

An aerial photograph of a river delta system. A central, dark brown river channel flows from the top center towards the bottom. It branches out into several smaller channels that surround a large, roughly diamond-shaped island in the center. The surrounding land is a mix of green and brown, indicating different vegetation or soil types. The overall pattern is highly irregular and fractal-like, characteristic of natural river networks.

The dynamics of spatial patterns in nature

2026-05-19, Deep Patterns workshop at Eurandom
Robbin Bastiaansen (r.bastiaansen@uu.nl)

Examples of spatial patterning – regular patterns



mussel beds



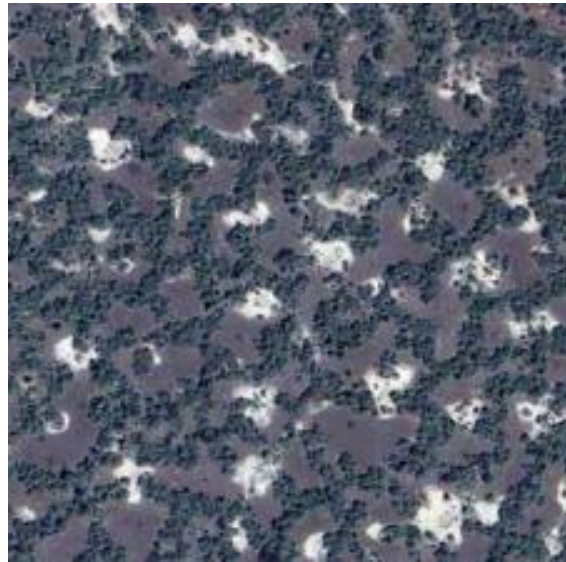
vegetation in coastal systems



marsh formation



melt ponds



savannas

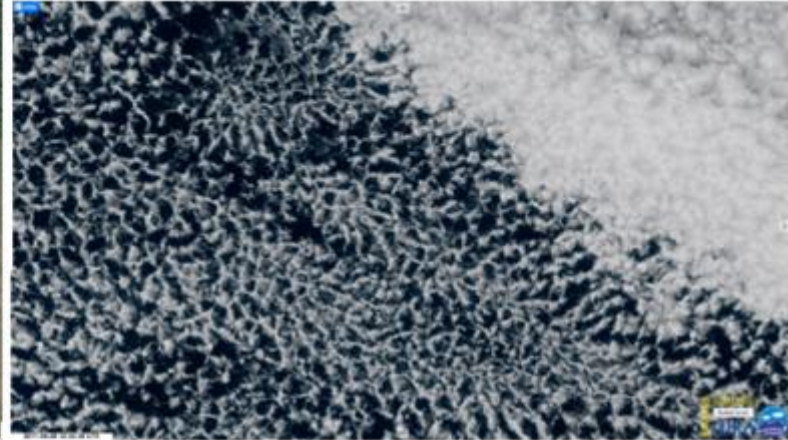


drylands

Examples of spatial patterning – spatial interfaces

tropical forest
& savanna
ecosystems

[Google Earth]



types of
stratocumulus
clouds

[RAMMB/CIRA SLIDER]

sea-ice & water
at Eltanin Bay

[NASA's Earth observatory]



algae bloom
in Lake St. Clair

[NASA's Earth observatory]

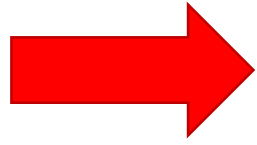
Examples of spatial patterning – mussels



Examples of spatial patterning – planetary rings



Why are spatial patterns important?



They affect system dynamics!

Example 1 – melt ponds



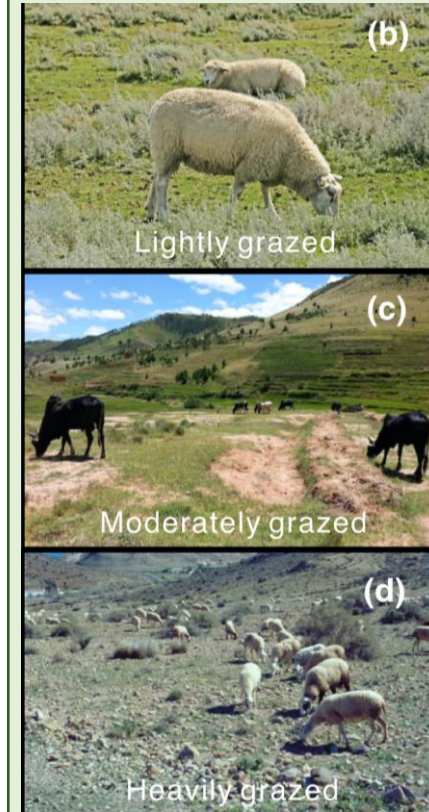
*higher temperatures → melt ponds
→ lower albedo → more sunlight
absorbed → higher temperatures*

Example 2 – clouds



*Many different types, all have
different effects on incoming and
outgoing radiation affecting e.g.
surface temperatures on Earth.
Much not properly understood*

Example 3 – grazing



*Grazers prefer
to eat younger
and shorter
vegetation.
Hence, patches
of older
vegetation
persist that can
grow further*

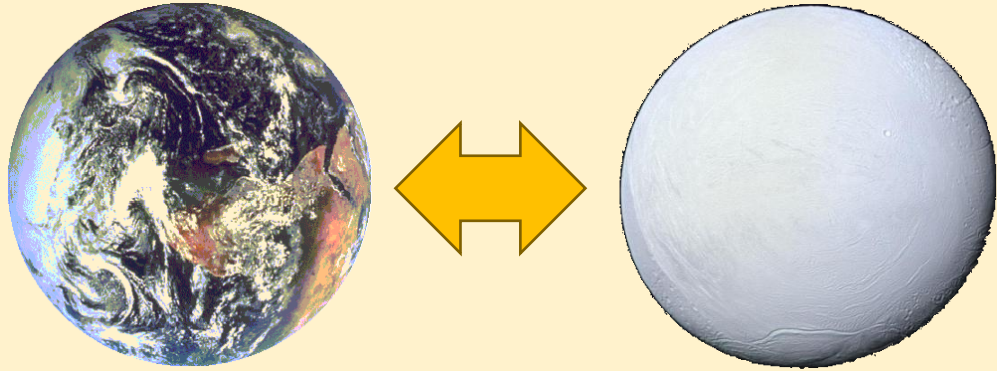
Many open questions here, often even quantifying these effects

An aerial photograph showing a fire front moving through a field of low-lying vegetation. The fire is visible as a bright orange and yellow line separating the unburned green and yellowish vegetation from a large, dark, charred area in the foreground. The text "Transitions in spatially patterned systems" is overlaid in white on the dark charred area.

Transitions in spatially patterned systems

Transitions and Tipping Points

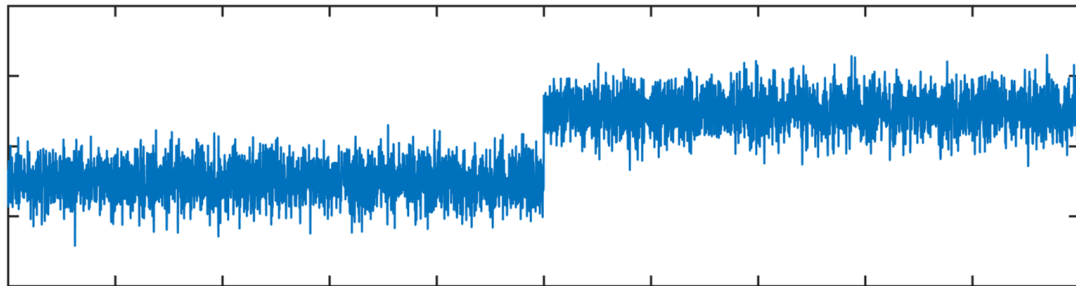
IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”



Planetary transitions

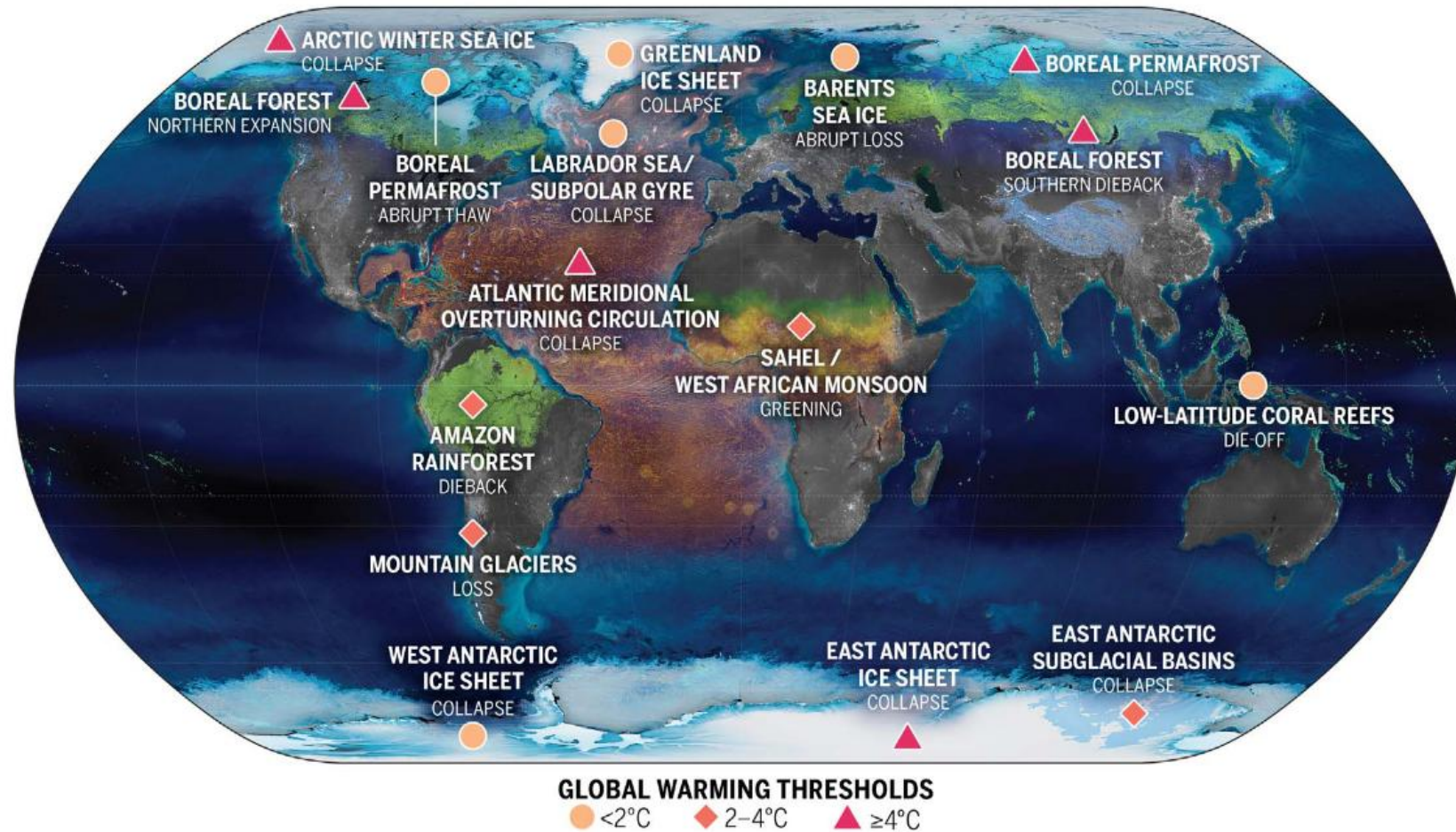


Ecosystem shifts



Tipping Points

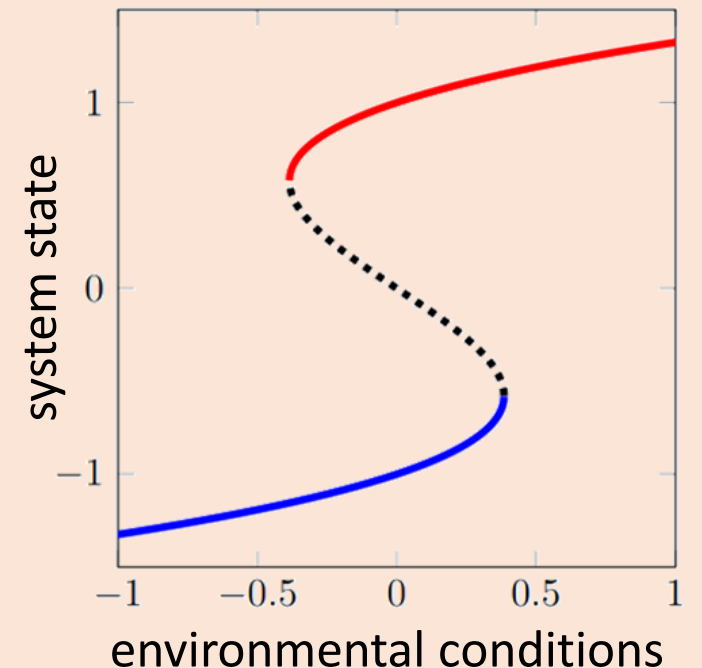
IPCC AR6 (2021) : “a critical threshold beyond which a system reorganizes, often abruptly and/or irreversibly”



Mathematics

Tipping points \leftrightarrow Bifurcations

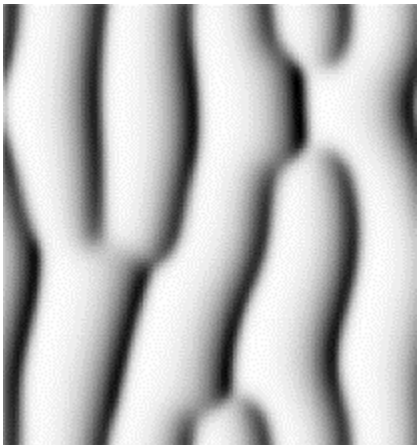
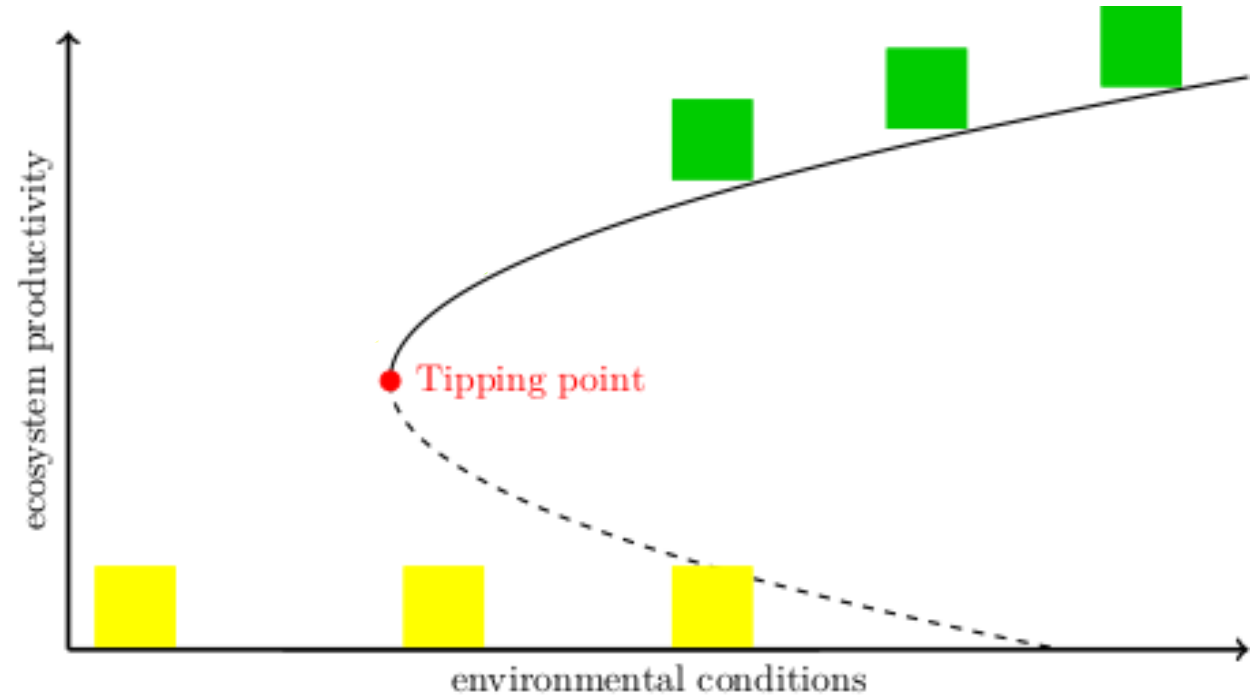
$$\frac{dy}{dt} = f(y, \mu)$$



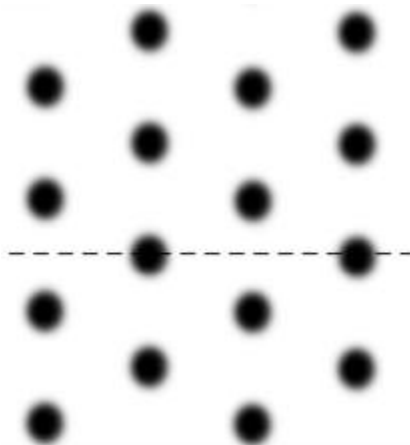
Patterns in models

Often modelled using
Reaction-Diffusion equations:

$$\begin{cases} \frac{du}{dt} = f(u, v) + D_u \Delta u \\ \frac{dv}{dt} = g(u, v) + D_v \Delta v \end{cases}$$



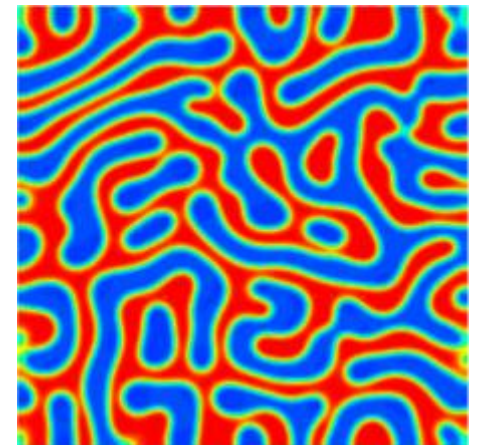
[Klausmeier, 1999]



[Gilad et al, 2004]



[Rietkerk et al, 2002]



[Liu et al, 2013]

Turing patterns

$$\begin{cases} \frac{du}{dt} = f(u, v) + D_u \Delta u \\ \frac{dv}{dt} = g(u, v) + D_v \Delta v \end{cases}$$

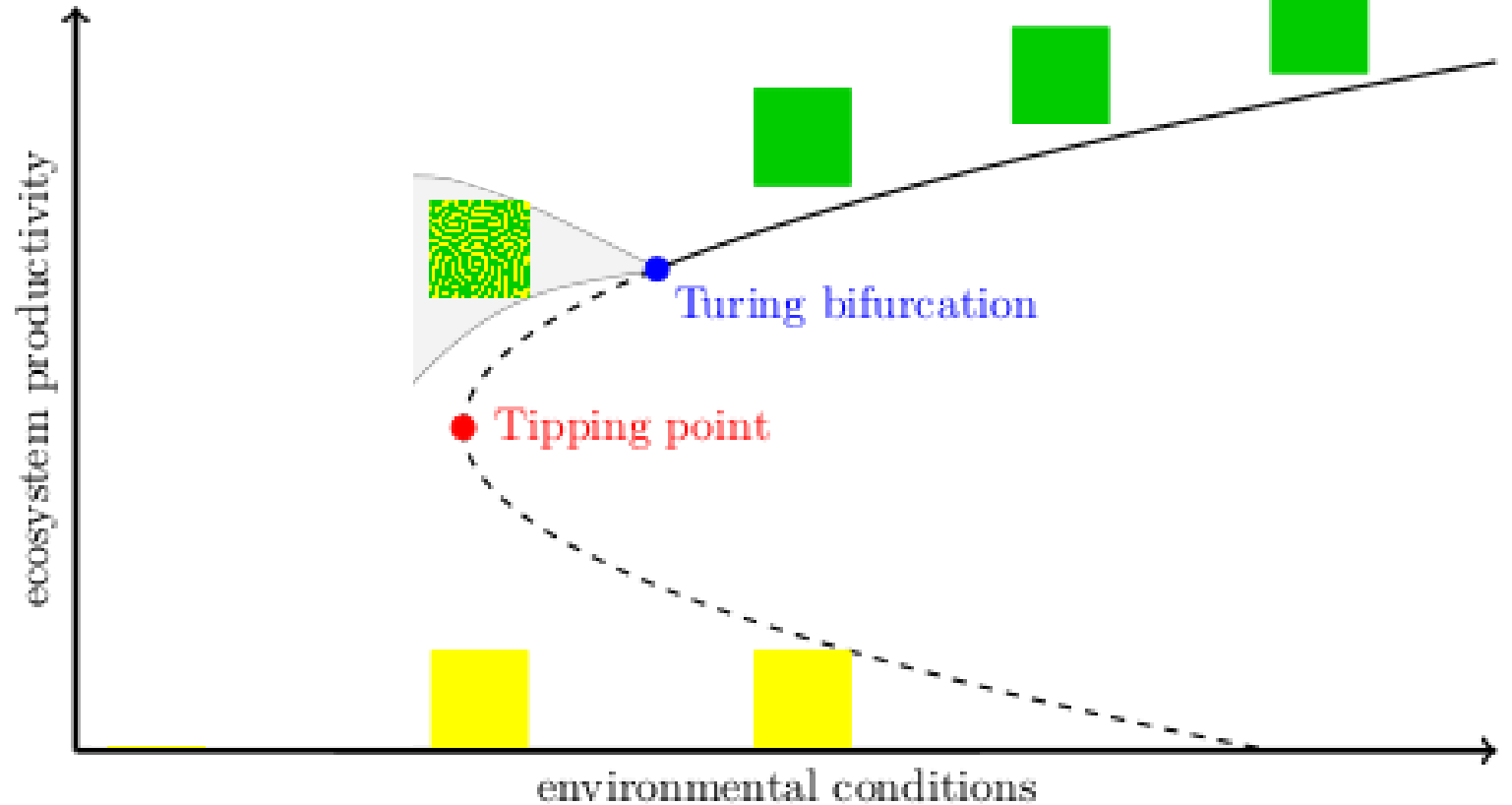
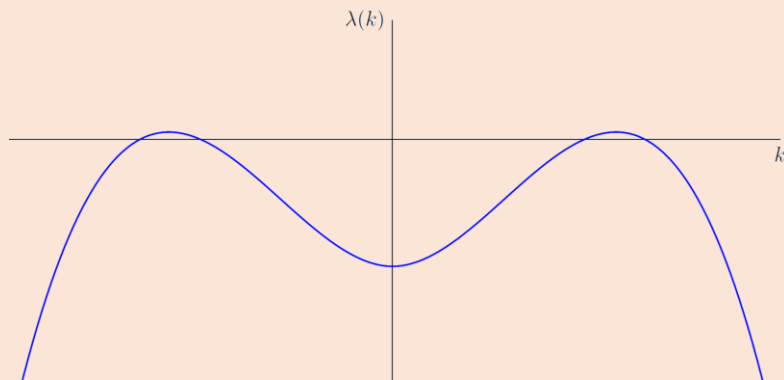
Turing bifurcation

Instability to non-uniform perturbations

$$\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} u_* \\ v_* \end{pmatrix} + e^{\lambda t} e^{ikx} \begin{pmatrix} \bar{u} \\ \bar{v} \end{pmatrix}$$

→ Dispersion relation

$$\lambda(k) = \dots$$



Weakly non-linear analysis

Ginzburg-Landau equation / Amplitude Equation
& Eckhaus/Benjamin-Feir-Newell criterion

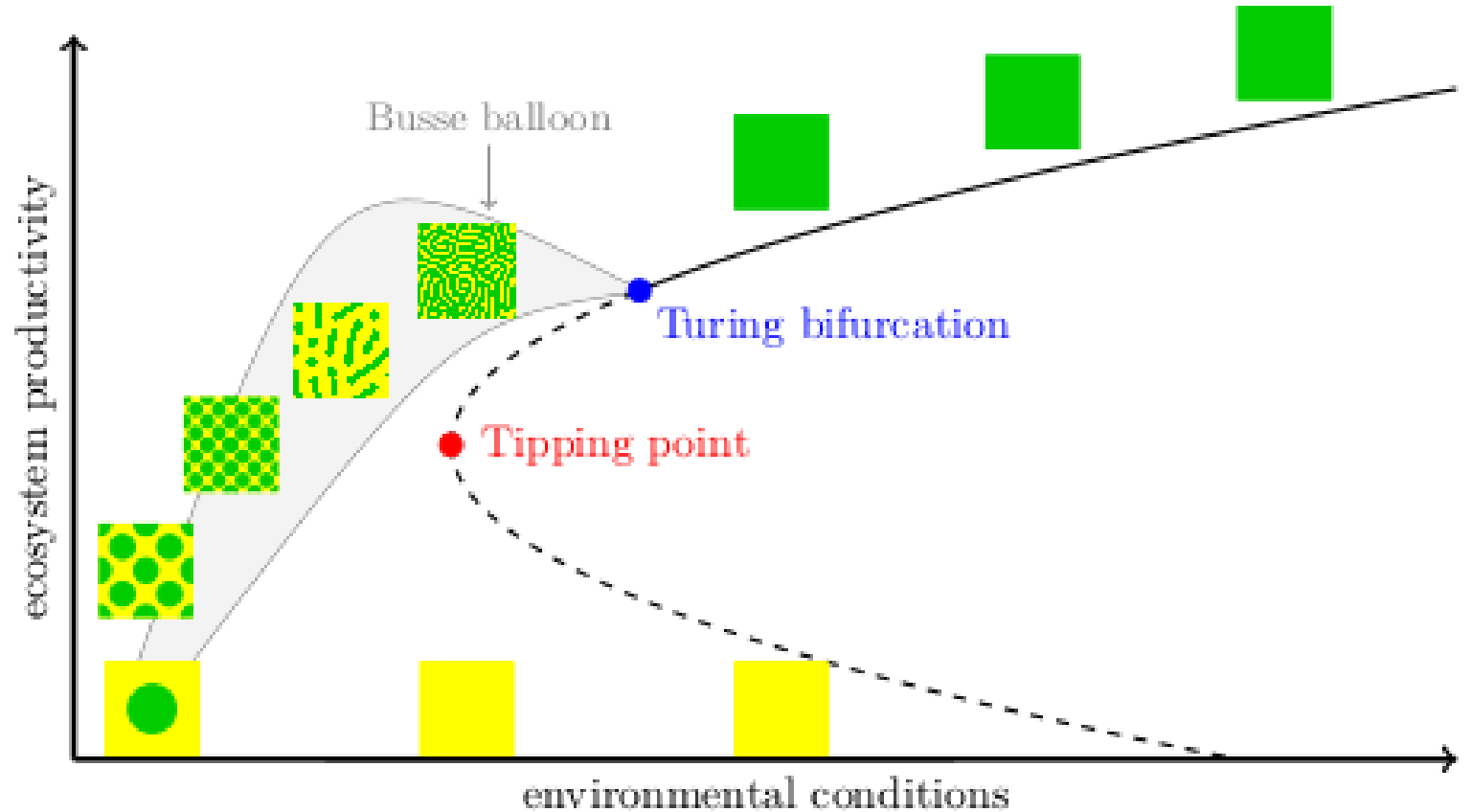
[Eckhaus, 1965; Benjamin & Feir, 1967; Newell, 1974]

Busse balloon

$$\begin{cases} \frac{du}{dt} = f(u, v) + D_u \Delta u \\ \frac{dv}{dt} = g(u, v) + D_v \Delta v \end{cases}$$

Busse balloon

A model-dependent shape in *(parameter, observable)* space that indicates all stable patterned solutions to the PDE.



Construction Busse balloon

Via numerical continuation

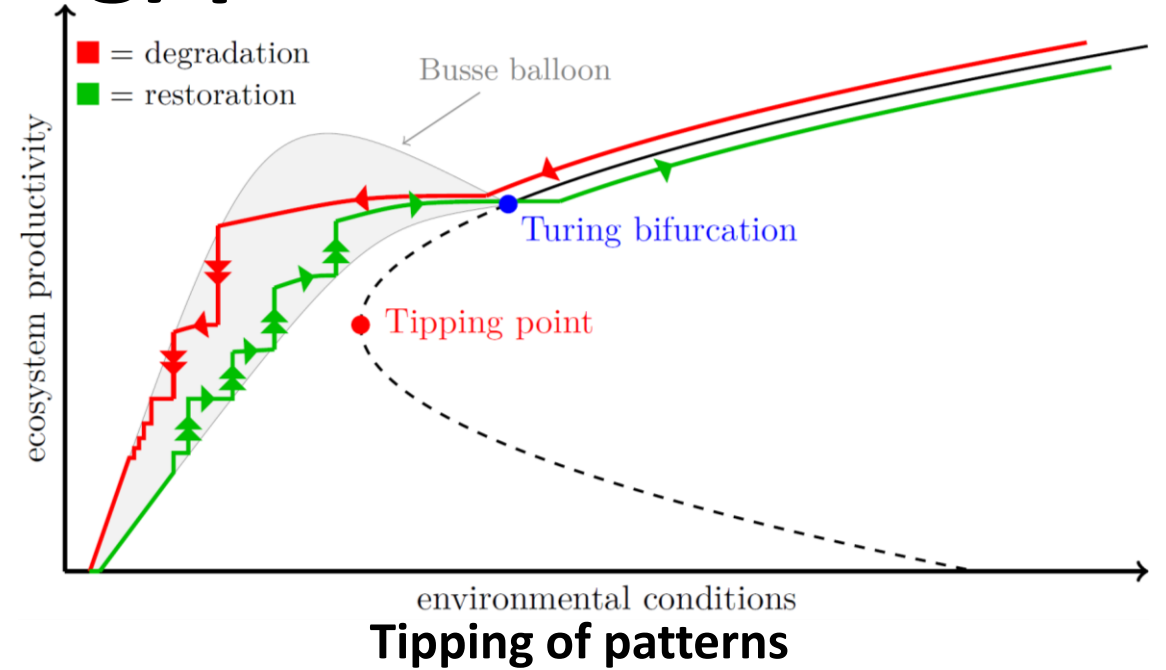
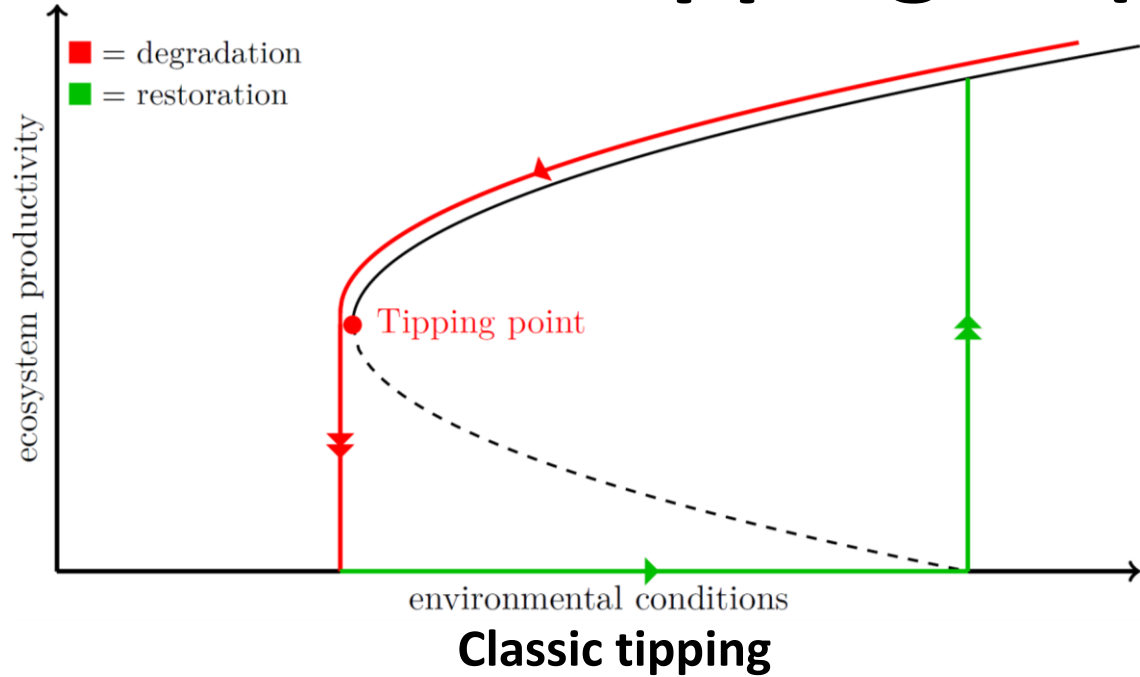
few general results on the shape of Busse balloon

Busse balloon

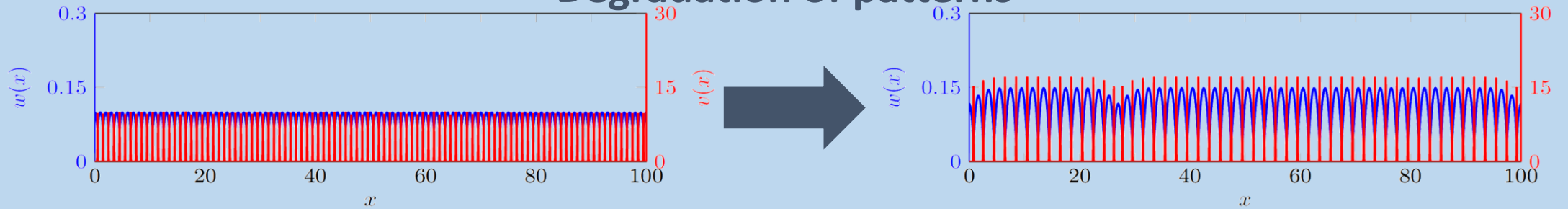
Idea originates from thermal convection

[Busse, 1978]

Tipping of (Turing) patterns



Degradation of patterns



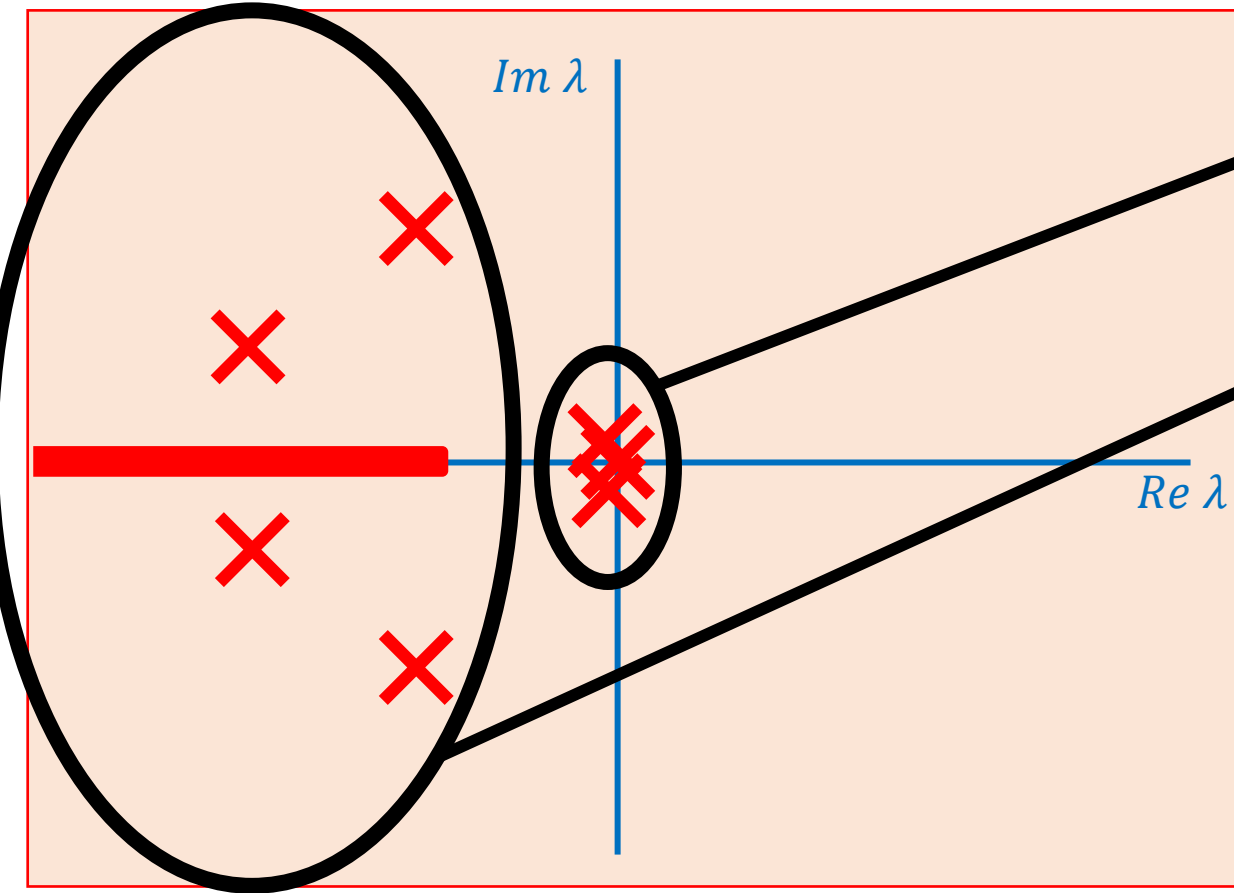
Open questions:

- transitions when and to what state?
- transient during transition?



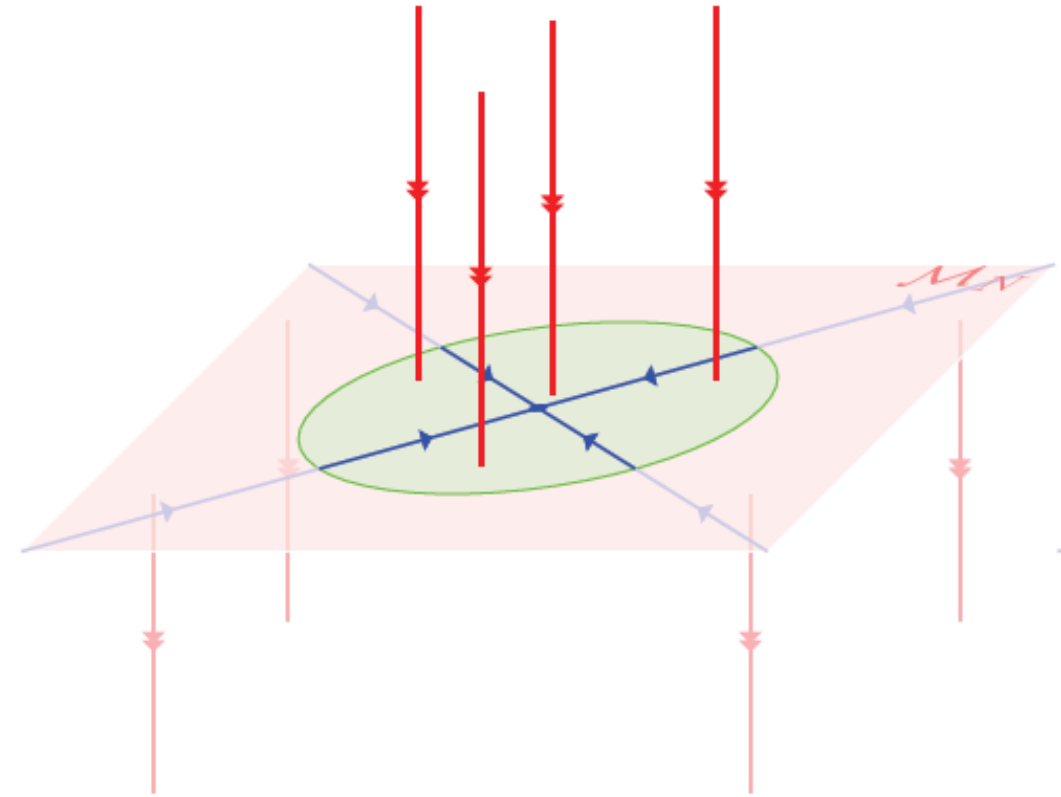
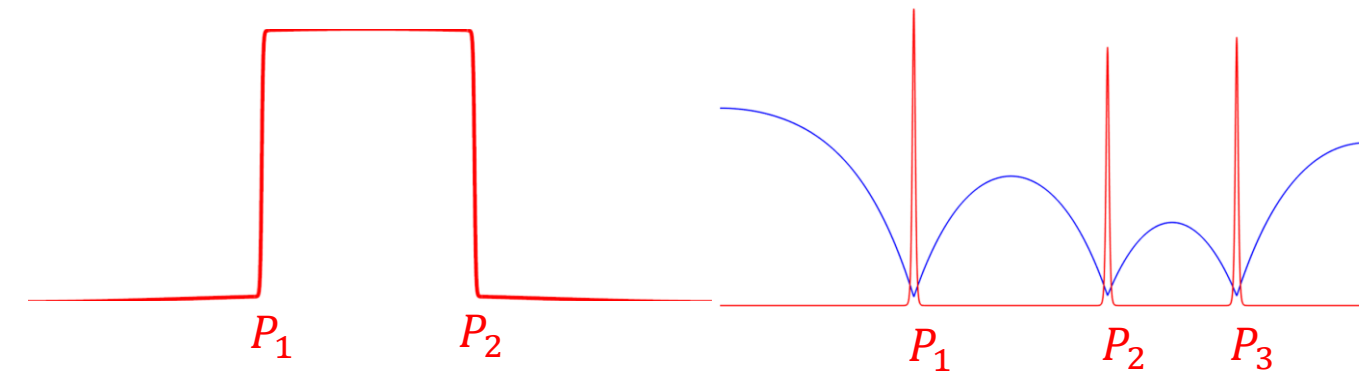
Dynamics & Bifurcations of Patterned States

Dynamics of Patterned States



1. SLOW Pattern Adaptation

2. FAST Pattern Degradation



1. SLOW pattern adaptation

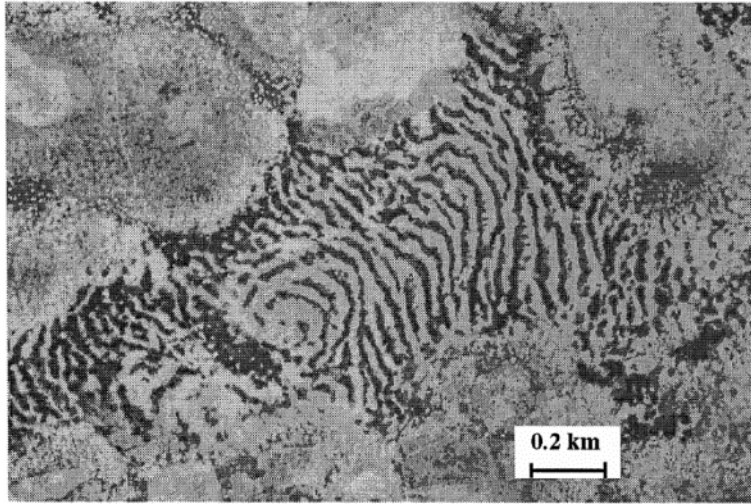


Somaliland, 1948 [Macfadyen, 1950]



Somaliland, 2008

2. FAST Pattern Degradation



Niger, 1950 [Valentin, 1999]



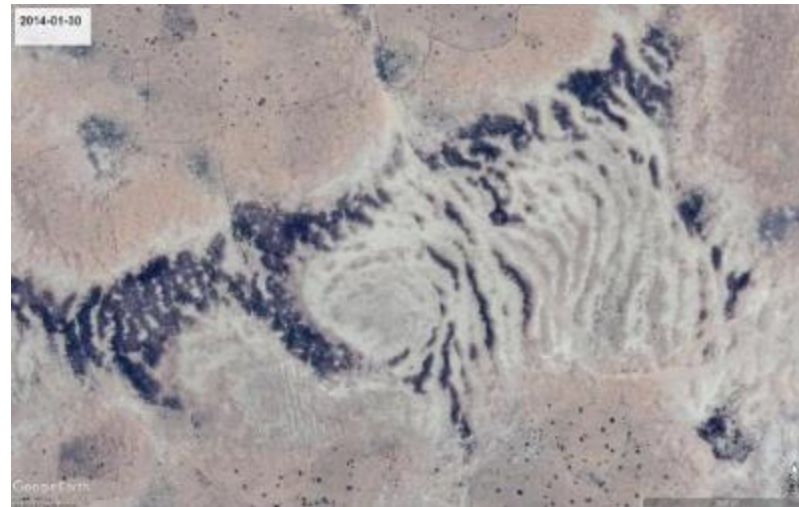
Niger, 2008



Niger, 2010



Niger, 2011



Niger, 2014

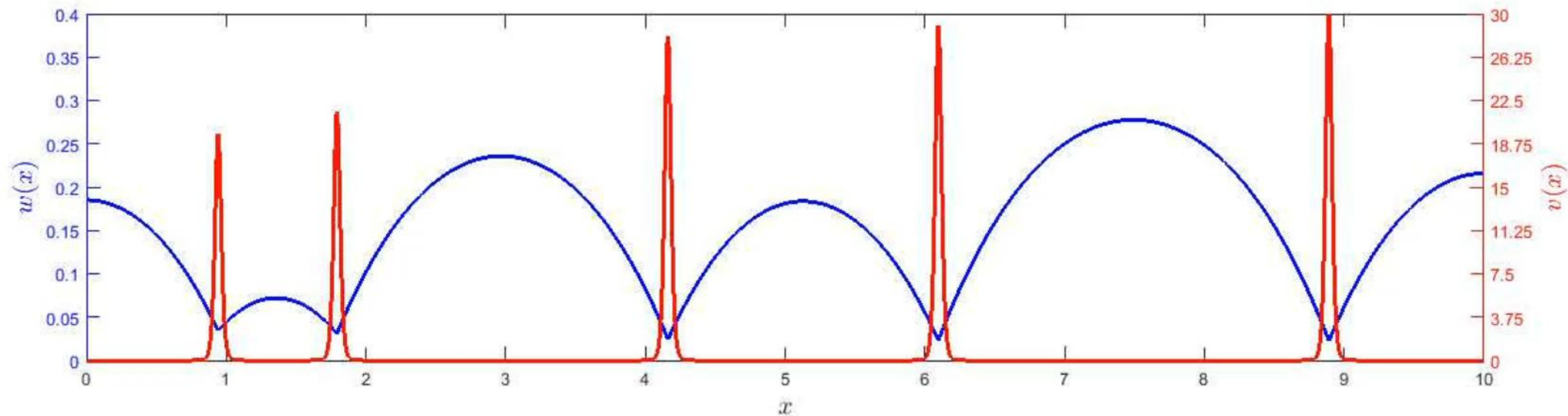


Niger, 2016

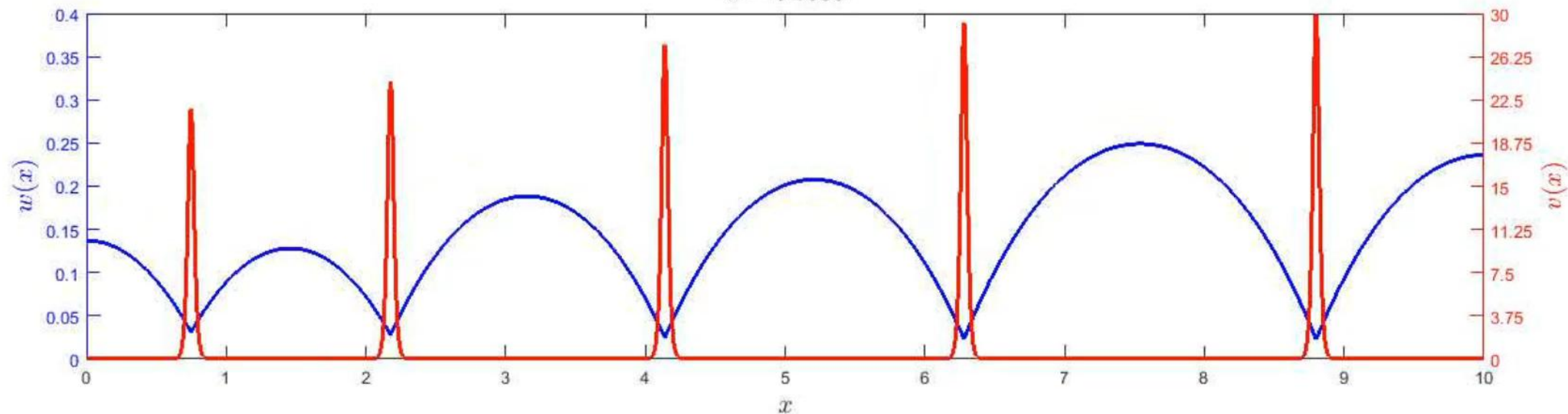
Vegetation patches under climate change

Rate of climate change

FAST



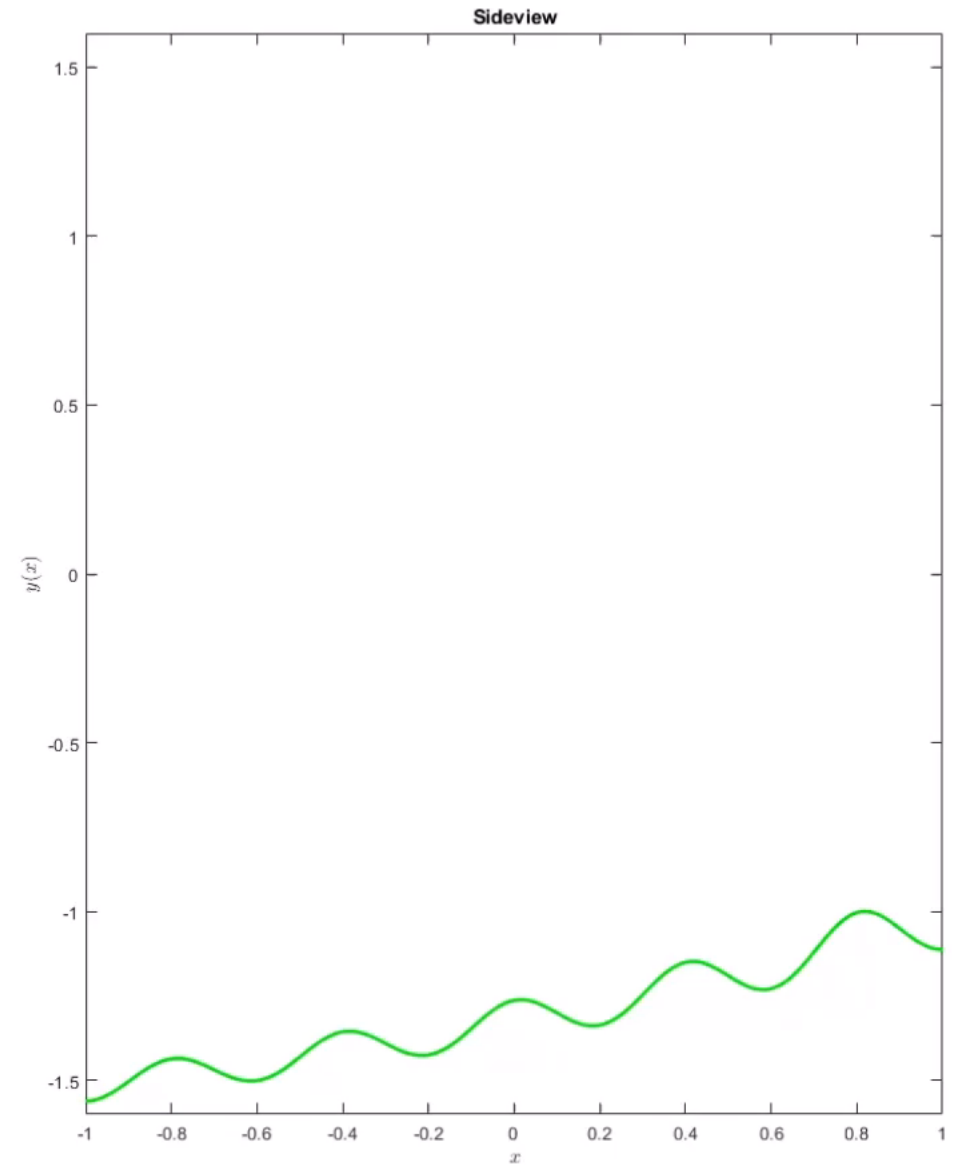
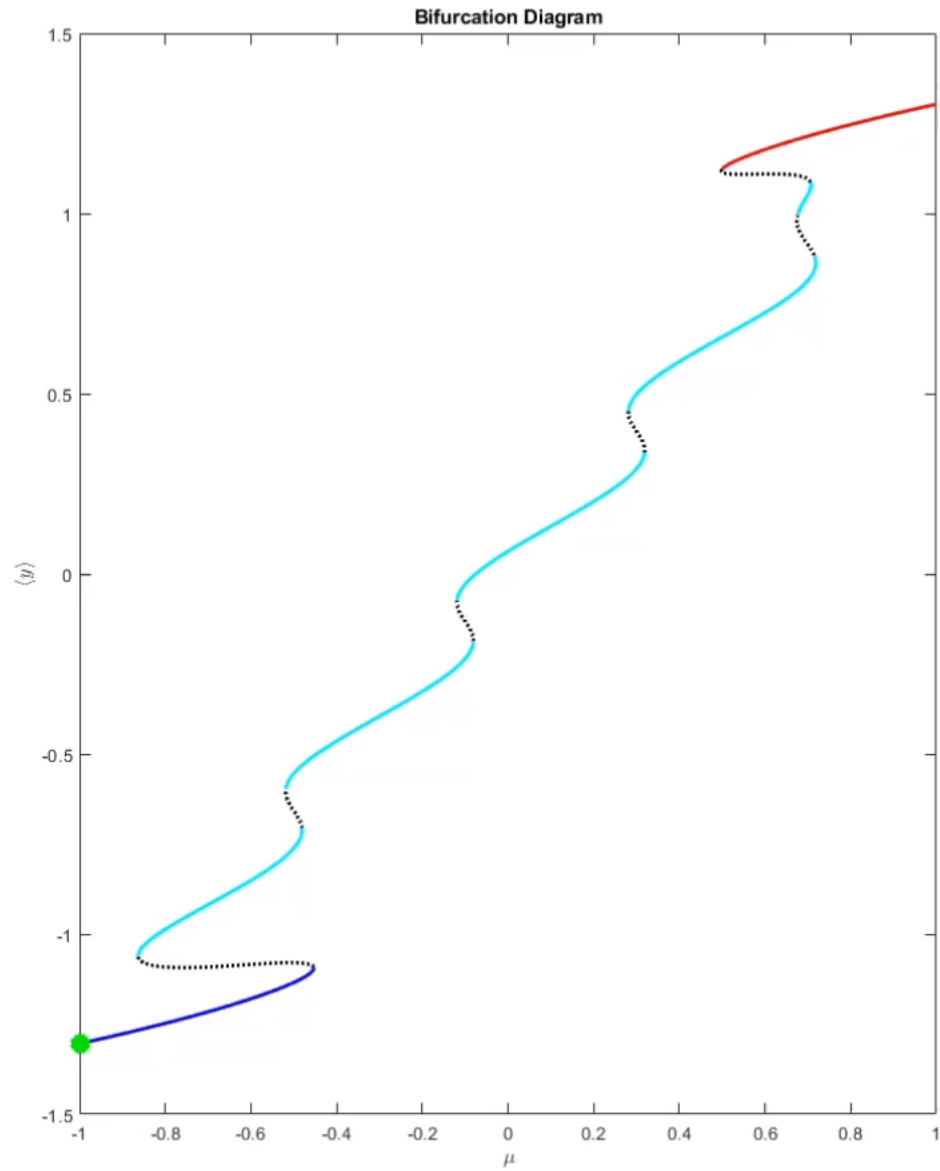
SLOW



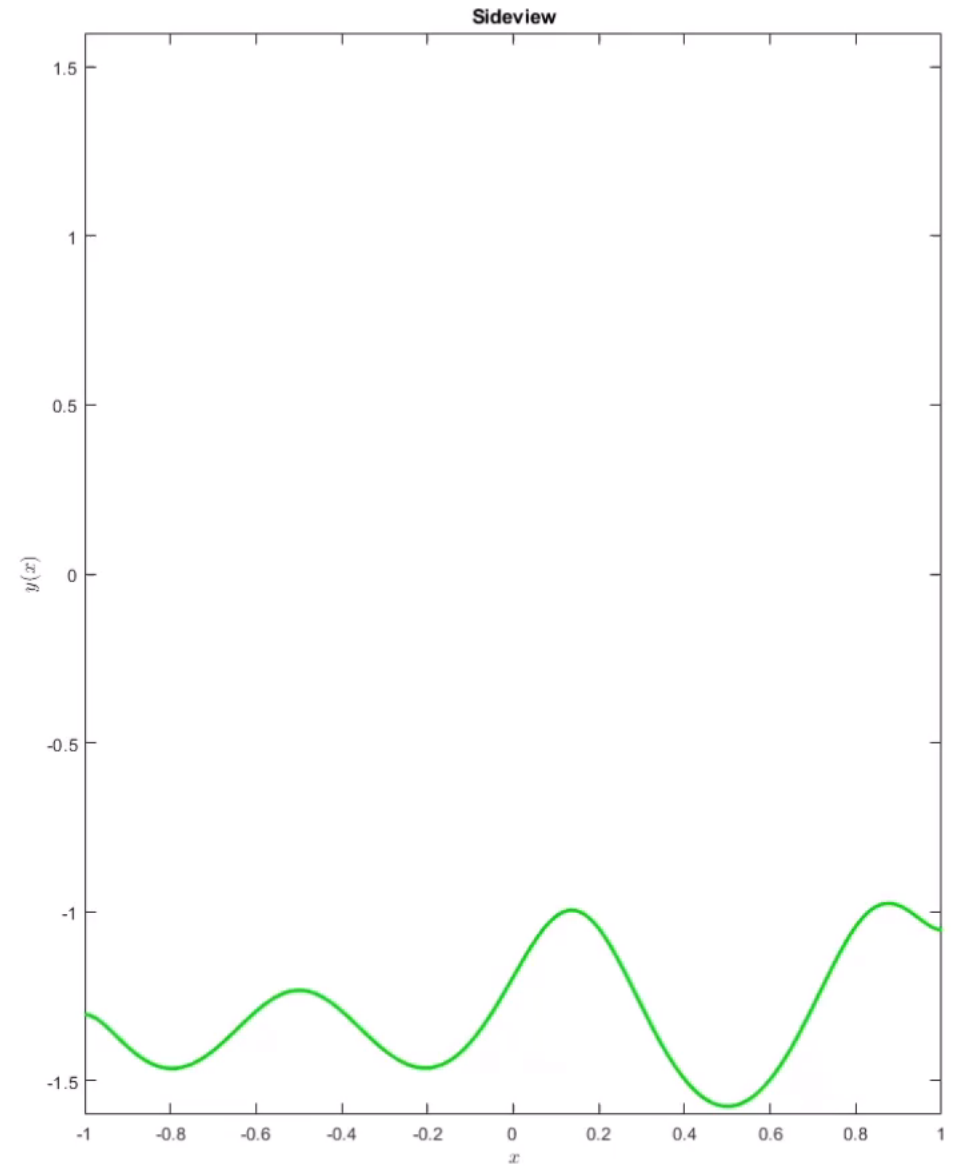
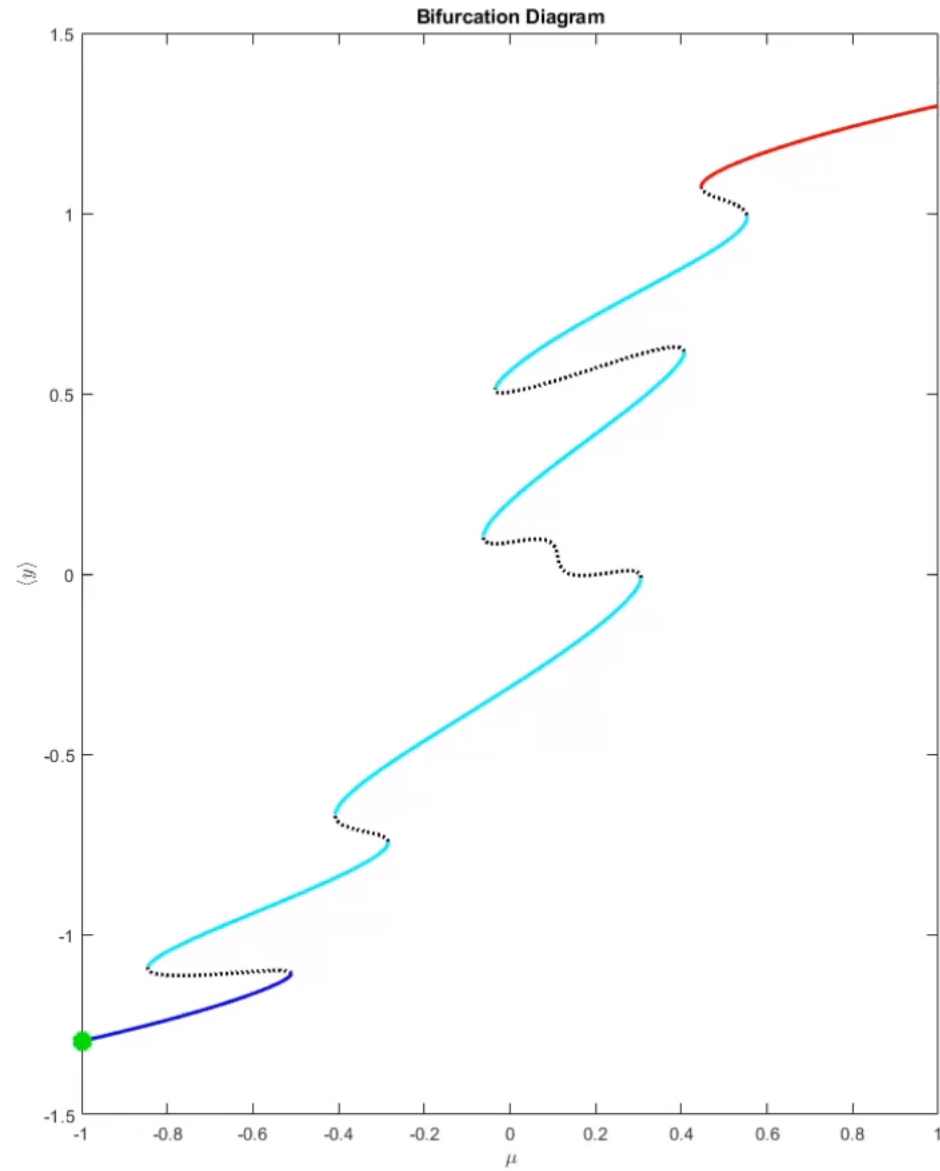


Some open questions

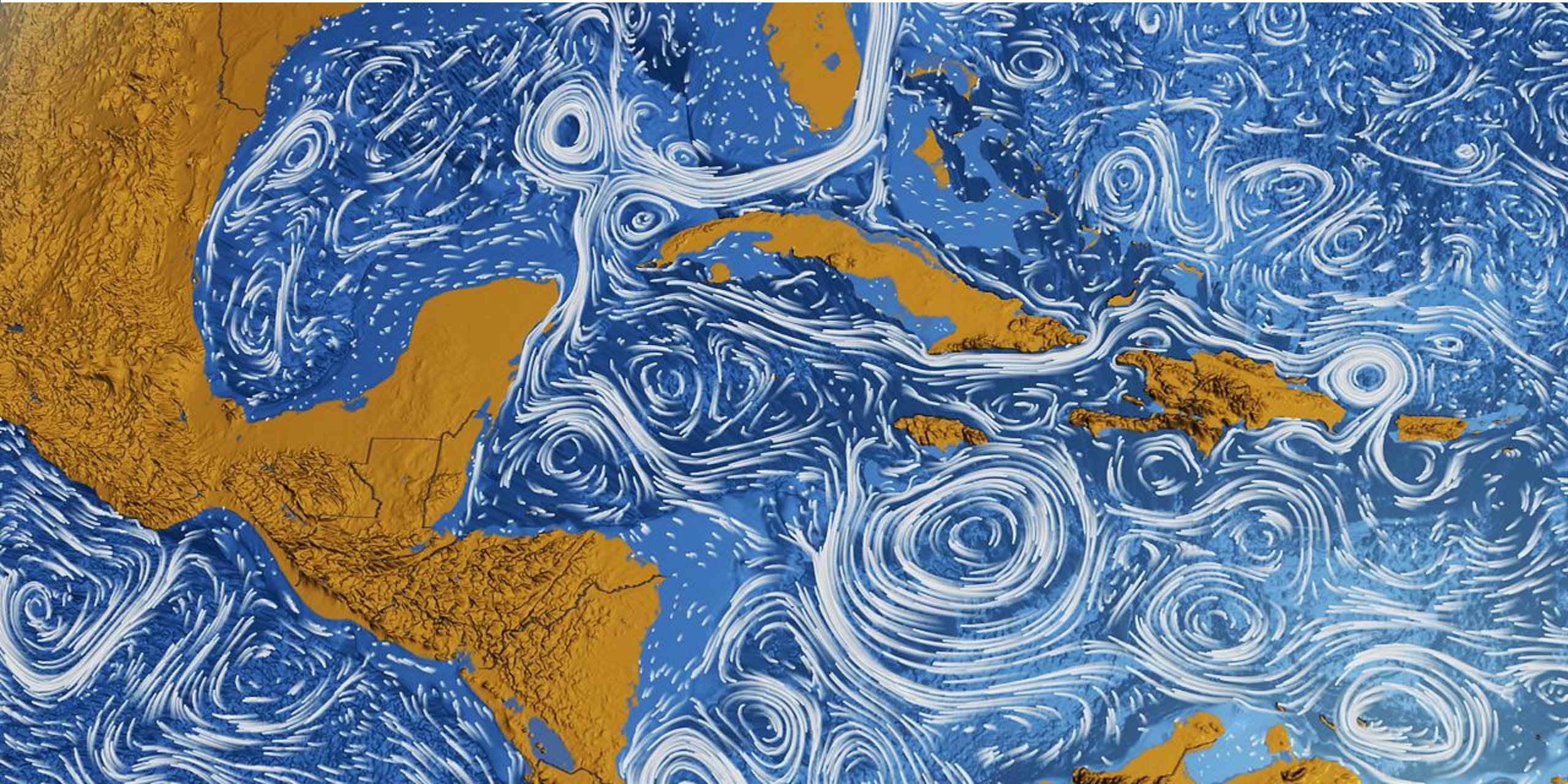
Open question: early warning for transition



Open question: early warning for transition



Open question: beyond reaction-diffusion systems?



Open question: pre-existing heterogeneity



Open question: pre-existing heterogeneity



Image © 2024 Maxar Technologies

Google Earth

1°19'47.14" S 13°52'48.66" E elev 594 m eye alt 1.73 km

1985

Data availability

SATELLITE DATA:

Up to **50 years**

SPATIAL RESOLUTION: **meters to kilometers**

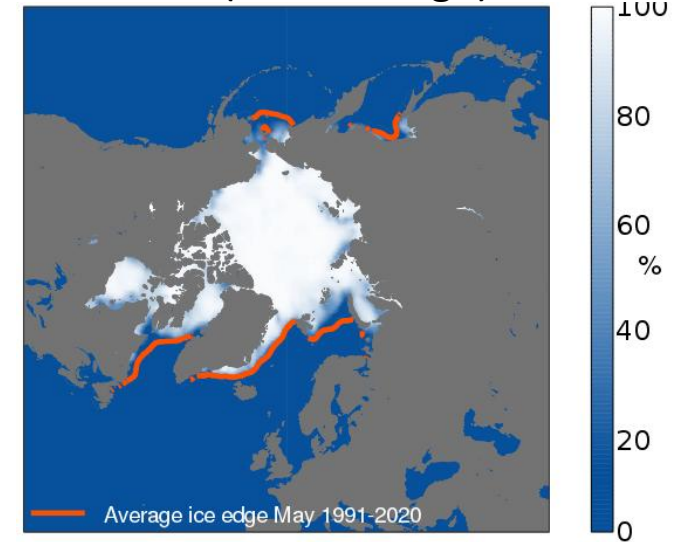
TEMPORAL RESOLUTION: **hours to days**

Site-specific data sometimes is available
(e.g. species inventory, drones)

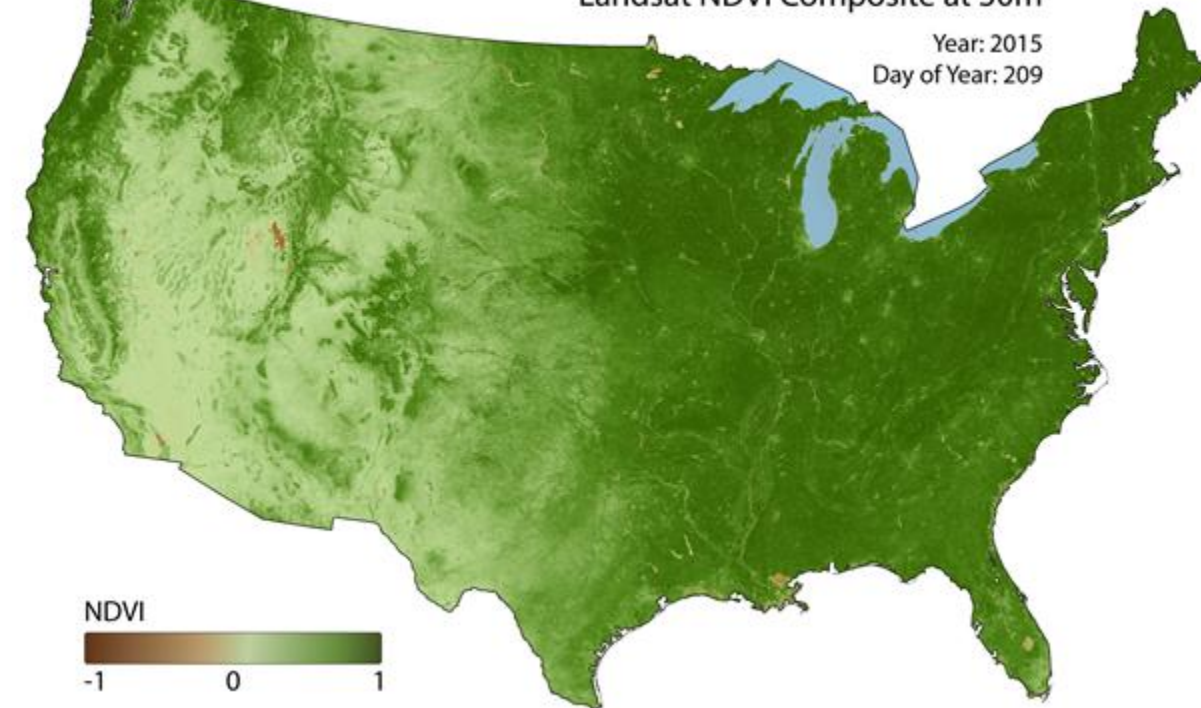
*Typically infrequently done
sometimes for up to decades*

However: transitions are infrequent!

ERA5 (ice coverage)



Landsat NDVI Composite at 30m



The dynamics of spatial patterns in nature

Spatial patterns affect dynamics of natural systems!
Often unclear how precisely

Dynamics of spatial patterns:
SLOW pattern adaption
FAST pattern degradation

Open questions regarding transitions include:

- Prediction of transitions (timing, kind etc.)
 - How do patterns transition?
- How to deal with pre-existing heterogeneity?
 - How in more complex systems

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