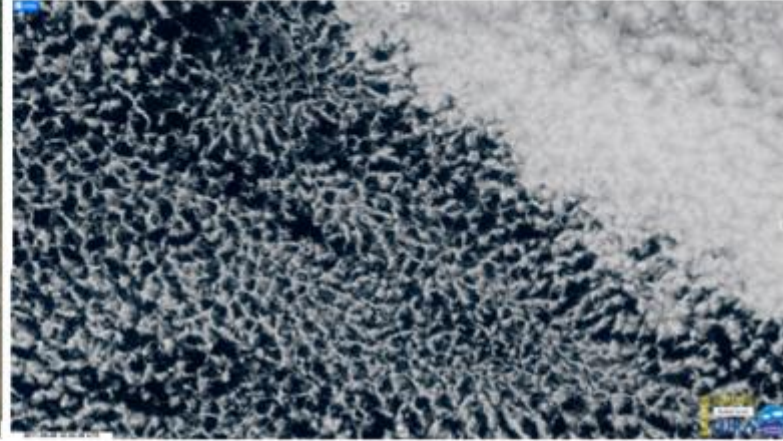
tropical forest  
& savanna  
ecosystems

[Google Earth]



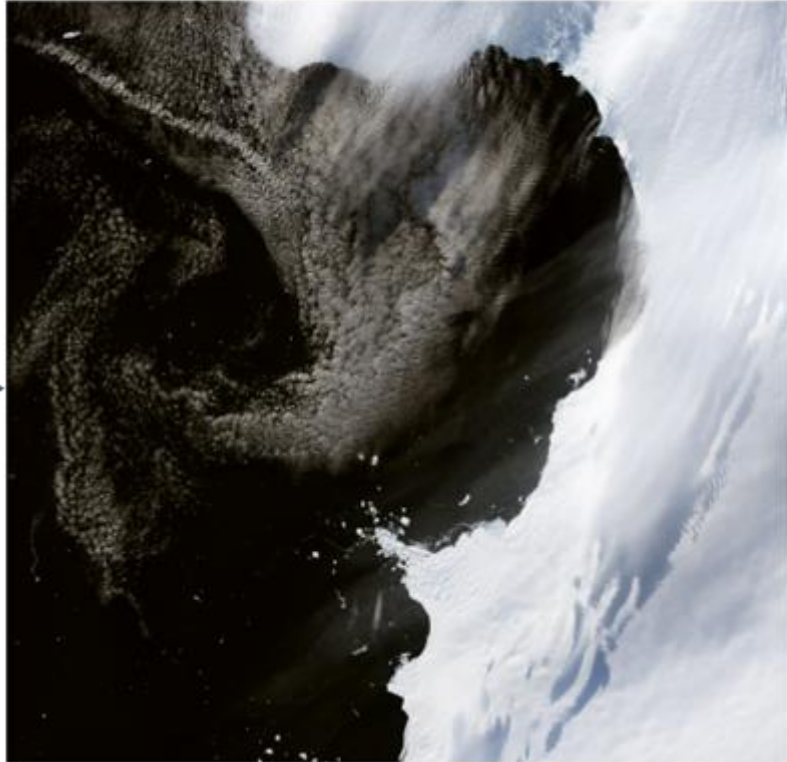
types of  
stratocumulus  
clouds

[RAMMB/CIRA SLIDER]



sea-ice & water  
at Eltanin Bay

[NASA's Earth observatory]

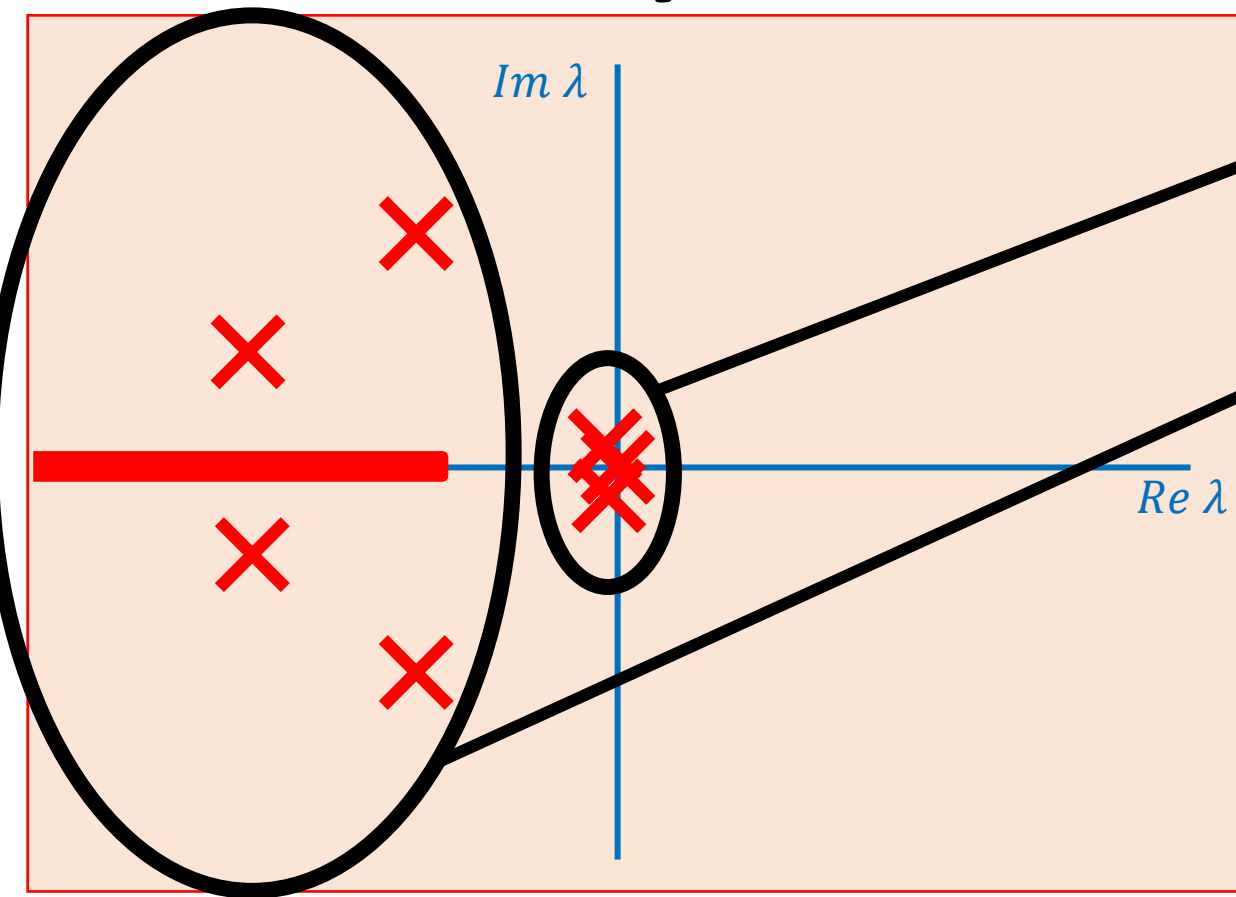


algae bloom  
in Lake St. Clair

[NASA's Earth observatory]

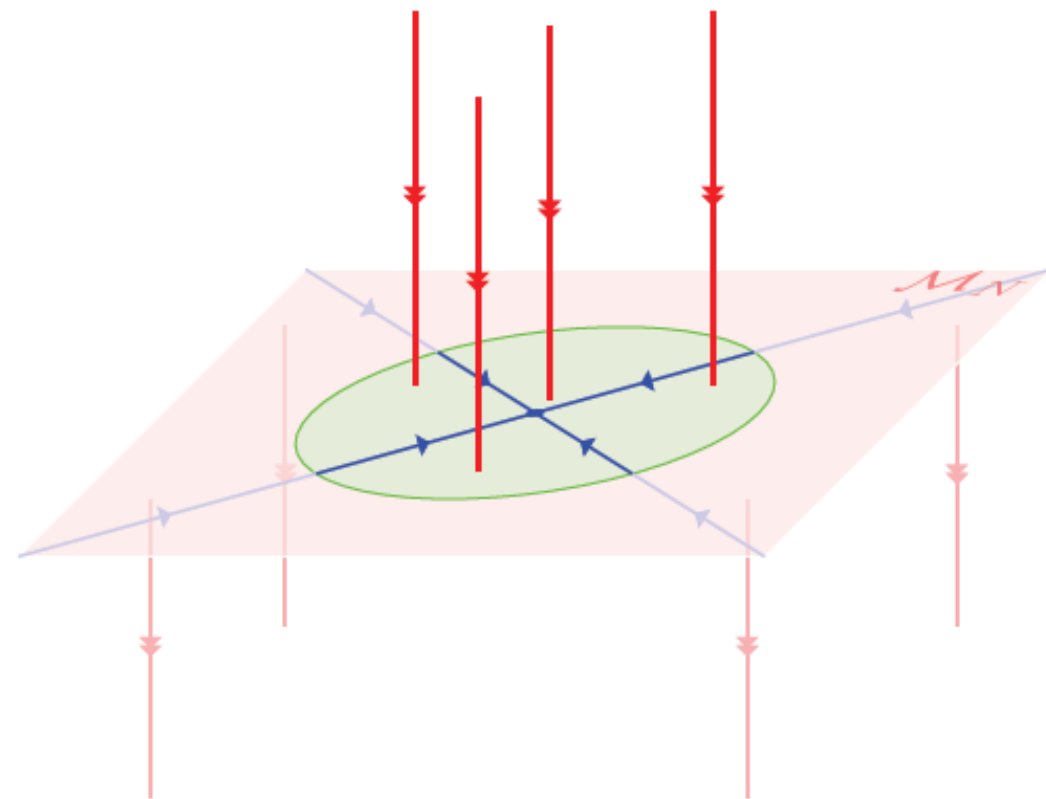
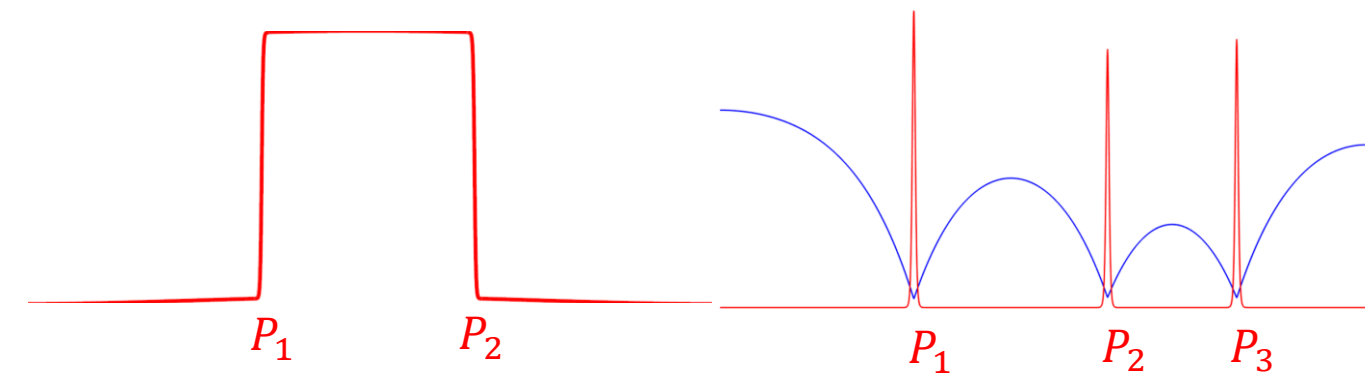


# Dynamics of Patterned States



1. SLOW Pattern Adaptation

2. FAST Pattern Degradation



# 1. SLOW pattern adaptation

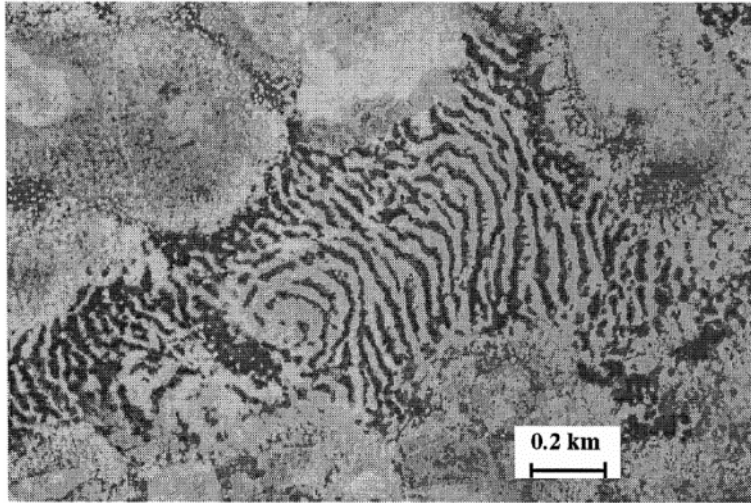


Somaliland, 1948 [Macfadyen, 1950]



Somaliland, 2008

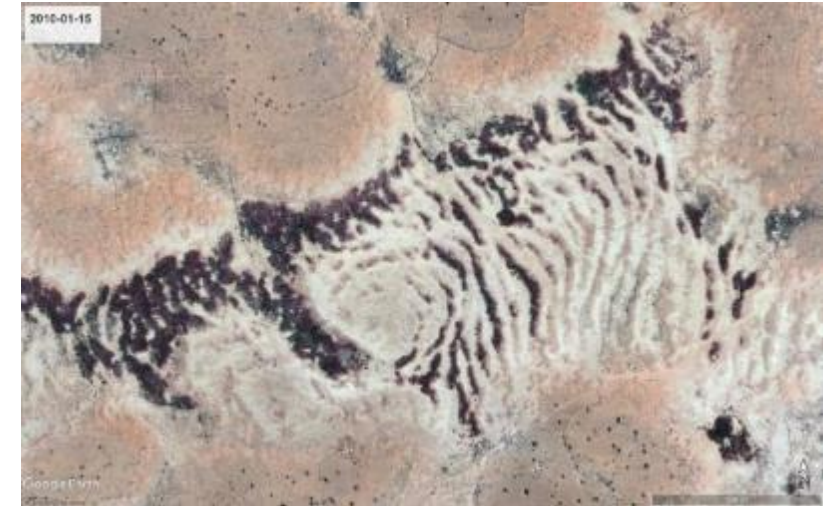
# 2. FAST Pattern Degradation



Niger, 1950 [Valentin, 1999]



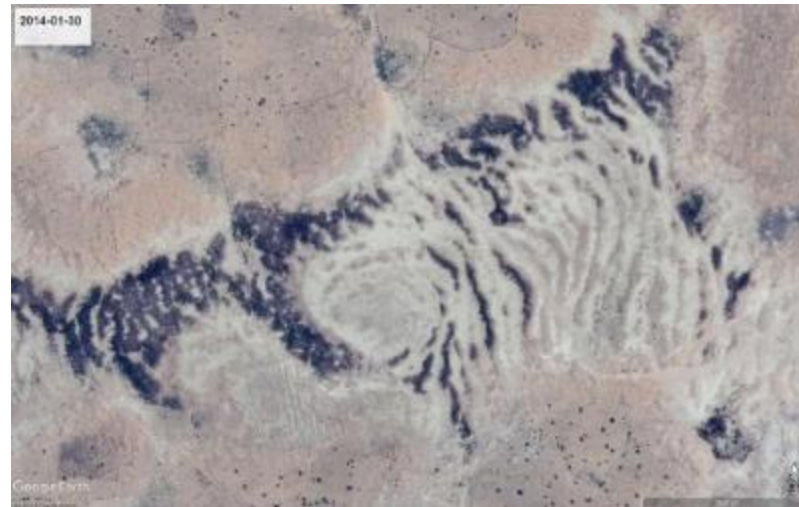
Niger, 2008



Niger, 2010



Niger, 2011



Niger, 2014

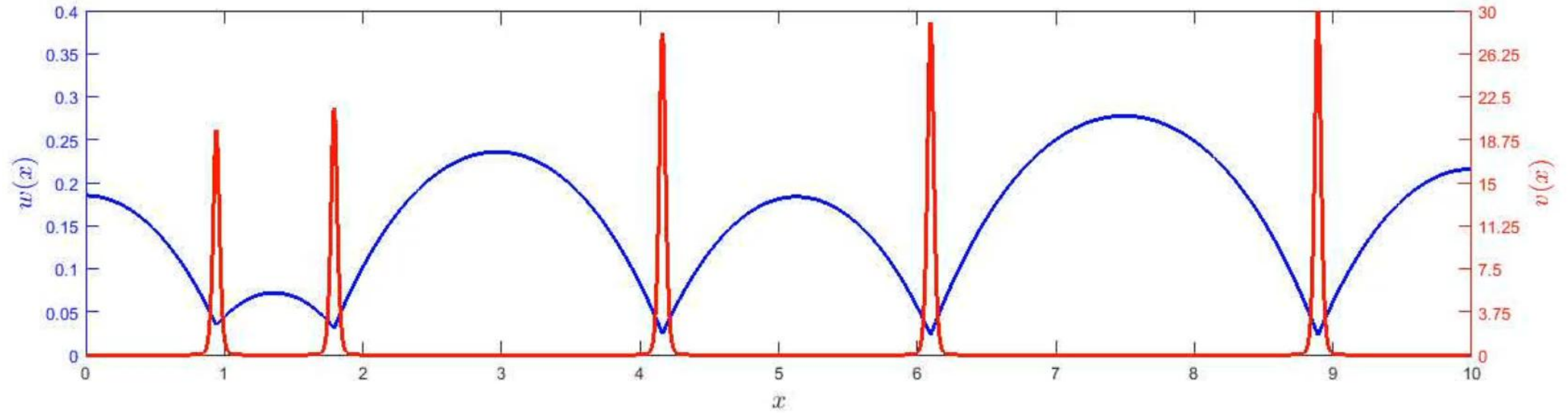


Niger, 2016

# Vegetation patches under climate change

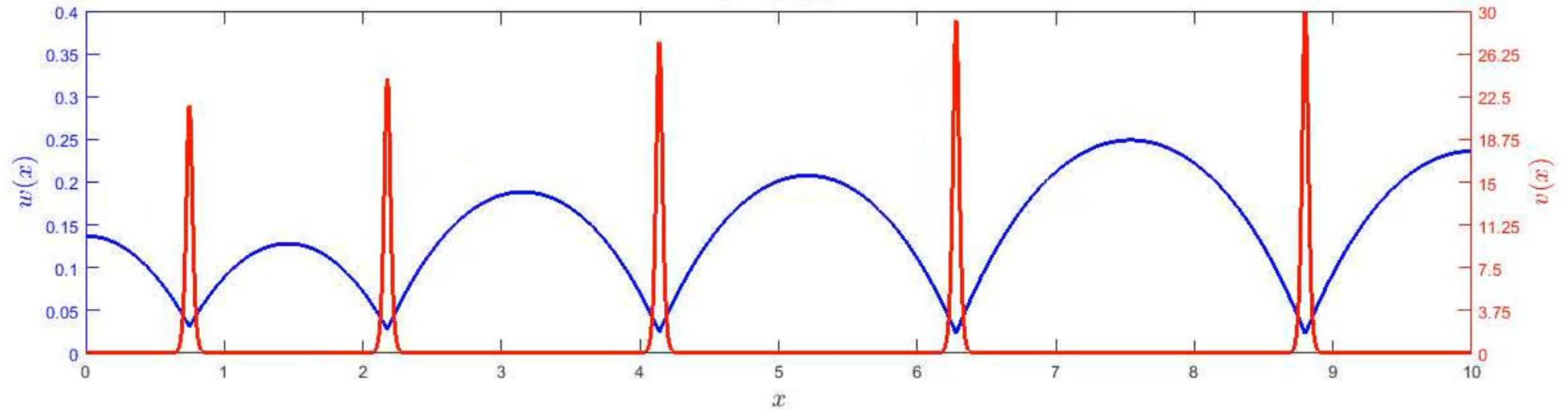
Rate of climate change

FAST



$a = 0.4995$

SLOW

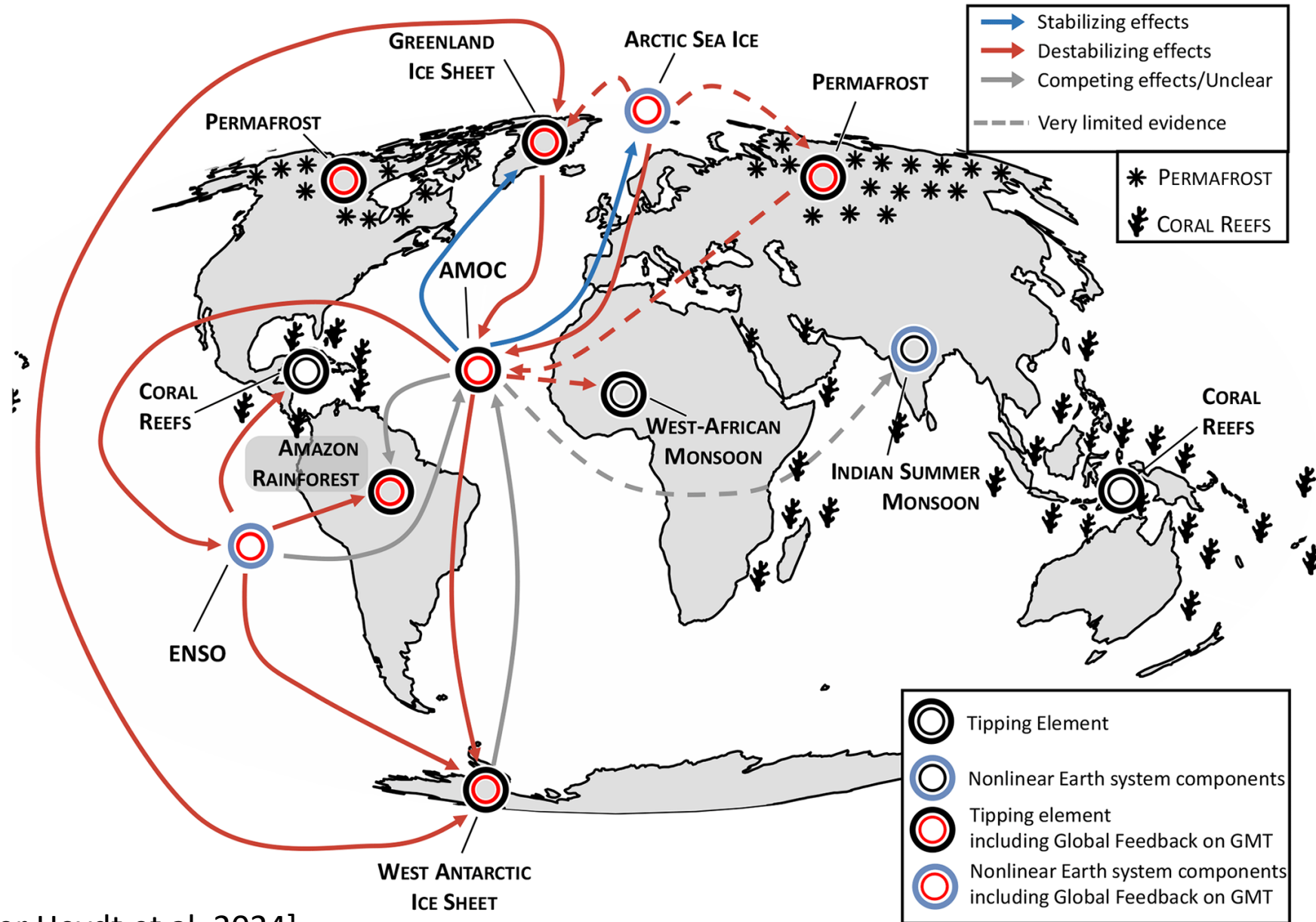


The background is a complex, blue-toned digital landscape. It features a central perspective view of a person's silhouette standing on a grid-like floor that recedes into the distance. The surrounding space is filled with various digital elements: floating rectangular panels, glowing lines, and abstract architectural structures, all rendered in shades of blue and white. The overall atmosphere is one of a high-tech, data-driven environment.

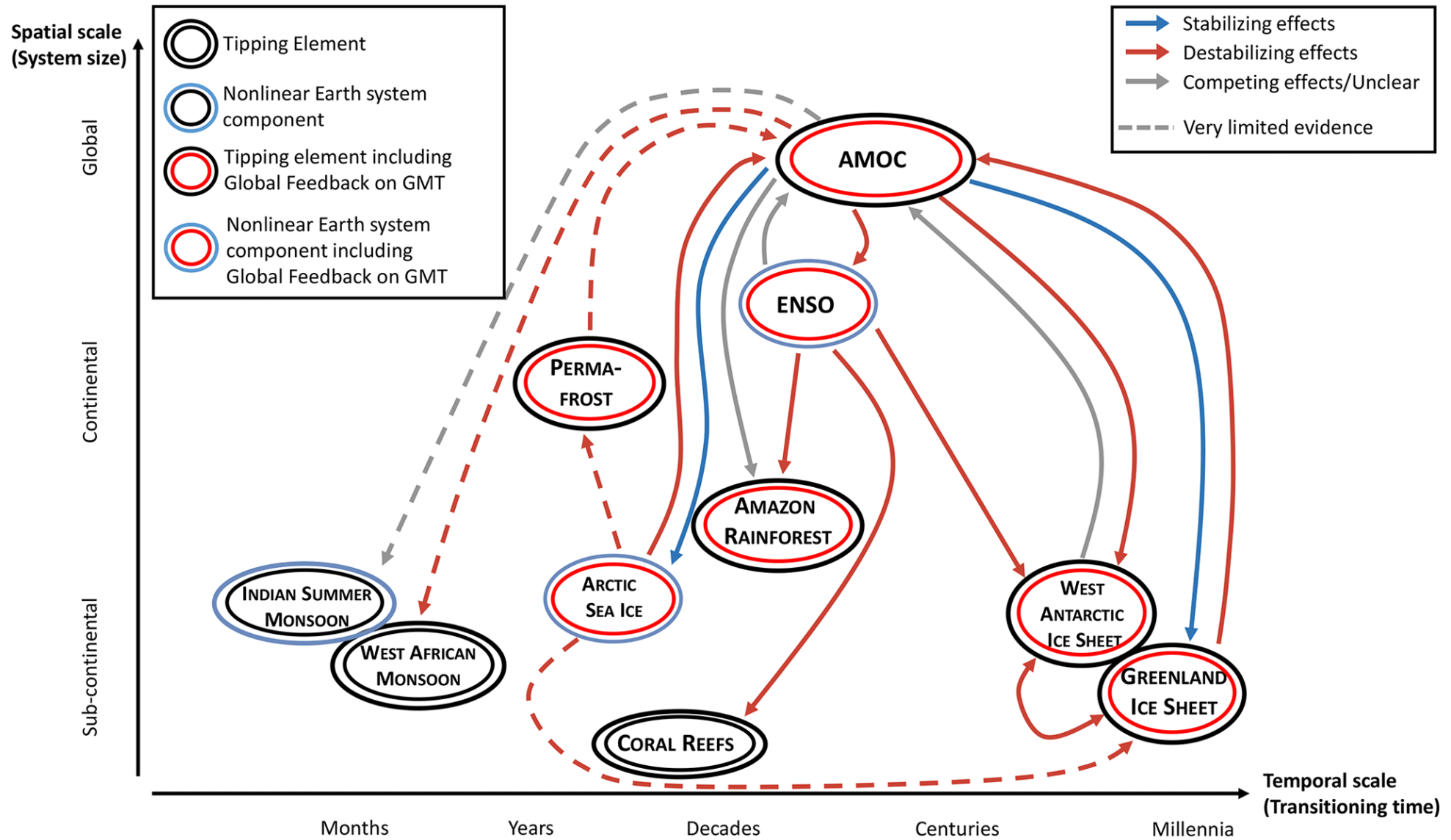
# Example 2

# Tipping Cascades

# Tipping Cascades



# Timescales of Tipping Cascades



# Conceptual model

upstream system:

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(r(t))$$

timescale of forcing ( $r \ll 1$ )

downstream system:

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$

unidirectional coupling

Timescale separation between systems:

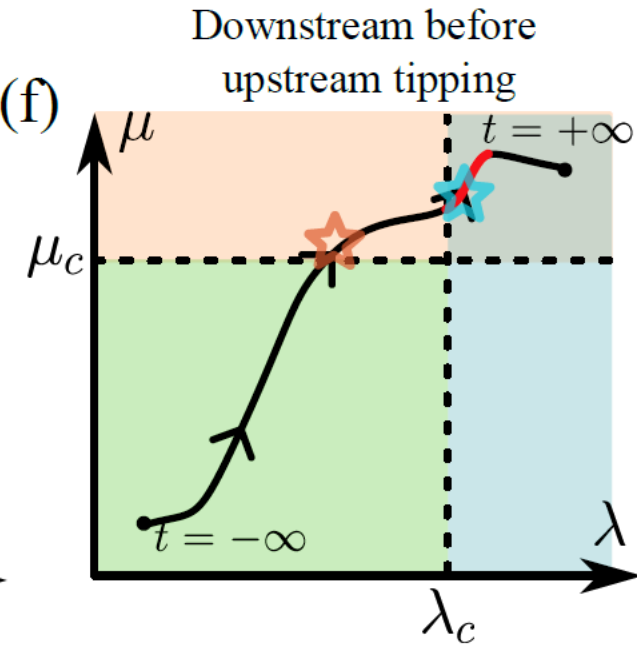
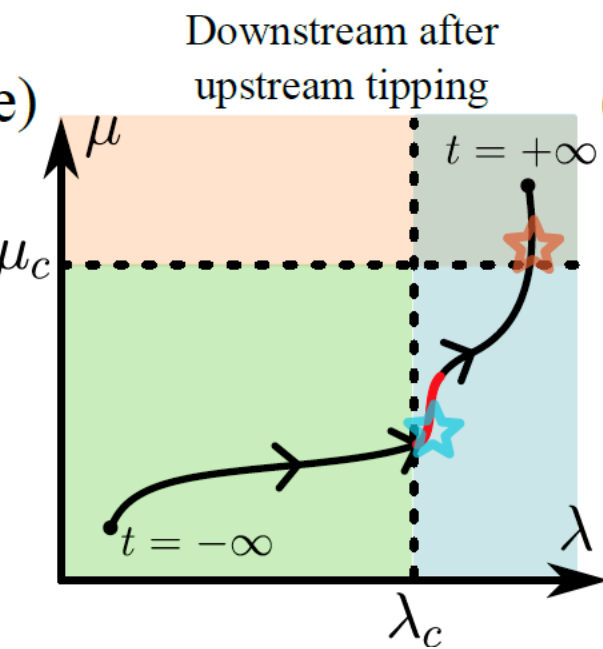
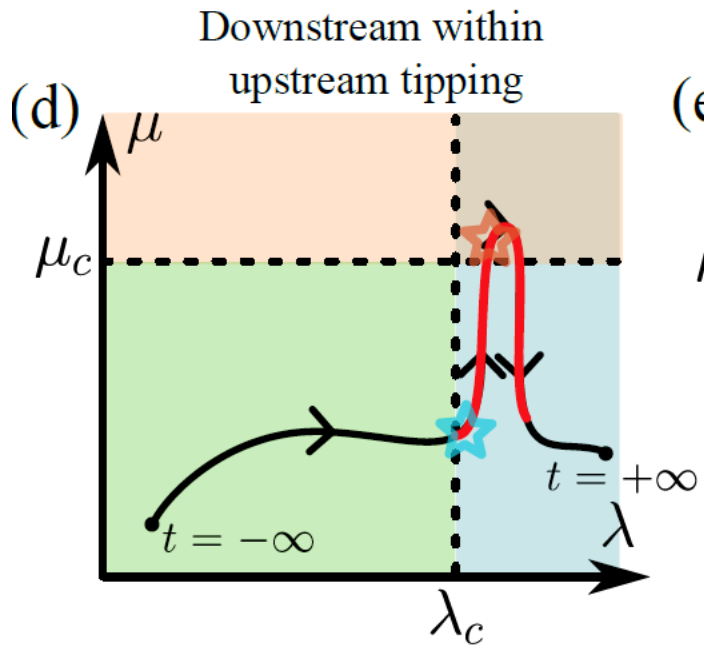
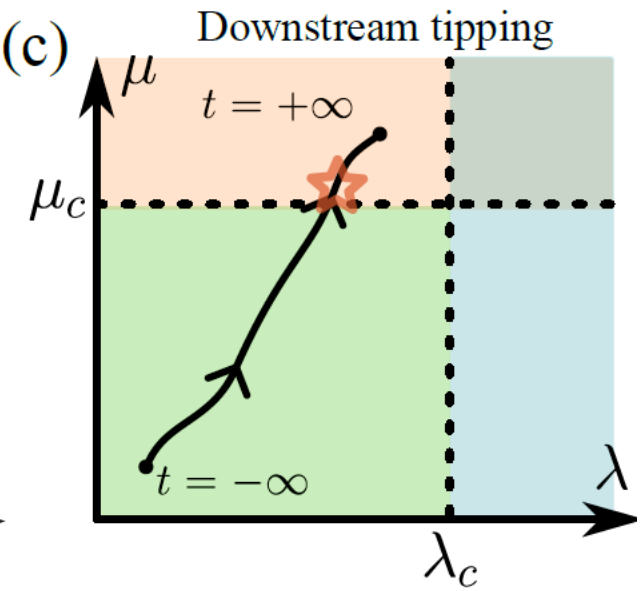
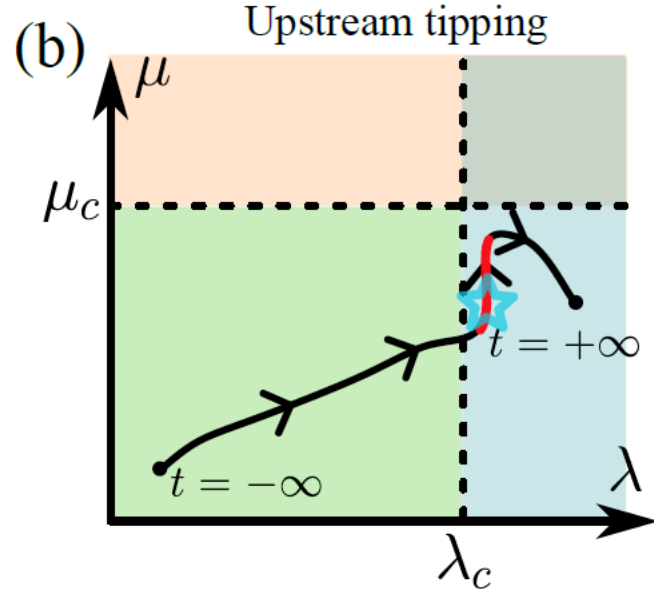
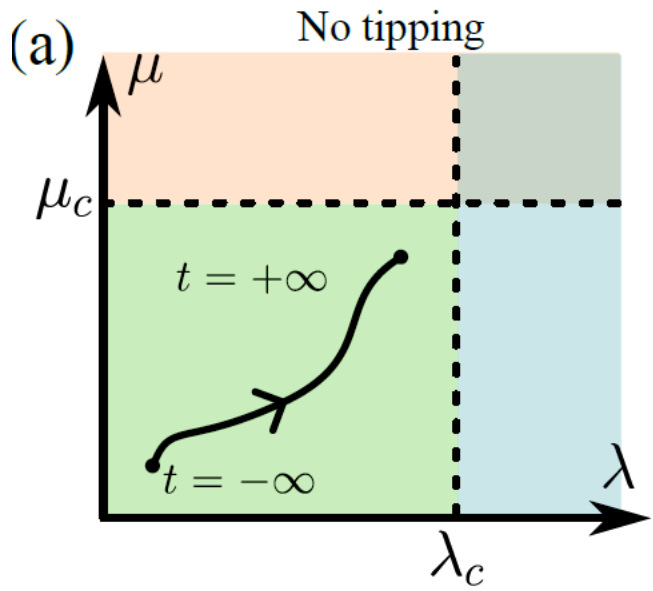
$\varepsilon \ll 1$  : downstream system acts on a **FASTER** time scale

$\varepsilon \gg 1$  : downstream system acts on a **SLOWER** time scale

# Potential tipping pathways

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

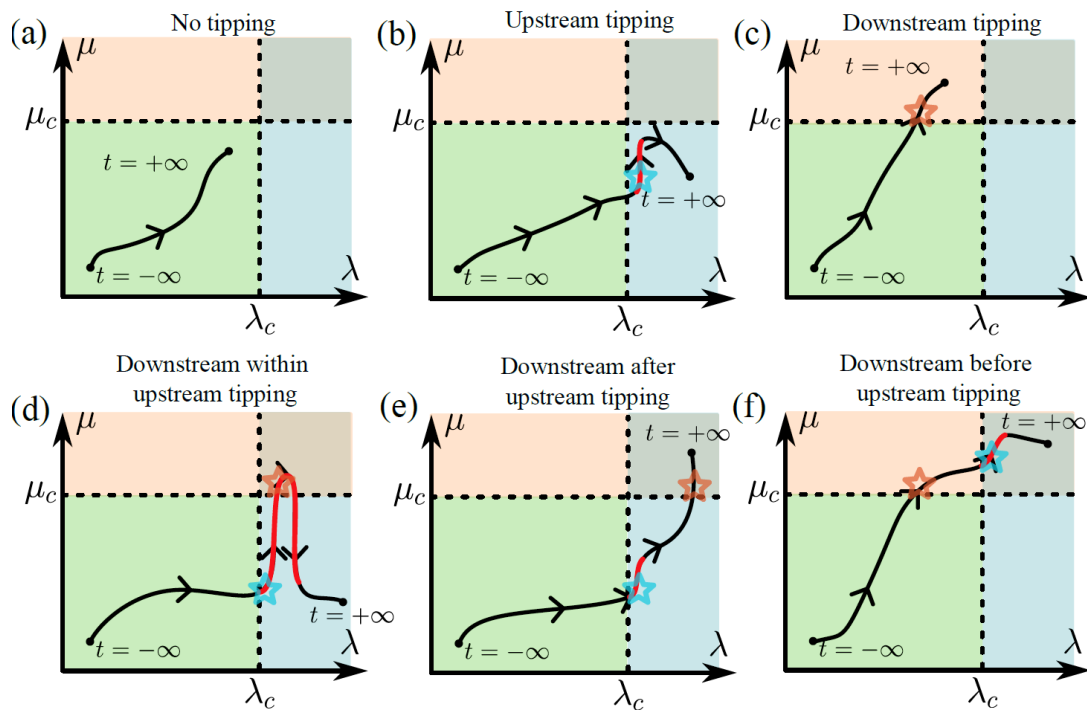
$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$



# Potential tipping pathways

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$

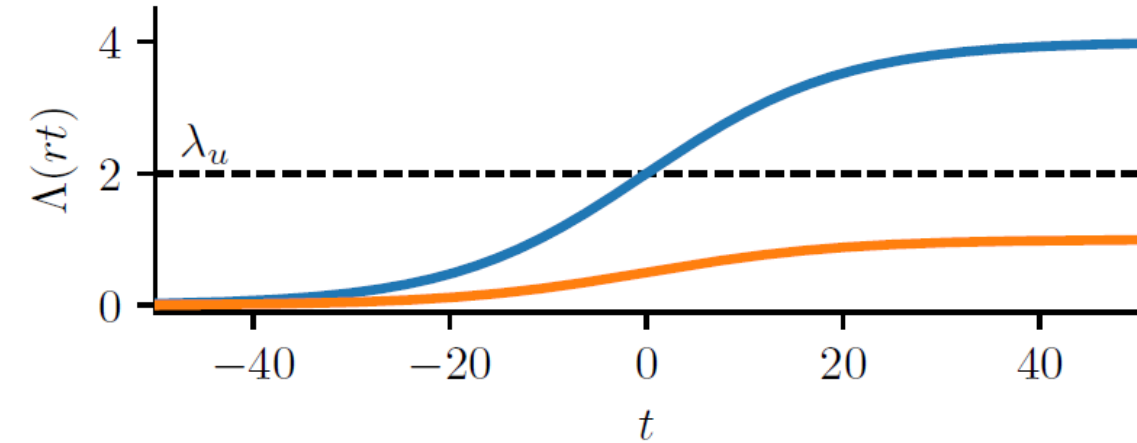


Tipping scenario	Tipping sequence		Accelerating cascade	Decelerating cascade
	Upstream	Downstream		
Upstream B-tipping Downstream tracking (UB)	█		✓	✓
Downstream after Upstream B-tipping (DaUB)	█	█	✓	✓
Downstream overlap Upstream B-tipping (DoUB)	█	█	✓	✓
Upstream overlap Downstream B-tipping (UoDB)		█	✓	✓
Upstream after Downstream B-tipping (UaDB)		█	✓	✓
Downstream within Upstream B-tipping (DwUB)	█	█	✓	✗
Upstream within Downstream B-tipping (UwDB)	█	█	✗	✓

Time →

# Upstream System Behaviour

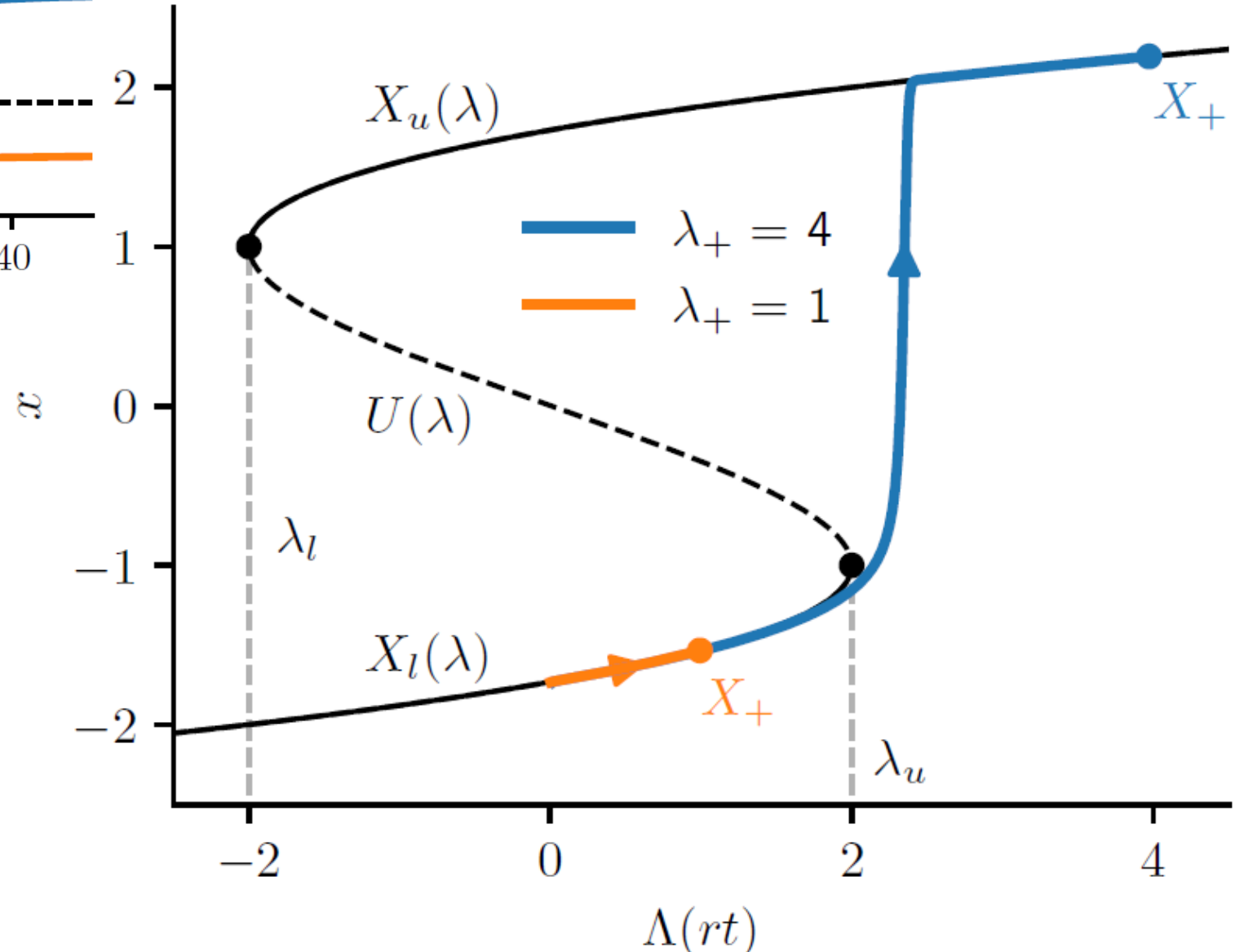
$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$



## Forcing scenario $\Lambda(rt)$

- $r \ll 1$
- $\lim_{t \rightarrow -\infty} \Lambda(rt) = \lambda_-$
- $\lim_{t \rightarrow +\infty} \Lambda(rt) = \lambda_+$

In remainder:  $\lambda_+ > \lambda_u := 2$   
so upstream **B-tipping**



# Linear Coupling: $M(x) = a + b x$

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

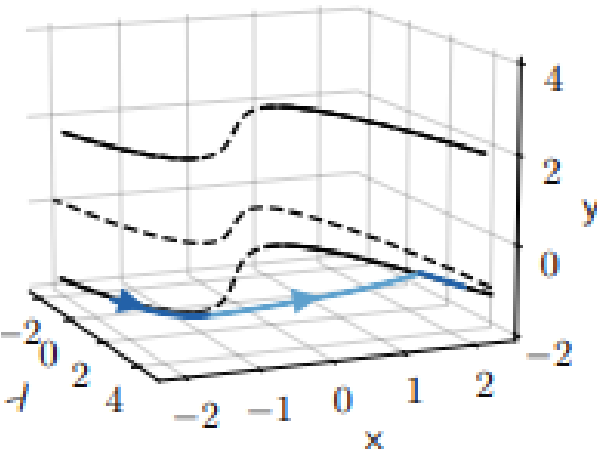
$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$

Upstream same!

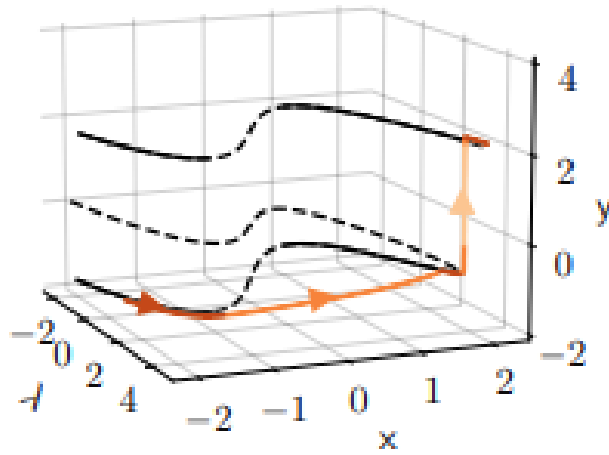
Order of tipping:

- (b)  $x$
- (c)  $x$  then  $y$
- (d)  $x$  and  $y$
- (e)  $y$  and/then  $x$
- (f)  $y$  then  $x$

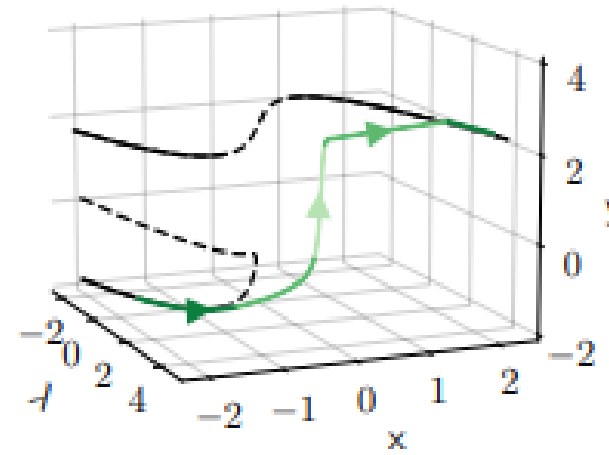
(a)  $b = 0.495$



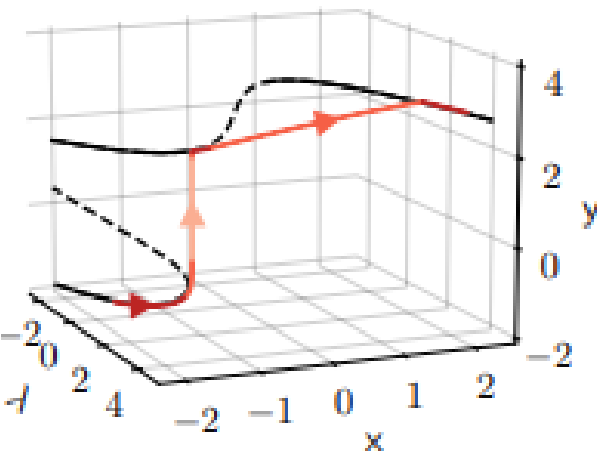
(b)  $b = 0.52$



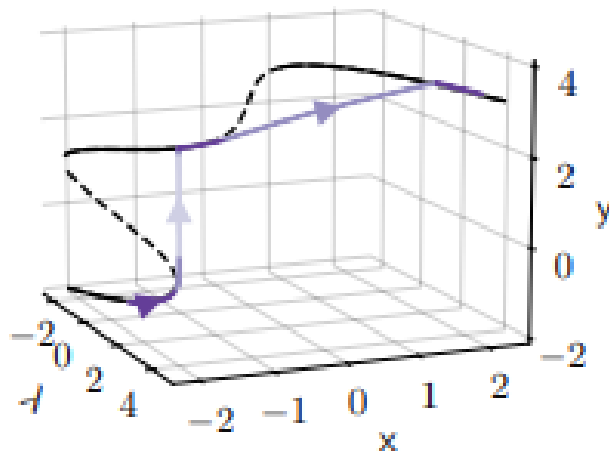
(c)  $b = 1.25$



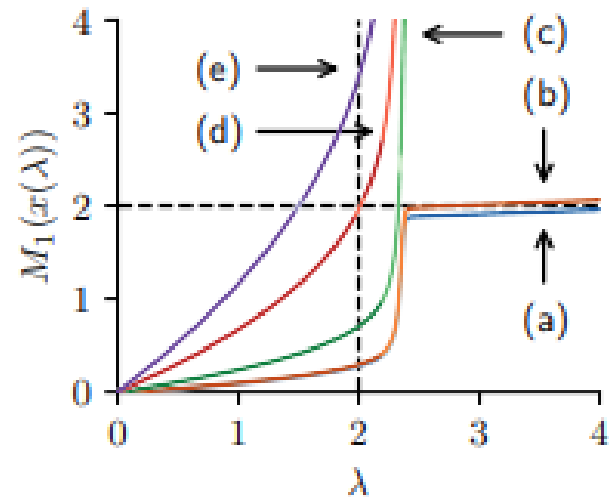
(d)  $b = 3.468$



(e)  $b = 6$



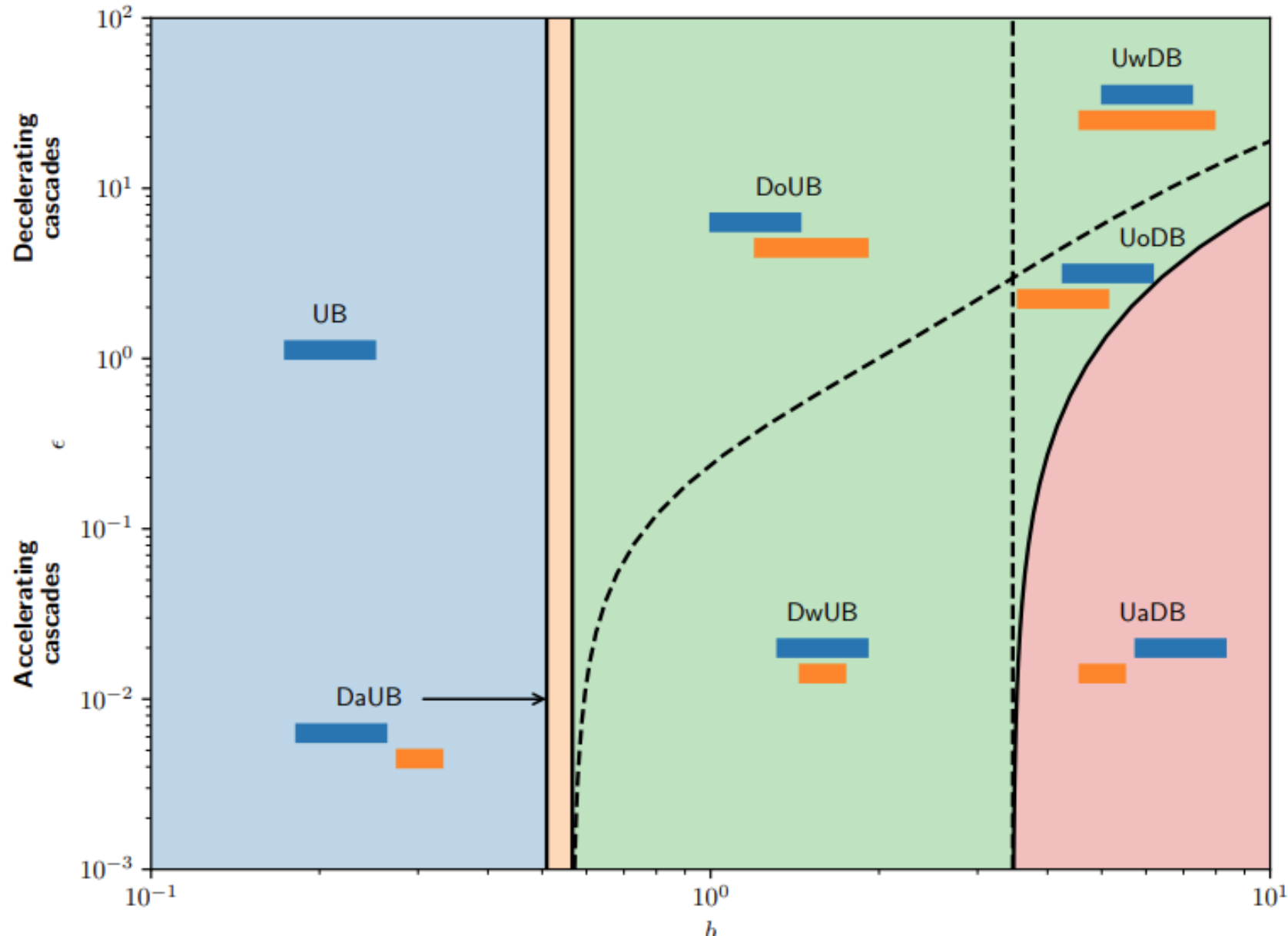
(f)



# Linear Coupling: $M(x) = a + b x$

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$



## Order of tipping:

UB:  $x$  only

DaUB:  $x$  then  $y$

propagation:  $x$  and  $y$

DBUB:  $y$  then  $x$

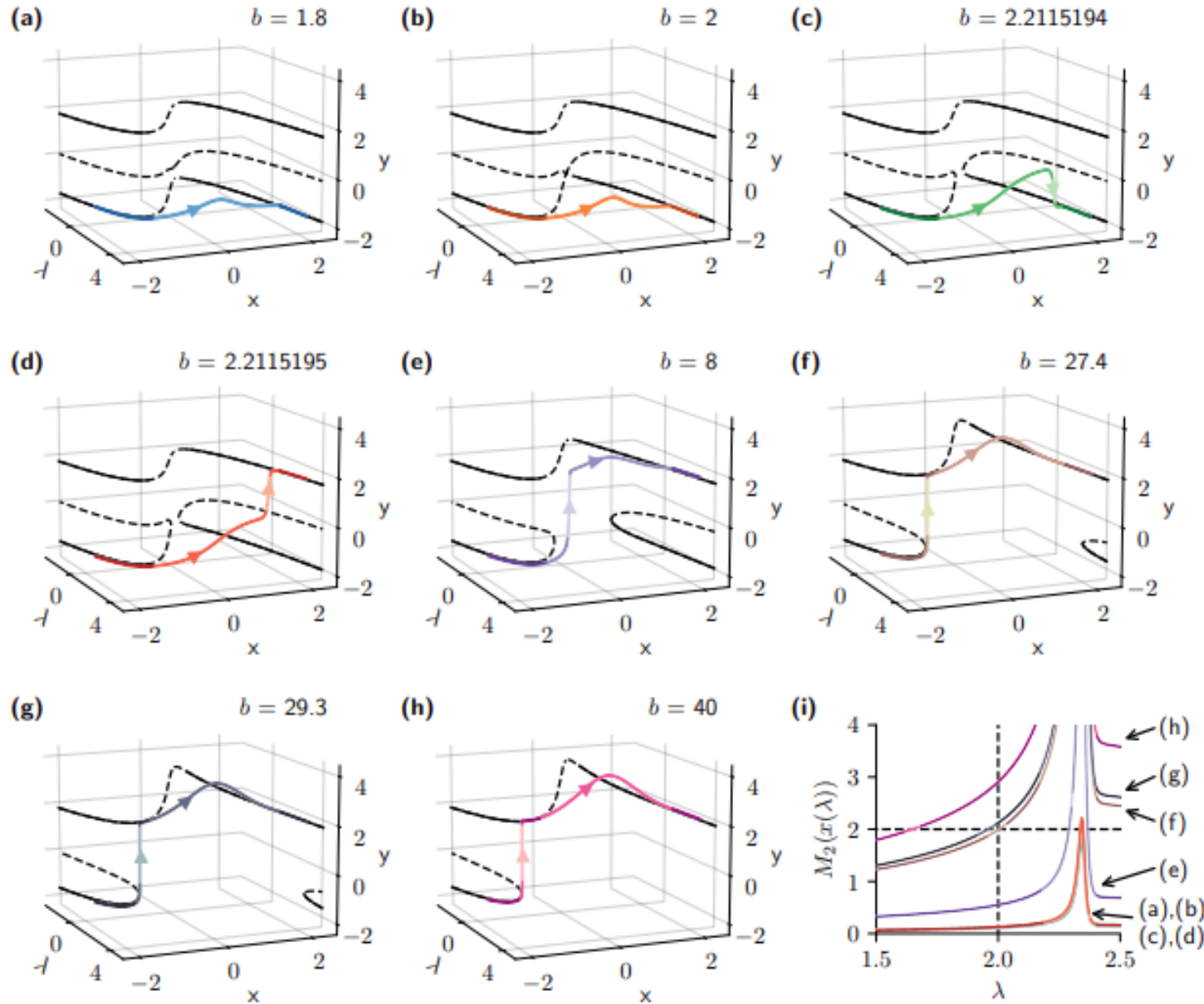
# Local Coupling: $M(x) = a + b \operatorname{sech}(c[x - d])$

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$

Upstream same!

- Order of tipping:
- (b)  $x$
  - (c)  $x$
  - (d)  $x$  ( $y$  overshoot)
  - (e)  $x$  and  $y$
  - (f)  $x$  and  $y$
  - (g)  $y$  then  $x$
  - (h)  $y$  then  $x$



# Local Coupling: $M(x) = a + b \operatorname{sech}(c[x - d])$

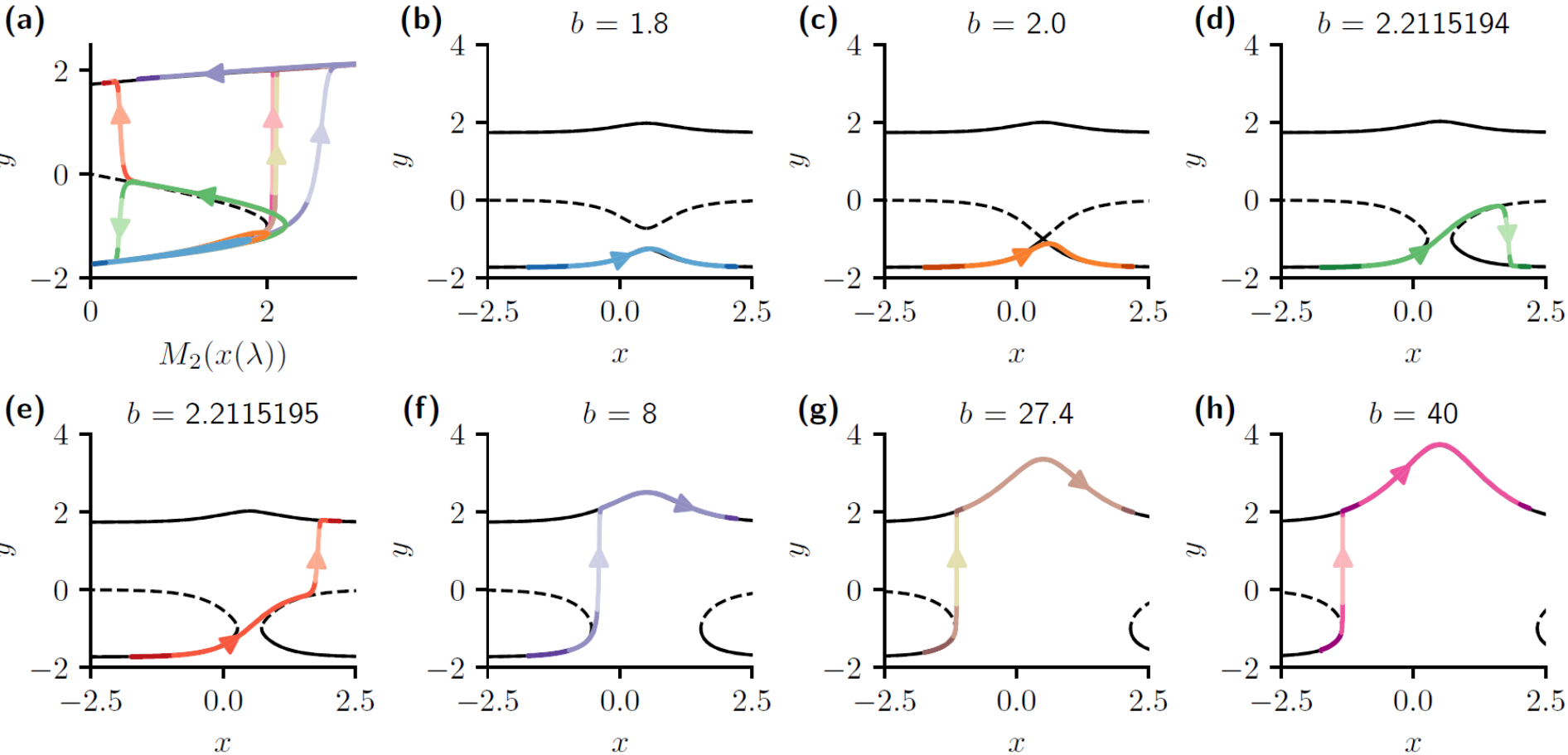
$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$

Upstream same!

Order of tipping:

- (b)  $x$
- (c)  $x$
- (d)  $x$  ( $y$  overshoot)
- (e)  $x$  and  $y$
- (f)  $x$  and  $y$
- (g)  $y$  then  $x$
- (h)  $y$  then  $x$



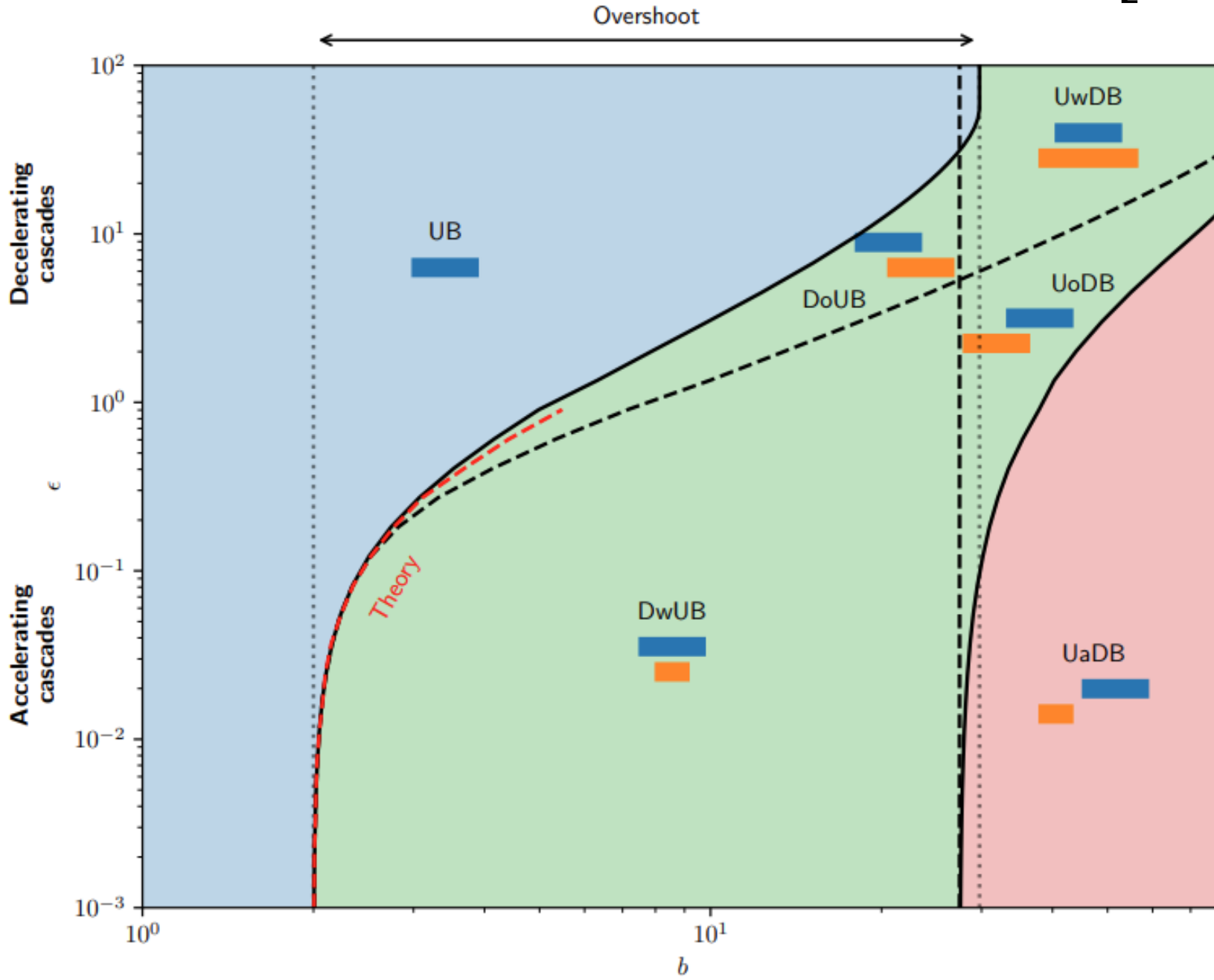
Phase portraits view

(c-e): slow passage during upstream tipping  
 Interpretation via slow passage through fold or transcritical bifurcation

# Local Coupling: $M(x) = a + b \operatorname{sech}(c[x - d])$

$$\frac{dx}{dt} = 3x - x^3 + \Lambda(rt)$$

$$\varepsilon \frac{dy}{dt} = 3y - y^3 + M(x)$$



## Order of tipping:

DT:  $x$  only

DOT:  $x$  ( $y$  overshoot)

DOB:  $x$  and  $y$

DB:  $y$  then  $x$

## Theory:

Using asymptotics, see e.g. [Ritchie et al, 2019]

Also matches with slow passage literature, e.g. [Krupa & Szmolyan, 2001]

# Early Warning Signs

- EWSs signal for imminent tipping
- Here: based on [Ditlevsen & Ditlevsen, 2024] estimation for return rate  $\alpha$
- At bifurcation:  $\alpha \downarrow 0$
- Linear extrapolation of trend in  $\alpha^2$  is an *early warning prediction*

## Setting:

With data up to time  $t = t_p$ ,  
predict whether tipping occurs within  $\Delta$  time

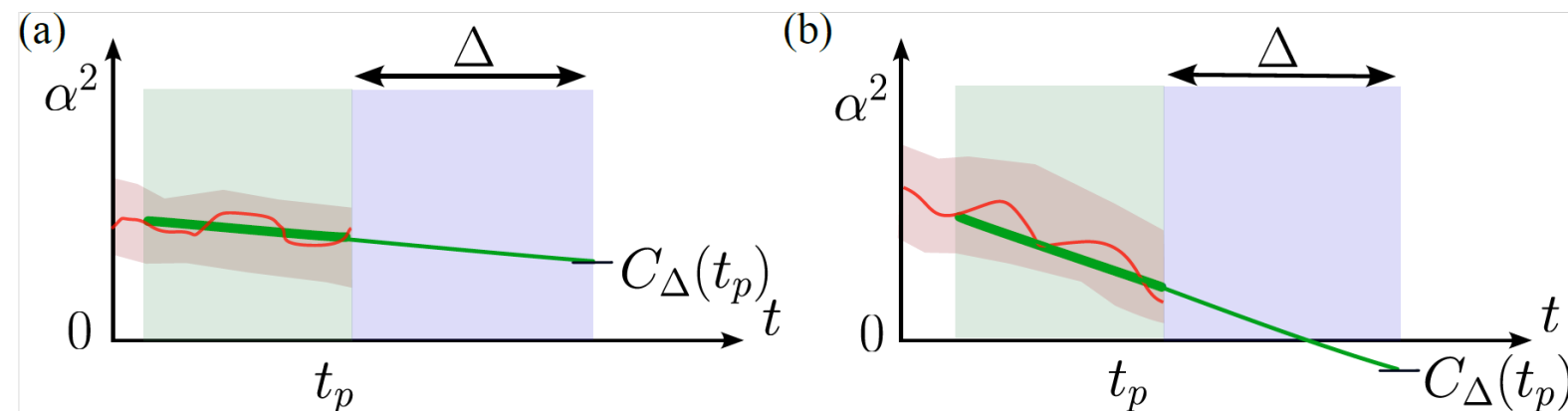
## Method:

Extrapolated  $\alpha$  at time  $t_p + \Delta$  is score  $C_\Delta(t_p)$

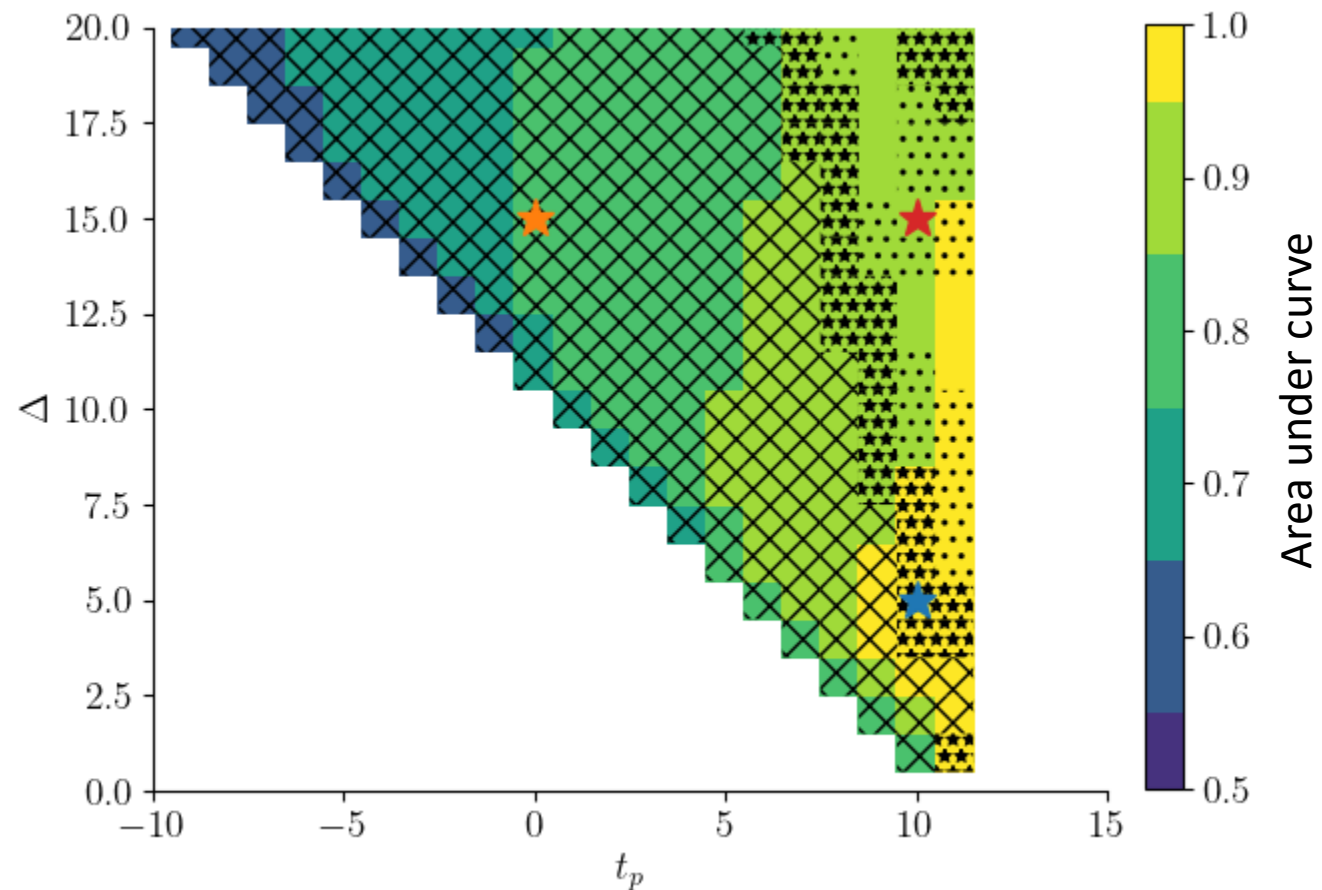
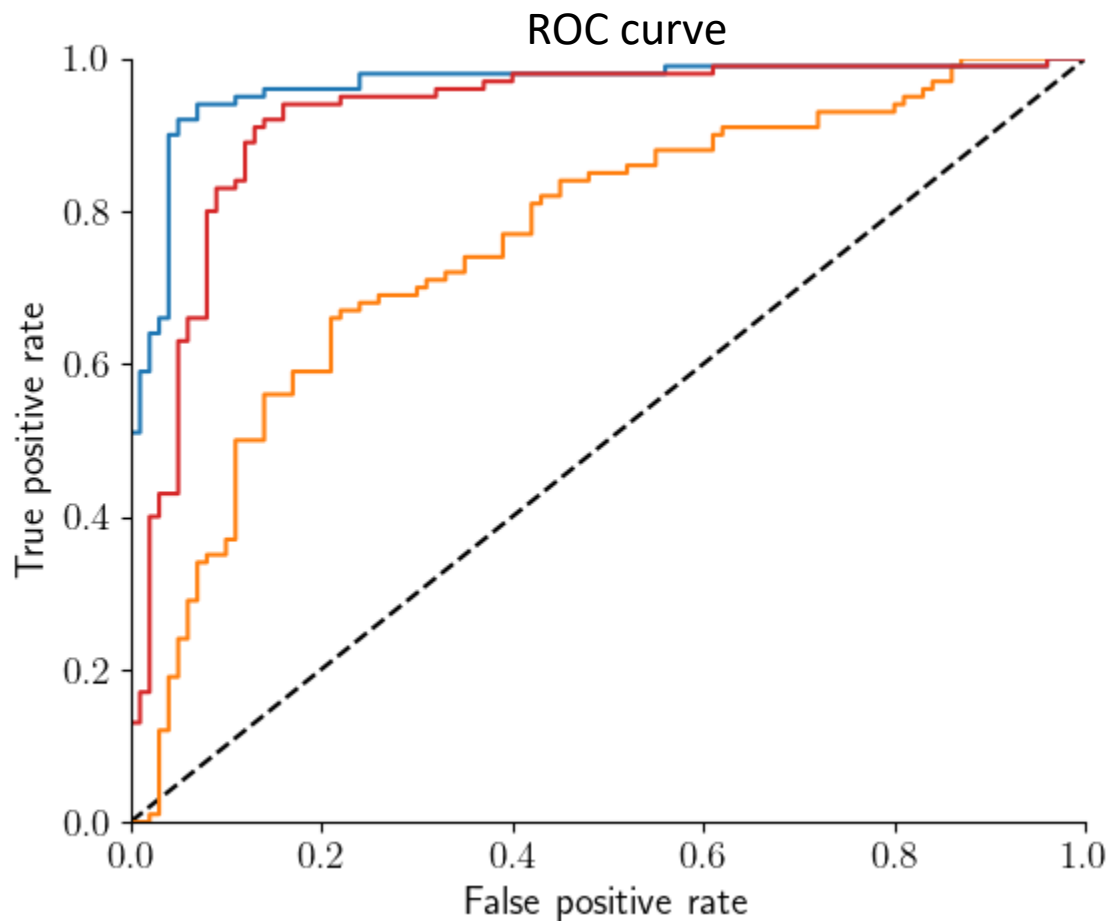
$$\begin{aligned} \frac{dx}{dt} &= 3x - x^3 + \Lambda(rt) + \sigma \xi_1 \\ \varepsilon \frac{dy}{dt} &= 3y - y^3 + M(x) + \eta \xi_2 \end{aligned}$$

Stochasticity added

Method is good if a score threshold can be found such that false predictions are minimal



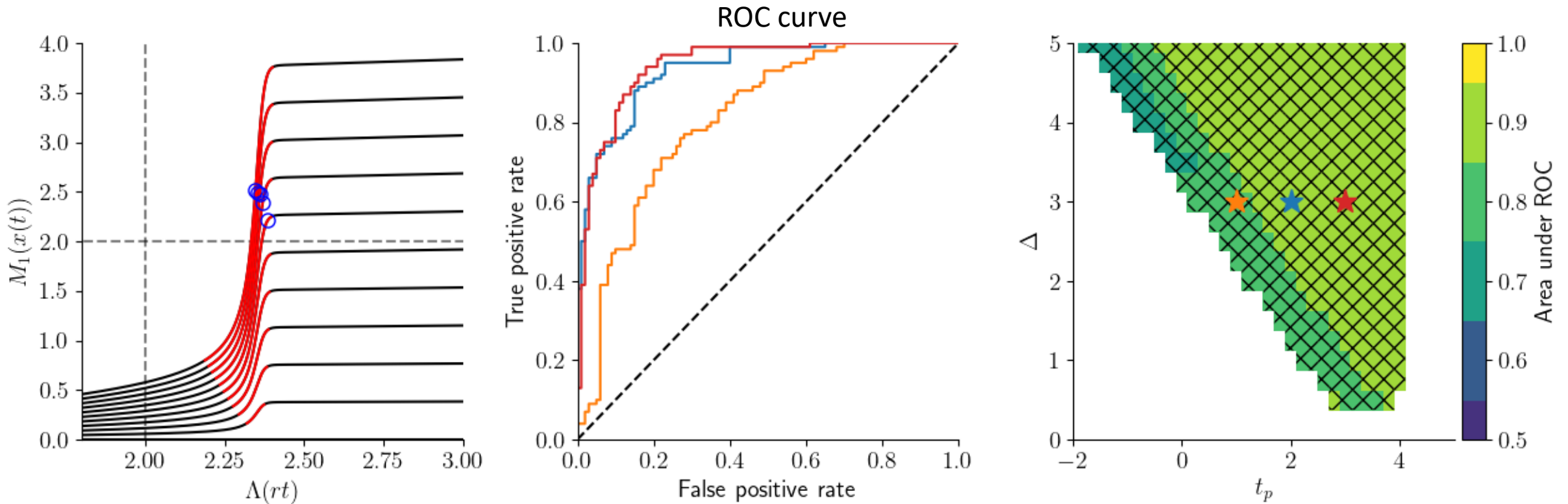
# EWS for upstream system



Test ensemble consists of different  $\lambda_+$  values between 1 and 3, so about 50% show B-tipping

Hatching relates to optimal threshold:  
Cross: above 10  
Star: between 5 and 10  
Dot: between 2 and 5  
Otherwise: below 2

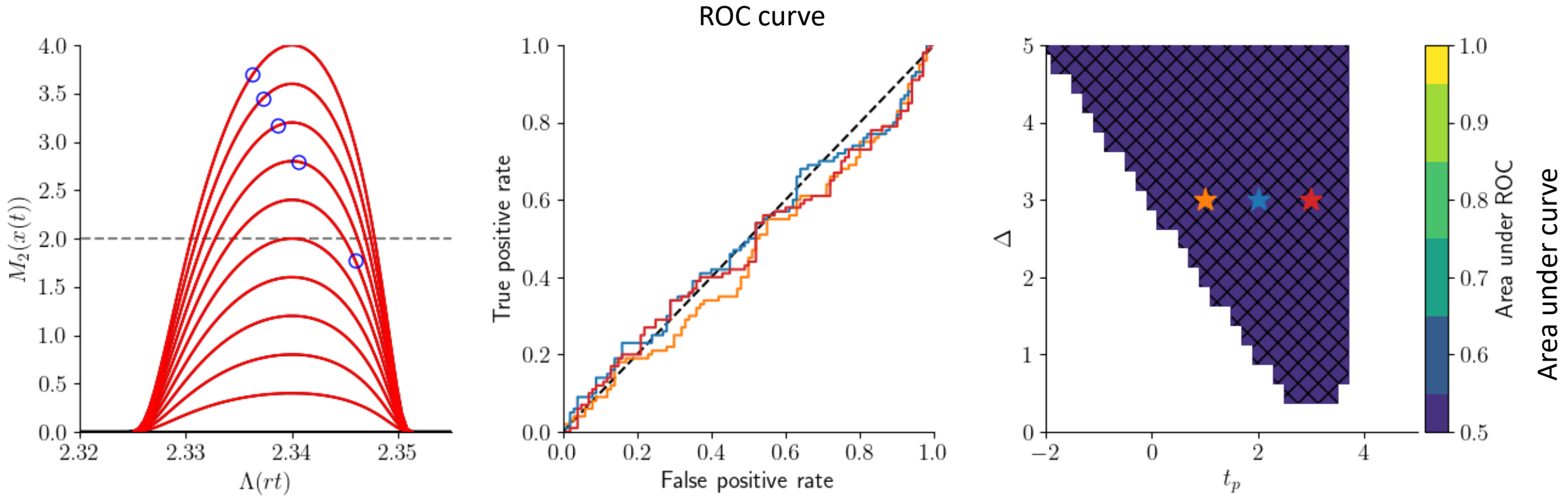
# EWS for downstream system (linear coupling)



Test ensemble consists of different linear coupling strength, so about 50% show cascade

Hatching relates to optimal threshold:  
Cross: above 10  
Star: between 5 and 10  
Dot: between 2 and 5  
Otherwise: below 2

# EWS for downstream system (local coupling)



Test ensemble consists of different localised coupling strength, so about 50% show cascade during tipping of the upstream system

Hatching relates to optimal threshold:  
Cross: above 10  
Star: between 5 and 10  
Dot: between 2 and 5  
Otherwise: below 2

The background is a complex, blue-toned digital cityscape or data environment. It features numerous floating rectangular panels, some with data-like patterns or text, and a grid of lines on the ground. A silhouette of a person stands in the center, looking towards the horizon. The overall aesthetic is high-tech and futuristic.

# Example 3

## Internal Feedbacks

# Time scale of feedback

resource

consumer

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + a - u - uv^2$$

$$\frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + uv^2(1 - bv) - mv$$



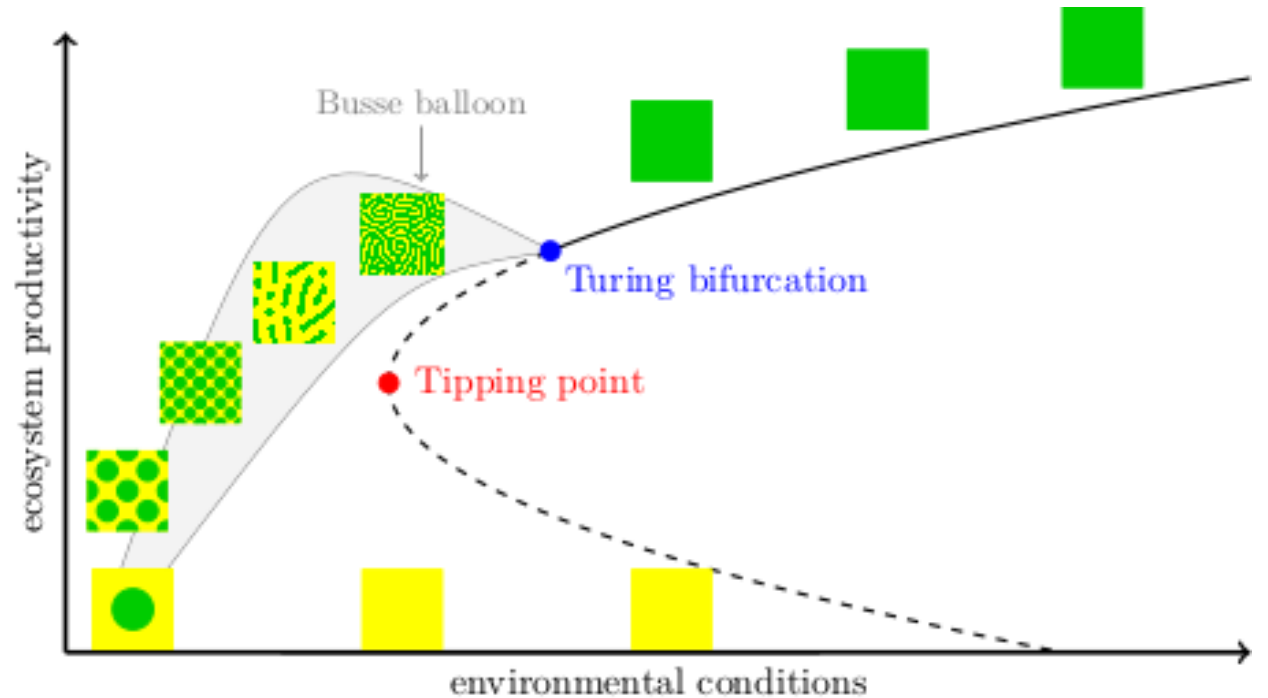
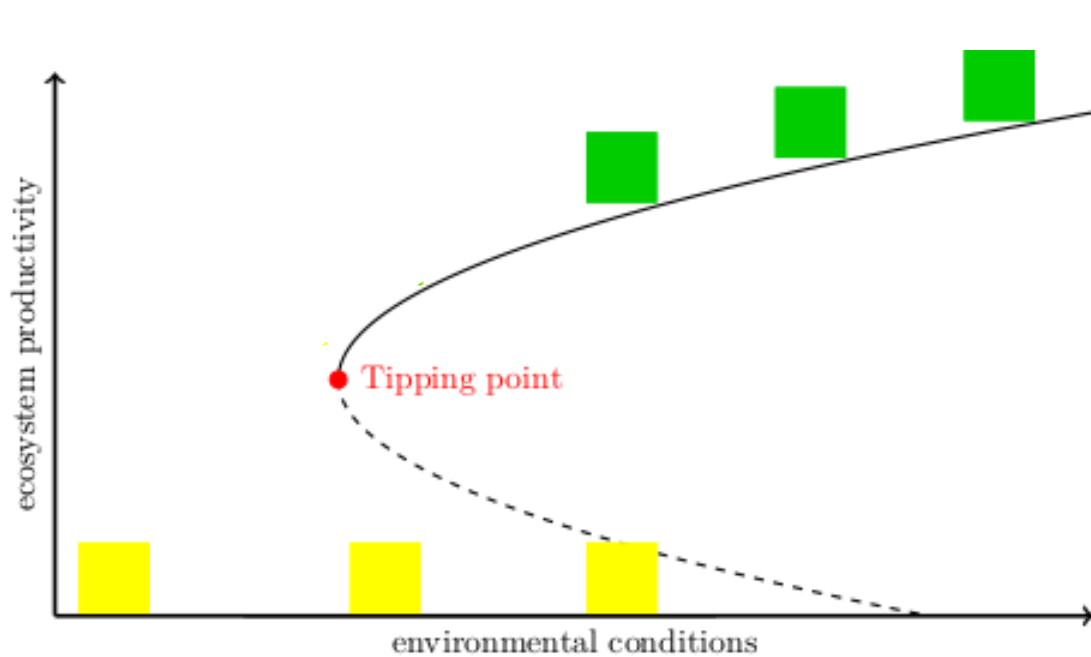
# Time scale of feedback

resource

consumer

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + a - u - uv^2$$

$$\frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + uv^2(1 - bv) - mv$$



# Time scale of feedback

resource

consumer

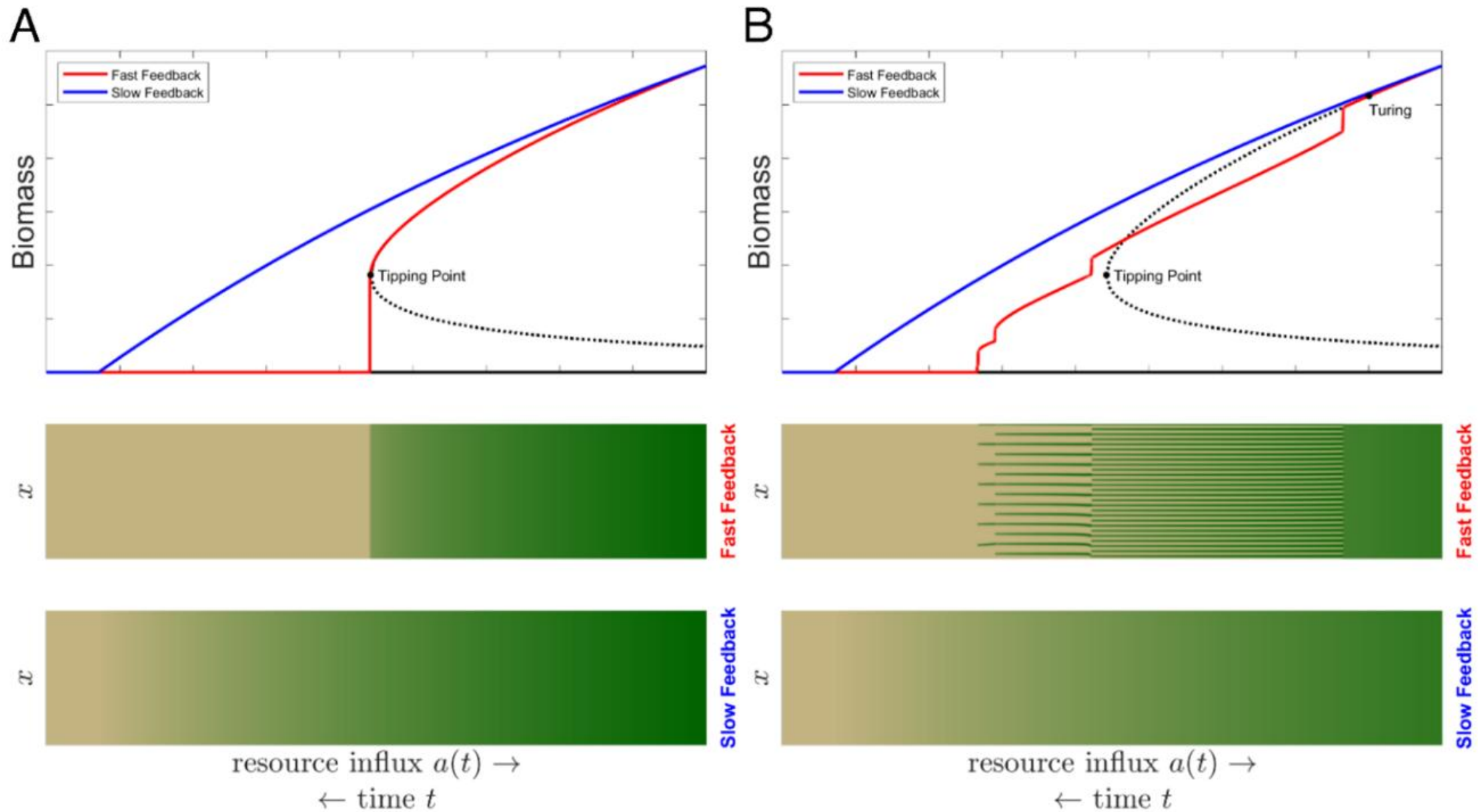
engineered construct

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + a - u - uv\mathbf{s}$$

$$\frac{\partial v}{\partial t} = D \frac{\partial^2 v}{\partial x^2} + uv\mathbf{s}(1 - bv) - mv$$

$$\frac{\partial \mathbf{s}}{\partial t} = (v - \mathbf{s})/\tau_s$$

# Time scale of feedback



# TIPPING DYNAMICS IN MULTISCALE SYSTEMS



ROBBIN BASTIAANSEN  
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DIEP seminar, 2026-05-28



# Tipping dynamics in multiscale systems

Time scales of internal processes and external forcing crucially affect tipping dynamics!

- LATE tipping can occur after long transients
- Early warning signs can fail in multiscale systems
- If external forcing is very rapid any critical behaviour might be absent!

## Primary papers discussed in this talk

Late tipping	Bastiaansen, R., Ashwin, P., & von der Heydt, A. S. (2023). Climate response and sensitivity: time scales and late tipping points. <i>Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences</i> , 479(2269).
Multiscale tipping in spatial systems	Bastiaansen, R., Doelman, A., Eppinga, M. B., & Rietkerk, M. (2020). The effect of climate change on the resilience of ecosystems with adaptive spatial pattern formation. <i>Ecology letters</i> , 23(3), 414-429.
Cascades [Early warning signs]	Ashwin, P., Bastiaansen, R., von der Heydt, A. S., & Ritchie, P. D. (2025). Early warning skill, extrapolation and tipping for accelerating cascades. <i>Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences</i> , 481(2321).
Cascades [Bifurcation analysis]	Ritchie, P. D., Bastiaansen, R., von der Heydt, A. S., & Ashwin, P. (2025). The role of coupling and timescales for interacting tipping elements. <i>arXiv preprint arXiv:2509.03996</i> .
Internal feedbacks	Mak, R., Van der Kaaden, A.S., Bastiaansen, R., Critical transitions and pattern formation might be irrelevant when external stressors outpace internal feedback processes. <i>submitted</i>

slides available at [bastiaansen.github.io](https://bastiaansen.github.io)



